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Computation of Inverse Sum Indeg Uphill Index and Its Polynomial of Certain Graphs

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ARTICLE INFO	ABSTRACT
Published Online: 21 June 2025 **Corresponding Author:** V.R. Kulli	In this paper, we introduce the inverse sum indeg uphill index and the inverse sum indeg uphill polynomial of a graph. Also we compute the inverse sum indeg uphill index and its corresponding polynomial of some standard graphs, wheel graphs, gear graphs and helm graphs.

KEYWORDS: inverse sum indeg uphill index, inverse sum indeg uphill polynomial, graph.

I. INTRODUCTION

We consider simple graphs which are finite, undirected, connected graphs without loops and multiple edges. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. For undefined notations and technology, we refer [1].

A topological index is a numerical parameter mathematically derived from the graph structure. Several topological indices have been considered in Theoretical Chemistry and many topological indices were defined by using vertex degree concept. The Zagreb, Nirmala, Sombor, Gourava, temperature indices are the most degree based topological indices in Chemical Graph Theory, see [2-28]. Topological indices have their applications in various disciplines in Science and Technology.

A u-v path P in G is a sequence of vertices in G, starting with u and ending at v, such that consecutive vertices in P are adjacent, and no vertex is repeated. A path $\pi = v_1, v_2, \ldots v_{k+1}$ in G is a downhill path if for every i, $1 \le i \le k$, $d_G(v_i) \ge d_G(v_{i+1})$.

A vertex v is downhill dominates a vertex u if there exists a downhill path originated from u to v. The downhill neighborhood of a vertex v is denoted by $N_{dn}(v)$ and defined as: $N_{dn}(v)$ = {u: v downhill dominates u}. The downhill degree $d_{dn}(v)$ of a vertex v is the number of downhill neighbors of v [29].

Recently, some downhill indices were studied in [30-32].

The uphill domination is introduced by Deering in [33].

A u-v path P in G is a sequence of vertices in G, starting with u and ending at v, such that consecutive vertices in P are adjacent, and no vertex is repeated. A path $\pi = v_1, v_2, \ldots v_{k+1}$ in G is a uphill path if for every i, $1 \le i \le k$, $d_G(v_i) \le d_G(v_{i+1})$.

A vertex v uphill dominates a vertex u if there exists an uphill path originated from u to v. The uphill neighborhood of a vertex v is denoted by $N_{up}(v)$ and defined as: $N_{up}(v)$ = {u: v uphill dominates u}. The uphill degree $d_{up}(v)$ of a vertex v is the number of uphill neighbors of v, see [34].

In [35], Vukičević et al. observed that many graph indices are defined simply as the sum of individual bound contributions. They have proposed a class of discrete Adriatic indices to study whether there other possibly significant graph indices of this form. One of these discrete Adriatic indices is the inverse sum indeg index, and this index is defined as

$$ISI(G) = \sum_{uv \in E(G)} \frac{d_G(u)d_G(v)}{d_G(u) + d_G(v)}.$$

Some inverse sum indices were studied in [36, 37].

Motivated by the inverse sum indeg index, we introduce the inverse sum indeg uphill index of a graph as follows:

The inverse sum indeg uphill index of a graph G is defined as

$$ISIU(G)=\sum_{uv\in E(G)}\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}.$$

In view of the inverse sum indeg uphill index, we propose the inverse sum indeg uphill polynomial of a graph G and it is defined as

$$ISIU(G,x)=\sum_{uv\in E(G)}x^{\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}}.$$

Recently, some uphill indices were studied such as the Nirmala uphill indices [38], Sombor uphill indices [39] and F-uphill index [40].

In this paper, we compute the inverse sum indeg uphill index and its corresponding polynomial of some standard graphs, wheel graphs, gear graphs and helm graphs.

II. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1. Let G be r-regular with n vertices and $r\geq 2$. Then

$$ISIU(G)=\frac{nr(n-1)}{4}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{up}(v)=n-1$ for every v in G. We obtain

$$ISIU(G)=\sum_{uv\in E(G)}\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}$$

$$=\frac{nr}{2}\frac{(n-1)(n-1)}{(n-1)+(n-1)}=\frac{nr(n-1)}{4}.$$

Corollary 1.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

$$ISIU(C_n)=\frac{n(n-1)}{2}.$$

Corollary 1.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$ISIU(K_n)=\frac{n(n-1)^2}{4}.$$

Proposition 2. Let G be r-regular with n vertices and $r\geq 2$. Then

$$ISIU(G,x)=\frac{nr}{2}x^{\frac{(n-1)}{2}}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $d_{up}(v)=n-1$ for every v in G. We obtain

$$ISIU(G,x)=\sum_{uv\in E(G)}x^{\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}}$$

$$=\frac{nr}{2}x^{\frac{(n-1)(n-1)}{(n-1)+(n-1)}}$$

$$=\frac{nr}{2}x^{\frac{(n-1)}{2}}.$$

Corollary 1.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

$$ISIU(C_n,x)=nx^{\frac{(n-1)}{2}}.$$

Corollary 1.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$ISIU(K_n,x)=\frac{n(n-1)}{2}x^{\frac{(n-1)}{2}}.$$

Proposition 3. Let P_n be a path with $n\geq 3$ vertices. Then

$$ISIU(P_n)=\frac{2(n^2-5n+6)}{2n-5}+\frac{(n-3)^2}{2}.$$

Proof: Let P_n be a path with $n\geq 3$ vertices. Clearly, $P=P_n$ has two types of edges based on the downhill degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(P) | d_{up}(u)= n-2, d_{up}(v) = n-3\}, |E_1| = 2.$

$E_2 = \{uv \in E(P) | d_{up}(u)= d_{up}(v) = n-3 \}, |E_2| = n-3.$

Then

$$ISIU(P)=\sum_{uv\in E(P)}\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}$$

$$=2\frac{(n-2)(n-3)}{(n-2)+(n-3)}+(n-3)\frac{(n-3)(n-3)}{(n-3)+(n-3)}$$

$$=\frac{2(n^2-5n+6)}{2n-5}+\frac{(n-3)^2}{2}.$$

Proposition 4. Let P_n be a path with $n\geq 3$ vertices. Then

$$ISIU(P_n,x)=2x^{\frac{(n^2-5n+6)}{2n-5}}+(n-3)x^{\frac{(n-3)}{2}}.$$

Proof: Let P_n be a path with $n\geq 3$ vertices. Clearly, $P=P_n$ has two types of edges based on the downhill degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(P) | d_{up}(u)= n-2, d_{up}(v) = n-3\}, |E_1| = 2.$

$E_2 = \{uv \in E(P) | d_{up}(u)= d_{up}(v) = n-3 \}, |E_2| = n-3.$

Then

$$ISIU(P_n,x)=\sum_{uv\in E(P_n)}x^{\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}}$$

$$=2x^{\frac{(n-2)(n-3)}{(n-2)+(n-3)}}+(n-3)x^{\frac{(n-3)(n-3)}{(n-3)+(n-3)}}$$

$$= 2x^{\frac{(n^2-5n+6)}{2n-5}} + (n-3)x^{\frac{(n-3)}{2}}.$$

$$= nx^0 + nx^{\frac{n}{2}}.$$

III. RESULTS FOR WHEEL GRAPHS

The wheel W_n is the join of C_n and K_1. Clearly W_n has n+1vertices and 2n edges. The vertex K_1 is called apex and the vertices of C_n are called rim vertices.

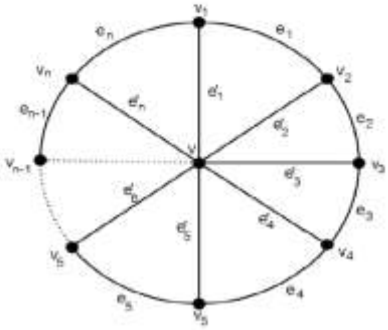

Figure 1. Wheel W_n

Let W_n be a wheel with n+1vertices and 2n edges, $n \geq 4$. Then there are two types of edges based on the downhill degree of end vertices of each edge as follows:

E_1 = {$uv \in E(W_n)$ | $d_{up}(u)$ = 0, $d_{up}(v)$ = n }, | E_1 | = n. | E_1 | = n.

E_2 = {$uv \in E(W_n)$ | $d_{up}(u)$ = $d_{up}(v)$ = n }, | E_2 | = n. | E_2 | = n.

Theorem 1. Let W_n be a wheel with n+1vertices and 2n edges, $n \geq 4$. Then

$$ISIU(W_n) = \frac{n^2}{2}.$$

Proof. We deduce

$$ISIU(W_n) = \sum_{uv \in E(W_n)} \frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}$$

$$= \frac{nn' \ 0}{0+n} + \frac{nnn}{n+n}$$

$$= \frac{n^2}{2}.$$

Theorem 2. Let W_n be a wheel with n+1vertices and 2n edges, $n \geq 4$. Then

$$ISIU(W_n, x) = nx^0 + nx^{\frac{n}{2}}.$$

Proof. We obtain

$$ISIU(W_n, x) = \sum_{uv \in E(W_n)} x^{\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}}$$

$$= nx^{\frac{0' \ n}{0+n}} + nx^{\frac{n' \ n}{n+n}}$$

IV. RESULTS FOR GEAR GRAPHS

A bipartite wheel graph is a graph obtained from W_n with n+1 vertices adding a vertex between each pair of adjacent rim vertices and this graph is denoted by G_n and also called as a gear graph. Clearly, $|V(G_n)|$ = 2n+1 and $|E(G_n)|$ = 3n. A gear graph G_n is depicted in Figure 2.

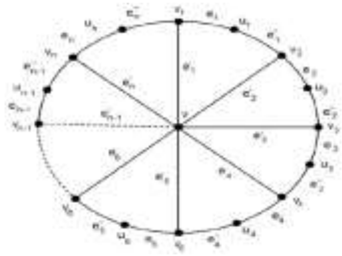

Figure 2. Gear graph G_n

Let G_n be a gear graph with 2n+1vertices, 3n edges, $n \geq 4$. Then G_n has two types of edges based on the downhill degree of the vertices of each edge as follows:

E_1 = {$u \in E(G_n)$ | $d_{up}(u)$ =1, $d_{up}(v)$ = 0}, | E_1 | = n.

E_2 = {$u \in E(G_n)$ | $d_{up}(u)$ = 1, , $d_{up}(v)$ = 3}, | E_2 | = 2n.

Theorem 3. Let G_n be a gear graph with 2n+1vertices, 3n edges, $n \geq 4$. Then the inverse sum indeg downhill index of G_n is

$$ISIU(G_n) = \frac{3n}{2}.$$

Proof: We deduce

$$ISIU(G_n) = \sum_{uv \in E(G_n)} \frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}$$

$$= n\left(\frac{1\times 0}{1+9}\right) + 2n\left(\frac{1\times 3}{1+3}\right)$$

$$= \frac{3n}{2}.$$

Theorem 4. Let G_n be a gear graph with 2n+1vertices, 3n edges, $n \geq 4$. Then the inverse sum indeg downhill polynomial of G_n is

$$ISIDW(G_n, x) = nx^{\frac{2n}{n+1}} + 2nx^0.$$

Proof: We deduce

$$ISIU(G_n, x) = \sum_{uv \in E(G_n)} x^{\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}}$$

$$= nx^{\frac{1\times 0}{1+0}} + 2nx^{\frac{1\times 3}{1+3}}$$

$$= nx^0 + 2nx^{\frac{3}{4}}.$$

V. RESULTS FOR HELM GRAPHS

The helm graph H_n is a graph obtained from W_n (with n+1 vertices) by attaching an end edge to each rim vertex of W_n. Clearly, $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. A graph H_n is shown in Figure 3.

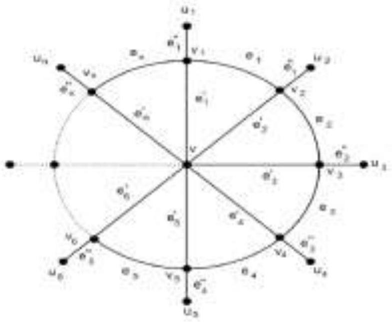

Figure 3. Helm graph H_n

Let H_n be a helm graph with 3n edges, $n \geq 5$. Then H_n has three types of edges based on the downhill degree of the vertices of each edge as follows:

$E_1 = \{uv \in E(H_n) \mid d_{up}(u) = n+1,\ d_{up}(v) = n\}$, $|E_1| = n$.
$|E_1| = n$.

$E_2 = \{uv \in E(H_n) \mid d_{up}(u) = d_{up}(v) = n\}$, $|E_2| = n$.
$|E_2| = n$.

$E_3 = \{uv \in E(H_n) \mid d_{up}(u) = n,\ d_{up}(v) = 0\}$, $|E_3| = n$.
$|E_3| = n$.

Theorem 5. Let H_n be a helm graph with 2n+1 vertices, $n \geq 5$. Then the inverse sum indeg downhill index of H_n is

$$ISIU(H_n) = \frac{n^2(n+1)}{2n+1} + \frac{n^2}{2}.$$

Proof: We obtain

$$ISIU(H_n) = \sum_{uv \in E(H_n)} \frac{d_{up}(u)d_{up}(v)}{d_{up}(u) + d_{up}(v)}$$

$$= \frac{n \times (n+1) \times n}{(n+1)+n} + \frac{n \times n \times n}{n+n} + \frac{n \times n \times 0}{n+0}$$

$$= \frac{n^2(n+1)}{2n+1} + \frac{n^2}{2}.$$

Theorem 6. Let H_n be a helm graph with 2n+1vertices, 3n edges, $n \geq 5$. Then the inverse sum indeg downhill polynomial of H_n is

$$ISIU(H_n, x) = nx^{\frac{n(n+1)}{2n+1}} + nx^{\frac{n}{2}} + nx^0.$$

Proof: We deduce

$$ISIU(H_n, x) = \sum_{uv \in E(H_n)} x^{\frac{d_{up}(u)d_{up}(v)}{d_{up}(u)+d_{up}(v)}}$$

$$= nx^{\frac{(n+1)\times n}{(n+1)+n}} + nx^{\frac{n \times n}{n+n}} + nx^{\frac{n \times 0}{n+0}}$$

$$= nx^{\frac{n(n+1)}{2n+1}} + nx^{\frac{n}{2}} + nx^0.$$

VI. CONCLUSION

In this paper, the inverse sum indeg uphill index and its corresponding polynomial of some standard graphs, wheel graphs, gear graphs, helm graphs are determined.

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