



# On Common Fixed-Point Results for Expansive Type Mappings in Cone Rectangular Metric Spaces

A. S. Saluja<sup>1</sup>, Asmita Yadav<sup>2</sup>

<sup>1,2</sup>Department of Mathematics, Institute for Excellence in Higher Education (IEHE), Bhopal-462016, Madhya Pradesh, India.

ARTICLE INFO	ABSTRACT
<p><b>Published Online:</b> 05 July 2025</p> <p><b>Corresponding Author:</b> A. S. Saluja</p>	<p>In this paper, we establish common fixed-point theorems for a class of self-mappings satisfying expansive-type conditions within the framework of cone rectangular metric spaces. These generalized metric spaces incorporate a cone structure in a real Banach space, allowing for a partial ordering and broader applicability beyond traditional metric settings. We provide sufficient conditions under which such mappings admit a unique common fixed point. The presented results extend and refine several known fixed-point theorems. Additionally, examples are included to demonstrate the applicability and effectiveness of the theoretical findings in generalized metric and nonlinear analysis contexts.</p> <p><b>MATHEMATICS SUBJECT CLASSIFICATION (2020):</b> Primary: 54H25; Secondary: 54E50, 47H10.</p>
<p><b>KEYWORDS:</b> Normal cone, cone rectangular metric spaces, coincidence point, common fixed point, expansive type mappings.</p>	

## 1. INTRODUCTION

Fixed point theorems concern maps  $f$  of a set  $X$  into itself that, under certain conditions, admit a fixed point, that is, a point  $x \in X$  such that  $f(x) = x$ . The knowledge of the existence of fixed points has relevant applications in many branches of analysis and topology. By a contraction on a metric space  $(X, d)$ , we understand a mapping  $T : X \rightarrow X$  satisfying for all  $x, y \in X : d(Tx, Ty) \leq kd(x, y)$ , where  $k$  is a real in  $[0, 1)$ .

Branciari [4] introduced a class of generalized (rectangular) metric spaces by replacing the triangular inequality of metric spaces with a similar one which involves four or more points instead of three points. The author also improved Banach Contraction Principle in such spaces. Recently many authors Cho, S.H.[6], Chaira K, et.al. [5] proved the existence and uniqueness of a fixed point for different types of mappings.

Azam A [2], introduced the concept of cone rectangular metric space by replacing the triangular inequality in the definition of cone metric space with a rectangular inequality and proved the Banach contraction principle on these spaces.

## 2. PRELIMINARIES

**Definition 2.1.** [2] Let  $X$  be a non-empty set. The mapping  $d_E : X \times X$  said to be cone rectangle metric space if it satisfies:

- (1)  $\alpha < d(x, y)$ , for all  $x, y \in X$  with  $x \neq y$  and  $d(x, y) = \alpha$  iff  $x = y$ ;
- (2)  $d(x, y) = d(y, x)$ , for all  $x, y \in X$ ;
- (3)  $d_E(x, y) \leq d_E(x, w) + d_E(w, z) + d_E(z, y)$ ;

for all  $x, y \in X$  and for all distinct points  $u, v \in X - \{x, y\}$ .

**Definition 2.2.** [10] Let  $P$  be a subset of a real Banach space  $E$  and  $\alpha$  is the zero vector of  $E$ .  $P$  is said to be a cone in  $E$  if it satisfies the following properties:

- (i)  $P$  is non-empty, closed and  $P \neq \alpha$
- (ii)  $x, y \in P$  implies a  $x + by \in P$ , where  $a$  and  $b$  are positive real numbers;
- (iii) The intersection of  $P$  and  $-P$  is  $\{\alpha\}$

**Definition 2.3.** [10] A cone  $P$  is said to be a solid cone if an interior of  $P$  is a non-empty subset of  $E$ .

**Definition 2.4.** [10] A partial order relation  $\leq$  with respect to a solid cone  $P \subseteq E$  is defined as  $x \leq y$  if  $y - x \in P$ , for  $x, y \in E$ .

**Definition 2.5.** [10] A cone  $P$  is called a normal cone if there is a number  $k > 1$  such that for all  $x, y \in X$ ,  $\alpha \leq x \leq y$  implies that  $\|x\| \leq k\|y\|$ .

**Definition 2.6.** [2] Let  $(X, d_E)$  be a cone rectangular metric space and  $P$  be a solid cone in  $E$ : Then the sequence  $\{x_n\}$  is said to converge to  $x$  if  $d_E(x_n, x) \rightarrow 0$  as  $n \rightarrow \infty$

**Definition 2.7** [2] Let  $(X, d_E)$  be a cone rectangular metric space and  $P$  be a solid cone in  $E$ : Then the sequence  $\{x_n\}$  is said to be Cauchy if for all  $p > 0$  we have  $d_E(x_n, x_{n+p}) \rightarrow 0$  as  $n \rightarrow \infty$

Throughout this paper,  $P$  is not necessarily a normal cone in  $E$ ; the relation  $x \ll y$  stands for  $y, x$  belongs to an interior of  $P$  and  $R$  denotes the set of all Real numbers.

### 3. MAIN RESULT

**Theorem 3.1:** Let  $(X, d_E)$  be a cone rectangular metric space. If the mappings  $P$  and  $Q : X \rightarrow X$  satisfying the condition:

$$d_E(Px, Py) \geq \alpha_1 d_E(Qx, Qy) + \alpha_2 d_E(Px, Qx) + \alpha_3 d_E(Py, Qx) + \alpha_4 d_E(Py, Qy) \quad (3.1.1)$$

for all  $x, y \in X$ , where  $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in R$  such that  $\alpha_1 \geq 2$  and  $0 < \alpha_2, \alpha_3, \alpha_4 < \frac{1}{2}$ .

If  $Q(X) \subseteq P(X)$  and either of  $Q(X)$  or  $P(X)$  is a complete subspace of  $X$ , then  $Q$  and  $P$  have a unique coincidence point in  $X$ . Further, if  $Q$  and  $P$  are weakly compatible self-maps then they have a unique common fixed point in  $X$ .

**Proof:** let  $x_0$  be an arbitrary point of  $X$ . Since  $Q(X) \subseteq P(X)$  and let  $x_1 \in X$  be such that  $Qx_0 = Px_1$ . Continuing this process, we can construct a sequence  $\{\beta_n\}$  in  $X$  such that  $\beta_n = Px_n = Qx_{n-1}$ , for all  $n \geq 1$ .

$$\text{If } \beta_{n-1} = \beta_n, \text{ for some } n \geq 1 \text{ then } \beta_{n-1} = Px_{n-1} = Qx_{n-1}$$

That is  $P$  and  $Q$  have a coincidence point  $x_{n-1}$  in  $X$ . Assume  $\beta_{n-1} \neq \beta_n$  for all  $n \geq 1$ . Then from (3.1.1)

$$\begin{aligned} d_E(\beta_{n-1}, \beta_n) &= d_E(Px_{n-1}, Px_n) \geq \alpha_1 d_E(Qx_{n-1}, Qx_n) + \alpha_2 d_E(Px_{n-1}, Qx_{n-1}) + \alpha_3 d_E(Px_n, Qx_{n-1}) + \alpha_4 d_E(Px_n, Qx_n) \\ &= \alpha_1 d_E(\beta_n, \beta_{n+1}) + \alpha_2 d_E(\beta_{n-1}, \beta_n) + \alpha_3 d_E(\beta_n, \beta_n) + \alpha_4 d_E(\beta_n, \beta_{n+1}) \\ &\geq (\alpha_1 + \alpha_4) d_E(\beta_n, \beta_{n+1}) + \alpha_2 d_E(\beta_{n-1}, \beta_n) \end{aligned}$$

Which implies that

$$(1 - \alpha_2) d_E(\beta_{n-1}, \beta_n) \geq (\alpha_1 + \alpha_4) d_E(\beta_n, \beta_{n+1})$$

$$\text{i.e. } (\alpha_1 + \alpha_4) d_E(\beta_n, \beta_{n+1}) \leq (1 - \alpha_2) d_E(\beta_{n-1}, \beta_n)$$

$$\text{Or, } d_E(\beta_n, \beta_{n+1}) \leq \frac{1 - \alpha_2}{\alpha_1 + \alpha_4} d_E(\beta_{n-1}, \beta_n)$$

Hence,

$$d_E(\beta_n, \beta_{n+1}) \leq \alpha d_E(\beta_{n-1}, \beta_n), \forall n \geq 1$$

$$\text{Where, } \alpha = \frac{1 - \alpha_2}{\alpha_1 + \alpha_4} < 1 \text{ (as } \alpha_1 + \alpha_2 + \alpha_4 > 1)$$

By induction for all  $n \geq 0$

$$d_E(\beta_n, \beta_{n+1}) \leq \alpha^n d_E(\beta_0, \beta_1) \quad (3.1.2)$$

Where  $0 < \alpha < 1$

Using (3.1.1) and (3.1.2) and the facts that,

$$\alpha_1 \geq 2, \alpha_2 < 1, 0 < \alpha_3 < 1 \text{ and } 0 < \alpha_4 < 1$$

That is  $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 > 1$  and  $0 < \alpha < 1$ ,

We have,

$$\begin{aligned} d_E(\beta_{n-1}, \beta_{n+1}) &= d_E(Px_{n-1}, Px_{n+1}) \\ &\geq \alpha_1 d_E(Qx_{n-1}, Qx_{n+1}) + \alpha_2 d_E(Px_{n-1}, Qx_{n-1}) + \alpha_3 d_E(Px_{n+1}, Qx_{n-1}) + \alpha_4 d_E(Px_{n+1}, Qx_{n+1}) \\ &\geq \alpha_1 d_E(\beta_n, \beta_{n+2}) + \alpha_2 d_E(\beta_{n-1}, \beta_n) + \alpha_3 d_E(\beta_{n+1}, \beta_n) + \alpha_4 d_E(\beta_{n+1}, \beta_{n+2}) \end{aligned}$$

Therefore,

$$\begin{aligned} \alpha_1 d_E(\beta_n, \beta_{n+2}) &\leq d_E(\beta_{n-1}, \beta_{n+1}) - \alpha_2 d_E(\beta_{n-1}, \beta_n) - \alpha_3 d_E(\beta_{n+1}, \beta_n) - \alpha_4 d_E(\beta_{n+1}, \beta_{n+2}) \\ &\leq [d_E(\beta_{n-1}, \beta_n) + d_E(\beta_n, \beta_{n+2}) + d_E(\beta_{n+2}, \beta_{n+1})] - \alpha_2 d_E(\beta_{n-1}, \beta_n) \\ &\quad - \alpha_3 d_E(\beta_{n+1}, \beta_n) - \alpha_4 d_E(\beta_{n+1}, \beta_{n+2}) \\ &\leq d_E(\beta_n, \beta_{n+2}) + (1 - \alpha_2) d_E(\beta_{n-1}, \beta_n) - \alpha_3 d_E(\beta_{n+1}, \beta_n) \\ &\quad + (1 - \alpha_4) d_E(\beta_{n+1}, \beta_{n+2}) \end{aligned}$$

Or,

$$\alpha_1 d_E(\beta_n, \beta_{n+2}) - d_E(\beta_n, \beta_{n+2}) \leq (1 - \alpha_2) d_E(\beta_{n-1}, \beta_n) - \alpha_3 d_E(\beta_{n+1}, \beta_n) + (1 - \alpha_4) d_E(\beta_{n+1}, \beta_{n+2})$$

$$(\alpha_1 - 1) d_E(\beta_n, \beta_{n+2}) \leq (1 - \alpha_2) d_E(\beta_{n-1}, \beta_n) - \alpha_3 d_E(\beta_{n+1}, \beta_n) + (1 - \alpha_4) d_E(\beta_{n+1}, \beta_{n+2})$$

Or,

$$d_E(\beta_n, \beta_{n+2})$$

$$\begin{aligned}
 &\leq \left(\frac{1-\alpha_2}{\alpha_1-1}\right) d_E(\beta_{n-1}, \beta_n) - \left(\frac{1-\alpha_2}{\alpha_1-1}\right) d_E(\beta_{n+1}, \beta_n) + \left(\frac{1-\alpha_2}{\alpha_1-1}\right) d_E(\beta_{n+1}, \beta_{n+2}) \\
 &\leq \left(\frac{1-\alpha_2}{\alpha_1-1}\right) d_E(\beta_{n-1}, \beta_n) - \left(\frac{\alpha_3}{\alpha_1-1}\right) d_E(\beta_n, \beta_{n+1}) + \left(\frac{1-\alpha_4}{\alpha_1-1}\right) d_E(\beta_{n+1}, \beta_{n+2}) \\
 &\leq \left(\frac{1-\alpha_2}{\alpha_1-1}\right) \alpha^{n-1} d_E(\beta_0, \beta_1) - \left(\frac{\alpha_3}{\alpha_1-1}\right) \alpha^n d_E(\beta_0, \beta_1) + \left(\frac{1-\alpha_4}{\alpha_1-1}\right) \alpha^{n+1} d_E(\beta_0, \beta_1) \\
 &\leq \left(\frac{1-\alpha_2}{\alpha_1-1} - \alpha \frac{\alpha_3}{\alpha_1-1} + \alpha^2 \frac{1-\alpha_4}{\alpha_1-1}\right) \alpha^{n-1} d_E(\beta_0, \beta_1) \\
 &\leq \left(\frac{1-\alpha_2}{\alpha_1-1} - \frac{\alpha_3}{\alpha_1-1} + \frac{1-\alpha_4}{\alpha_1-1}\right) \alpha \alpha^{n-1} d_E(\beta_0, \beta_1) \\
 &\leq \left(\frac{2-(\alpha_2+\alpha_3+\alpha_4)}{\alpha_1-1}\right) \alpha \alpha^{n-1} d_E(\beta_0, \beta_1) \\
 &\leq \left(\frac{1+\alpha_1}{\alpha_1-1}\right) \alpha^n d_E(\beta_0, \beta_1)
 \end{aligned}$$

Hence,

$$d_E(\beta_n, \beta_{n+2}) \leq \lambda \alpha^n d_E(\beta_0, \beta_1) \tag{3.1.3}$$

For  $\{\beta_n\}$  we consider  $d_E(\beta_n, \beta_{n+l})$  in two cases

If  $l$  is odd say  $2p+1$ , for  $p \geq 1$  then by using rectangular inequality and (3.1.2)

$$\begin{aligned}
 d_E(\beta_n, \beta_{n+2p+1}) &\leq d_E(\beta_{n+2p+1}, \beta_{n+2p}) + d_E(\beta_{n+2p}, \beta_{n+2p-1}) + d_E(\beta_{n+2p-1}, \beta_n) \\
 &\leq d_E(\beta_{n+2p}, \beta_{n+2p+1}) + d_E(\beta_{n+2p-1}, \beta_{n+2p}) + d_E(\beta_{n+2p-1}, \beta_{n+2p-2}) \\
 &\quad + d_E(\beta_{n+2p-2}, \beta_{n+2p-3}) + \dots + d_E(\beta_{n+2}, \beta_{n+1}) + d_E(\beta_{n+1}, \beta_n) \\
 &= d_E(\beta_n, \beta_{n+1}) + d_E(\beta_{n+1}, \beta_{n+2}) + \dots + d_E(\beta_{n+2p-1}, \beta_{n+2p}) + d_E(\beta_{n+2p}, \beta_{n+2p+1}) \\
 &\leq \alpha^n d_E(\beta_0, \beta_1) + \alpha^{n+1} d_E(\beta_0, \beta_1) + \dots + \alpha^{n+2p-1} d_E(\beta_0, \beta_1) + \alpha^{n+2p} d_E(\beta_0, \beta_1) \\
 &\leq [1 + \alpha + \alpha^2 + \alpha^3 + \dots] \alpha^n d_E(\beta_0, \beta_1) \\
 &\leq \frac{1}{1-\alpha} [\alpha^n d_E(\beta_0, \beta_1)] \\
 &\leq \frac{\alpha^n}{1-\alpha} d_E(\beta_0, \beta_1) \\
 &\leq \left[\lambda + \frac{1}{1-\alpha}\right] \alpha^n d_E(\beta_0, \beta_1)
 \end{aligned}$$

Hence,

$$d_E(\beta_n, \beta_{n+2p+1}) \leq \left[\lambda + \frac{1}{1-\alpha}\right] \alpha^n d_E(\beta_0, \beta_1) \tag{3.1.4}$$

For all  $n \geq 1, p \geq 1, \lambda = \frac{1+\alpha_1}{\alpha_1-1} \geq 0$

If  $l$  is even say  $2p$  for  $p \geq 1$ , then by using rectangular inequality(3.1.2), (3.1.3) and the fact that  $0 < \lambda < 1$ , we get

$$\begin{aligned}
 d_E(\beta_n, \beta_{n+2p}) &\leq d_E(\beta_{n+2p}, \beta_{n+2p-1}) + d_E(\beta_{n+2p-1}, \beta_{n+2p-2}) + d_E(\beta_{n+2p-2}, \beta_n) \\
 &\leq d_E(\beta_{n+2p-1}, \beta_{n+2p}) + d_E(\beta_{n+2p-2}, \beta_{n+2p-1}) + \dots + d_E(\beta_{n+3}, \beta_{n+2}) + d_E(\beta_{n+2}, \beta_n) \\
 &= d_E(\beta_n, \beta_{n+2}) + d_E(\beta_{n+2}, \beta_{n+3}) + d_E(\beta_{n+3}, \beta_{n+4}) + \dots + d_E(\beta_{n+2m-2}, \beta_{n+2m-1}) \\
 &\quad + d_E(\beta_{n+2m-1}, \beta_{n+2m}) \\
 &\leq \lambda \alpha^n d_E(\beta_0, \beta_1) \\
 &\quad + [\alpha^{n+2} d_E(\beta_0, \beta_1) + \alpha^{n+3} d_E(\beta_0, \beta_1) + \dots + \alpha^{n+2m-2} d_E(\beta_0, \beta_1) + \alpha^{n+2m-1} d_E(\beta_0, \beta_1)] \\
 &= \lambda \alpha^n d_E(\beta_0, \beta_1) + [\alpha^2 + \alpha^3 + \dots + \alpha^{2m-1}] \alpha^n d_E(\beta_0, \beta_1) \\
 &\leq \lambda \alpha^n d_E(\beta_0, \beta_1) + [1 + \alpha + \alpha^2 + \alpha^3 + \dots] \alpha^n d_E(\beta_0, \beta_1) \\
 &\leq \lambda \alpha^n d_E(\beta_0, \beta_1) + \frac{1}{1-\alpha} [\alpha^n d_E(\beta_0, \beta_1)] \\
 &\leq \lambda \alpha^n d_E(\beta_0, \beta_1) + \frac{\alpha^n}{1-\alpha} [d_E(\beta_0, \beta_1)]
 \end{aligned}$$

Hence,

$$d_E(\beta_n, \beta_{n+2p}) \leq \left[\lambda + \frac{1}{1-\alpha}\right] \alpha^n d_E(\beta_0, \beta_1) \tag{3.1.5}$$

For all  $n \geq 1, m \geq 1$  and

$$\lambda = \frac{1 + \alpha_1}{\alpha_1 - 1} \geq 0$$

From (3.1.4) and (3.1.5)

$$d_E(\beta_n, \beta_{n+l}) \leq \left[\lambda + \frac{1}{1-\alpha}\right] \alpha^n d_E(\beta_0, \beta_1)$$

For all  $n \geq 1, p \geq 1$  and  $\lambda = \frac{1+\alpha_1}{\alpha_1-1} \geq 0$

Assume that  $\theta \ll k$ . Since,

$$\alpha^n d_E(\beta_0, \beta_1) \rightarrow 0 \text{ as } n \rightarrow \infty$$

$\{\beta_n\}$  is a Cauchy sequence in  $X$ . since  $Q(X)$  is a complete subspace of  $X$ , then there exists a point  $z \in Q(X) \subseteq P(X)$  such that

$$\lim_{n \rightarrow \infty} \beta_n = \lim_{n \rightarrow \infty} Px_n = \lim_{n \rightarrow \infty} Qx_{n-1} = z$$

Also we can find  $x \in X$  such that  $z=Px$ .

Let  $\theta \ll l$  be given, we can choose natural numbers  $N_1$  and  $N_2$  such that  $d_E(z, \beta_{n-1}) \ll \frac{\lambda_1 l}{2(\lambda_1+1)}$

For all  $n > N_1$  and  $d_E(\beta_{n-1}, \beta_n) \ll \frac{l}{2}$

For all  $n > N_2$ . Let  $N = \max \{N_1, N_2\}$ , from (3.1.1)

$$\begin{aligned} d_E(\beta_{n-1}, z) &= d_E(Px_{n-1}, Px) \\ &\geq \alpha_1 d_E(Qx_{n-1}, Qx) + \alpha_2 d_E(Px_{n-1}, Qx_{n-1}) + \alpha_3 d_E(Px, Qx_{n-1}) + \alpha_4 d_E(Px, Qx) \\ &\geq \alpha_1 d_E(\beta_n, Qx) + \alpha_2 d_E(\beta_{n-1}, \beta_n) + \alpha_3 d_E(z, \beta_n) + \alpha_4 d_E(z, Qx) \\ &\geq \alpha_1 d_E(\beta_n, Qx) \end{aligned}$$

$$\text{Hence } d_E(\beta_n, Qx) \leq \frac{1}{\alpha_1} d_E(\beta_{n-1}, z)$$

Using rectangular inequality

$$\begin{aligned} d_E(z, Qx) &\leq d_E(z, \beta_{n-1}) + d_E(\beta_{n-1}, \beta_n) + \frac{1}{\alpha_1} d_E(\beta_{n-1}, z) \\ &\leq \left[1 + \frac{1}{\alpha_1}\right] d_E(\beta_{n-1}, z) + d_E(\beta_{n-1}, \beta_n) \end{aligned}$$

Hence,

$$d_E(z, Qx) \ll \frac{l}{2} + \frac{l}{2} = l$$

i.e.

$$d_E(z, Qx) = l$$

Therefore

$$Px = Qx = z$$

That is,  $z$  is a point of coincidence of  $P$  and  $Q$ . if  $z^*$  is another point of coincidence of  $P$  and  $Q$  then

$$P\gamma = Q\gamma = z^*$$

For some  $\gamma \in X$  then  $d_E(z, z^*) = d_E(Px, P\gamma)$

$$\begin{aligned} d_E(z, z^*) &\geq \alpha_1 d_E(Qx, Q\gamma) + \alpha_2 d_E(Px, Qx) + \alpha_3 d_E(P\gamma, Qx) + \alpha_4 d_E(P\gamma, Q\gamma) \\ &\geq \alpha_1 d_E(z, z^*) + \alpha_2 d_E(z, z) + \alpha_3 d_E(z^*, z) + \alpha_4 d_E(z^*, z^*) \\ &\geq (\alpha_1 + \alpha_3) d_E(z, z^*) \end{aligned}$$

$$\text{Hence } d_E(z, z^*) \leq \frac{1}{(\alpha_1 + \alpha_3)} d_E(z, z^*)$$

Since,  $\alpha_1 + \alpha_3 > 1$

Therefore, we have  $d_E(z, z^*) = \theta$  i.e.  $z = z^*$

$P$  and  $Q$  have a unique point of coincidence in  $X$ .

Suppose  $P$  and  $Q$  are weakly compatible mappings, then we have

$$Pz = PQx = QPx = Qz$$

Therefore,  $Pz = Qz = w$  (say)

This shows that  $w$  is another point of coincidence between  $P$  and  $Q$ .

Therefore, by the uniqueness of the point of coincidence we must have  $z = w$

Hence  $z$  is a unique common fixed point of  $P$  and  $Q$  in  $X$ .

## REFERENCES

1. Asim, M., Imdad, M., & Radenovic, S. (2019). Fixed point results in extended rectangular b-metric spaces with an application. *UPB Sci. Bull. Ser. A*, 81(2), 43-50
2. Azam, A., Arshad, M., & Beg, I. (2009). Banach contraction principle on cone rectangular metric spaces. *Applicable Analysis and Discrete Mathematics*, 3(2), 236-241.
3. Bisht, R. K. (2023). An overview of the emergence of weaker continuity notions, various classes of contractive mappings and related fixed point theorems. *Journal of Fixed Point Theory and Applications*, 25(1), 11.
4. Branciari, A. (2000). A fixed point theorem of Banach-Caccioppoli type on a class of generalized metric spaces. *Publ. Math. Debrecen*, 57(1-2), 31-37.
5. Chaira, K., Eladraoui, A., Kabil, M., & Kamouss, A. (2020). Fisher fixed point results in generalized metric spaces with a graph. *International Journal of Mathematics and Mathematical Sciences*, 2020(1), 7253759.

6. Cho, S. H. (2018, January). Fixed point theorems for L-contractions in generalized metric spaces. In *Abstract and Applied Analysis* (Vol. 2018, pp. 1-6). Hindawi
7. Gandhi, M. P., & Aserkar, A. A. (2025). Non-expansive Mapping in S-Metric Space. *a a*, 2, 3.
8. George, R., Radenovic, S., Reshma, K. P., & Shukla, S. (2015). Rectangular b-metric space and contraction principles. *J. Nonlinear Sci. Appl*, 8(6), 1005-1013.
9. Gu, F. E. N. G., & Shatanawi, W. A. S. F. I. (2019). Some new results on common coupled fixed points of two hybrid pairs of mappings in partial metric spaces. *Journal of Nonlinear Functional Analysis*, 2019(1).
10. Huang, H., & Xu, S. (2013). Fixed point theorems of contractive mappings in cone b-metric spaces and applications. *Fixed Point Theory and Applications*, 2013, 1-10.
11. Mustafa, Z., Shahkoochi, R. J., Parvaneh, V., Kadelburg, Z., & Jaradat, M. M. M. (2019). Ordered  $S_p$ -metric spaces and some fixed point theorems for contractive mappings with application to periodic boundary value problems. *Fixed Point Theory and Applications*, 2019, 1-20.
12. Rao, C. S., Kumar, S. R., & Sarma, K. K. M. (2024). Fixed Point Theorems On 4-Dimensional Ball Metric Spaces And Their Applications. *Journal of Applied Science and Engineering*, 27(11), 3583-3588.
13. Reddy, P. M. (2023). A Common Fixed Point Theorem in Cone Rectangular Metric Space Under Expansive Type Condition. *Indian Journal Of Science And Technology*, 16(32), 2510-2517.
14. Samuel, B. W., Mani, G., Ganesh, P., Thabet, S. T., & Kedim, I. (2025). Fixed Point Theorems on Controlled Orthogonal  $\delta$ -Metric-Type Spaces and Applications to Fractional Integrals. *Journal of Function Spaces*, 2025(1), 5560159.
15. Vujaković, J., Mitrović, S., Mitrović, Z. D., & Radenović, S. (2020). On F-contractions for weak  $\alpha$ -admissible mappings in metric-like spaces. *Mathematics*, 8(9), 1629.
16. Samuel, B., Mani, G., Thabet, S. T., Kedim, I., & Vivas-Cortez, M. (2024). New fixed point theorems in extended orthogonal S-metric spaces of type  $(\mu, \sigma)$  with applications to fractional integral equations. *Results in Nonlinear Analysis*, 7(4), 146-162.