



SOME APPLICATIONS OF VEDIC PRINCIPLES IN MODERN MATHEMATICS

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Abstract: Vedic mathematics is ancient culture traditional mathematics, especially derived from 'Atharva Veda'. It deals with the branches of mathematics, and all other sciences with which we are today aware of. It was first discovered by Sri Bharati Krishna Tirthaji Maharaja from ancient Sanskrit text Veda is known today as Vedic Mathematics, and based on 16 sutras and 13 sub – sutras which can be applied in almost every branch of mathematics. It has a bright future ahead of it, and will aid in teaching and learning of mathematics depth with clear explanations. The main objective of this paper discusses some potential applications of Vedic principles in various branches of modern mathematics i.e. trigonometry, algebra etc. with examples and proofs. Using Vedic principles, it has described how it saves the time consumption, easy and fast calculation of the problems.

Keywords: *Mathematics; Vedic Principles; Applications; Trigonometry*

1. Introduction

Veda is a Sanskrit word which means knowledge or fountain head. Vedas are not the text on Mathematics but maintain a lot of mathematical concepts. Vedic mathematics has originated from the “Atharva Veda”. It deals the branches of modern mathematics which we are today aware of [1]. Founder of Vedic Mathematics is Jagat Guru, Shankracharya Sri Bharati Krishna Tirthaji Maharaj (1884 – 1960) a scholar of Mathematics, Sanskrit & Philosophy. He discovered 16 sutras (aphorisms) and 13 sub – sutras (corollaries) between the periods 1911 to 1918 from the Atharva Veda, involved sutras that can be applied in the modern system of mathematics to ease up any complicated problems. These sutras and sub – sutras collectively were later named as Vedic Mathematics [2]. The Ganit Sutras' also

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known as 'Sulabh Sutras' is the simple system of Mathematics. These Sutras can be applied to cover almost every branch of Mathematics. These formulae can be applied even to complex problems involving a large number of mathematical operations saves a lot of time and efforts in solving these problems. Applications of the sutras can be applied in different branches of mathematics. These techniques make mathematical calculation fast, easy, great confidence, and booster for students [4].

Vedic Mathematics processing systems are in tune with the maturity process of 'innocence fulfilled your minds'. It is because of it that Vedic mathematics teacher while interacts with the pure innocence during teaching of first principles of mathematics to young minds [5].

2. Some Vedic Principles and its Application in Modern Mathematics

2.1 Urdhva – Tiryagbhyam (In Sanskrit, ऊर्ध्वतिर्यग्भ्याम्)

Which means “vertically and crosswise” and this formula can be applied in different branches of mathematics. For example;

1. Proof of $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Proof: We have, $\tan A = \frac{\sin A}{\cos A}$, $\tan B = \frac{\sin B}{\cos B}$. Then by using Vedic formula 4.1

Table 1. Findings the problem of tan (A + B)

Angles	P	b	h
A	↑ sin A	cos A ↑	1 ↑
B	sin B	cos B ↑	1 ↑
A + B	sin A cos B + cos A sin B	cos A cos B – sin A sin B	1
A – B	sin A cos B – cos A sin B	cos A cos B + sin A sin B	1

$$\tan(A + B) = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

After simplification, we obtain

$$\therefore \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

2. Generalized form of $\tan(A+B+C)$

Proof: We have, $\tan A = \frac{\tan A}{1}$, $\tan B = \frac{\tan B}{1}$, $\tan C = \frac{\tan C}{1}$. Then

Table 2. Findings the problem of $\tan(A+B+C)$

Angles	p	b	h
A	$\tan A$	1	$\sec A$
B	$\tan B$	1	$\sec B$
A + B	$\tan A + \tan B$	$1 - \tan A \tan B$	$\sec A \sec B$
C	$\tan C$	1	$\sec C$
A + B + C	$\tan A + \tan B + \tan C(1 - \tan A \tan B)$	$1 - \tan A \tan B - \tan C(\tan A + \tan B)$	$\sec A \sec B \sec C$

After simplification, we obtain

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan A \tan C - \tan B \tan C}$$

3. Similarly, proof of $\cos(A+B)$ and its generalized form

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

We have, $\cos A = \frac{\cos A}{1}$, $\cos B = \frac{\cos B}{1}$. Then

Table 3. Findings the problems of $\cos(A+B)$ and $\sin(A-B)$

Angles	p	b	h
A	$\sin A$	$\cos A$	1
B	$\sin B$	$\cos B$	1

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A + B	$\sin A \cos B + \cos A \sin B$	$\cos A \cos B - \sin A \sin B$	1
A - B	$\sin A \cos B - \cos A \sin B$	$\cos A \cos B + \sin A \sin B$	1

$\therefore \cos(A + B) = \cos A \cos B - \sin A \sin B$

$\therefore \sin(A - B) = \sin A \cos B - \cos A \sin B$

4. Proof of $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}(xy \mp \sqrt{1-x^2}\sqrt{1-y^2})$

Proof: let $\cos^{-1} x = A, \cos^{-1} y = B$. Then $\cos A = x, \cos B = y$.

Table 4. Findings the problems of inverse circular functions

Angles	p	b	h
A			1
B			1
A + B	$y\sqrt{1-x^2} + x\sqrt{1-y^2}$	$xy - \sqrt{1-x^2}\sqrt{1-y^2}$	1
A - B	$y\sqrt{1-x^2} - x\sqrt{1-y^2}$	$xy + \sqrt{1-x^2}\sqrt{1-y^2}$	1

$A + B = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$

$\therefore \cos^{-1} x + \cos^{-1} y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$

Similarly, other remaining inverse circular functions $\sin^{-1}x \pm \sin^{-1}y, \tan^{-1}x \pm \tan^{-1}y$ also can be proved easily in this way.

5. Proof of $\sin 3A, \cos 3A, \sin (-A),$ and $\cos (-A)$

Proof: We have, $\sin A = \frac{\sin A}{1}, \cos A = \frac{\cos A}{1}$. Then

Table 5. findings the problems of $\sin 3A, \sin (-A), \cos 3A, \cos (-A)$

Angles	p	b	h
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A	$\sin A$	$\cos A$	1
A	$\sin A$	$\cos A$	1
A + A = 2A	$2 \sin A \cos A$	$\cos^2 A - \sin^2 A$	1
A + 2A = 3A	$\sin A(\cos^2 A - \sin^2 A) + 2 \sin A \cos^2 A$	$\cos A(\cos^2 A - \sin^2 A) - 2 \sin^2 A \cos A$	1
A - 2A = -A	$\sin A(\cos^2 A - \sin^2 A) - 2 \sin A \cos^2 A$	$\cos A(\cos^2 A - \sin^2 A) + 2 \sin^2 A \cos A$	1

$\therefore \sin 3A = \sin A(\cos^2 A - \sin^2 A) + 2 \sin A \cos^2 A = 3 \sin A - 4 \sin^3 A$

$\therefore \sin(-A) = \sin A(\cos^2 A - \sin^2 A) - 2 \sin A \cos^2 A = -\sin A$

Similarly, $\cos 3A = 4 \cos^3 A - 3 \cos A$

$\therefore \cos 3A = \cos A(\cos^2 A - \sin^2 A) - 2 \sin^2 A \cos A = 4 \cos^3 A - 3 \cos A$

$\therefore \cos(-A) = \cos A(\cos^2 A - \sin^2 A) + 2 \sin^2 A \cos A = \cos A$

6. If $\sin \alpha = \frac{3}{5}$, $\cos \beta = \frac{3}{\sqrt{13}}$, find $\sin(\alpha \pm \beta)$, $\cos(\alpha \pm \beta)$, $\tan(\alpha \pm \beta)$, $\sin 2\alpha$, $\sin 3\alpha$ and

so on.

Table 6. Findings the given problems

Angles	p	b	h
α	3	4	5
β	2	3	$\sqrt{13}$
$\alpha + \beta$	+ 9 + 8 = 17	- 12 - 6 = 6	$5\sqrt{13}$ = $5\sqrt{13}$

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$\alpha - \beta$	- 9 - 8 = 1	+ 12 + 6 = 18	$5\sqrt{13}$ = $5\sqrt{13}$
$2\alpha = \alpha + \alpha$	12 + 12 = 24	16 - 9 = 7	25
$3\alpha = 2\alpha + \alpha$	21 + 96 = 117	28 - 72 = - 44	125
$4\alpha = 2\alpha + 2\alpha$ or $3\alpha + \alpha$	168 + 168 = 336	49 - 576 = -527	625
$-\alpha = \alpha - 2\alpha$	21 - 96 = - 75	28 + 72 = 100	125

We have, $\sin(\alpha + \beta) = \frac{p}{h} = \frac{17}{5\sqrt{13}}$, $\sin(\alpha - \beta) = \frac{1}{5\sqrt{13}}$, $\cos(\alpha + \beta) = \frac{b}{h} = \frac{6}{5\sqrt{13}}$,

$\cos(\alpha - \beta) = \frac{18}{5\sqrt{13}}$, $\tan(\alpha + \beta) = \frac{17}{6}$, $\tan(\alpha - \beta) = \frac{1}{18}$, $\sin 2\alpha = \frac{24}{25}$, $\sin 3\alpha = \frac{117}{125}$, $\sin(-\alpha) =$

$\frac{-75}{125} = \frac{-3}{5}$.

Absolutely, trigonometry can indeed be tackled efficiently with some streamlined approaches. Traditional methods, while thorough, can sometimes be cumbersome. However, there are several Vedic techniques which is helpful as well as that can be simplified and speed up the process to solve trigonometric problems more efficiently and easily in calculations.

7. Find Adj. (A), where $A = \begin{pmatrix} 1 & 0 & 4 \\ -2 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

Solution:

$$\begin{array}{c|cccc}
 1 & 0 & 4 & 1 & 0 \\
 \hline
 -2 & 3 & 1 & -2 & 3 \\
 0 & 1 & 2 & 0 & 1 \\
 1 & 0 & 4 & 1 & 0 \\
 -2 & 3 & 1 & -2 & 3
 \end{array}$$

$$= \begin{pmatrix} 6-1 & 4-0 & 0-12 \\ 0+4 & 2-0 & -8-1 \\ -2+0 & 0-1 & 3+0 \end{pmatrix}$$

$$\therefore \text{Adj. (A)} = \begin{pmatrix} 5 & 4 & -12 \\ 4 & 2 & -9 \\ -2 & -1 & 3 \end{pmatrix}$$

8. Find the area of a triangle whose vertices are A (4, 7), B (-2, 11), and C (12, -6).

$$\begin{array}{ccccccc}
 4 & -2 & 12 & 4 & -2 \\
 7 & 11 & -6 & 7 & 11
 \end{array}$$

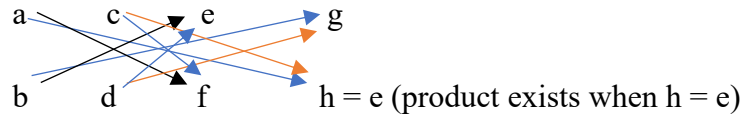
Solution: Area of a triangle = $\frac{1}{2} (12 - 132 + 84 + 24 + 44 + 14) = 23$ sq. units

9. Find the product of two-by-two square matrix.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, find AB.

Procedure:

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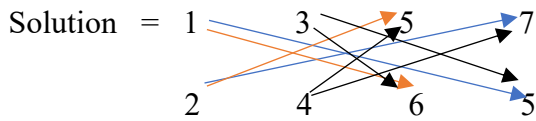


$$\therefore AB = \begin{pmatrix} ae+bg & af+be \\ ce+dg & cf+de \end{pmatrix} \quad (\text{From first to second column by cross wise method,}$$

and place the value in first and second row)

Example: Find the product of two matrix

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 5 \end{pmatrix}$$



$$AB = \begin{pmatrix} 5+14 & 6+10 \\ 15+28 & 18+20 \end{pmatrix}$$

$$= \begin{pmatrix} 19 & 16 \\ 43 & 38 \end{pmatrix}$$

2.2. Gunitasamuccayah Samuccayagunitah (In Sanskrit गुणितसमुच्चयः समुच्चयगुणितः)

Which means “the product of the sum of coefficients in the factors is equal to the sum of coefficients in the product”.

Is $(x+2)(x-3)$ factors of $x^2 - 5x + 6$?

Solution: Sum of the coefficients in the factors = $(1+2)(1-3) = -6$.

Sum of the coefficients in the product = $1 - 5 + 6 = 2$.

Which shows that $(x+2)(x-3)$ is not factors of $x^2 - 5x + 6$.

Consider the cubic equation $x^3 + ax^2 + bx + c = 0$

Let $\alpha, \beta,$ and γ be the roots of equation. Then

$$a = \alpha + \beta + \gamma = \text{coefficient of } x^2$$

$$b = \alpha\beta + \beta\gamma + \alpha\gamma = \text{coefficient of } x$$

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$c = \alpha \beta \gamma =$ constant term. Let us choose the value of α , β , and γ from the constant so that which satisfies the given conditions.

1. Find the factors of $x^3 + 6x^2 + 11x + 6$

By the given formula, we obtain

Sum of the coefficients in the product $= 1 + 6 + 11 + 6 = 24$.

Possible factors of 24 are $(\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \text{ and } \pm 24)$

Sum of the coefficients in the factors $= (x + \alpha)(x + \beta)(x + \gamma)$

$\therefore (x + \alpha)(x + \beta)(x + \gamma) = 24 \dots (1)$ (by using the given Vedic formula)

$$(1 + \alpha)(1 + \beta)(1 + \gamma) = 24 \dots (2)$$

Choose α , β , and γ which satisfies the eqⁿ (2)

So, $(x + 1)(x + 2)(x + 3)$ is a factor of $x^3 + 6x^2 + 11x + 6$

2. Find the factors of $x^3 - 7x + 6$

The possible factors of 6 are $(\pm 1, \pm 2, \pm 3, \text{ and } \pm 6)$, then choose α , β , and γ

$a = -1 - 2 + 3 = 0 =$ Coefficient of x^2 , $c = -1 \times -2 \times 3 = 6 =$ constant term.

Verification:

$$b = \alpha\beta + \beta\gamma + \alpha\gamma = \text{coefficient of } x = 2 - 6 - 3 = -7$$

Hence, the required factors of $x^3 - 7x + 6 = (x - 1)(x - 2)(x + 3)$

This sutra is actually used to find the factors of the polynomials and the roots of equations.

2.3 Lopanasthapanabhyam (In Sanskrit लोपनस्थापनाभ्याम्)

Which means "by alternate elimination and retention".

Consider the case of factorization of this expression as $ax^2 + by^2 + cz^2 + dxy + eyz + fzx$

This is homogeneous expression of second degree in three variables. To find the factors of this types of expression, follow three steps.

Step – 1. Eliminate z by putting $z = 0$, remaining expression is in the variables of x and y . Then factorize it.

Step – 2. Eliminate y by putting $y = 0$, remaining expression is in the variables of x and z . Then factorize it.

Step – 3. Use these two sets of factors, fill the gaps caused by the elimination process of z and y respectively. This gives actual factors of the expression.

This formula can be used to find the factors of harder polynomial.

1. Find the factors of $3x^2 + 2y^2 + 6z^2 + 7xy + 7yz + 11xz$

Solution: Put $z = 0$. Then $3x^2 + 7xy + 2y^2 = (3x + y)(x + 2y) \dots (1)$

Put $y = 0$, then we obtain $3x^2 + 11xz + 6z^2 = (3x + 2z)(x + 3z) \dots (2)$

Fill the gaps from eqⁿ (1), and (2). We obtain the required factors of the given expression $(3x + y + 2z)(x + 2y + 3z) \dots (3)$

Hence, the required factors of $3x^2 + 2y^2 + 6z^2 + 7xy + 7yz + 11xz$ is $(3x + y + 2z)(x + 2y + 3z)$

Alternative,

Put $x = 0$, and $y = 0$. We obtain the same result.

2. Find the factors of $3x^2 + 4y^2 - z^2 + 7xy - 3yz - 2xz + 17x + 21y - z + 20$

Solution: Put $z = 0$, $y = 0$, $x = 0$, $z = 0$, and $x = 0$, $y = 0$.

Put $z = 0$, $y = 0$, then

$3x^2 + 17x + 20 = (x + 4)(3x + 5) \dots (1)$

Put $x = 0$, $z = 0$, then

$4y^2 + 21y + 20 = (y + 4)(4y + 5) \dots (2)$

Put $x = 0$, $y = 0$, then

$20 - z - z^2 = (4 - z)(z + 5)$

$= (-z + 4)(z + 5) \dots (3)$

Fill in the gaps from eqⁿ (1), (2), and (3), we obtain the required factors of the given expression

$(x + y - z + 4)(3x + 4y + z + 5) \dots (4)$

Hence, the required factors of $3x^2 + 4y^2 - z^2 + 7xy - 3yz - 2xz + 17x + 21y - z + 20$ is

$(x + y - z + 4)(3x + 4y + z + 5)$

3. Find the pair of lines of this equation $3x^2 - 2xy - 21y^2 + 13x + 41y - 10 = 0$

Given polynomial is $3x^2 - 2xy - 21y^2 + 13x + 41y - 10$

By given sutra, put $y = 0$

$$3x^2 + 13x - 10 = (x + 5)(3x - 2) \dots (1)$$

Put $x = 0$

$$\begin{aligned} -21y^2 + 41y - 10 &= -(3y - 5)(7y - 2) \dots (2) \\ &= (-3y + 5)(7y - 2) \end{aligned}$$

Fill the gaps from equation (1) and (2), we obtain the required factors $(x - 3y + 5)(3x + 7y - 2) \dots$
(3)

Factors of the given polynomial is $(x - 3y + 5)(3x + 7y - 2)$

Hence, the pair of lines of this equation is $(x - 3y + 5)(3x + 7y - 2) = 0$

2.4. Paravartya Yojayet (In Sanskrit, परावर्त्य योजयेत्)

Which means "Transpose and Adjust".

This formula can be applied to find the equation of line passing through two points.

$$A(x_1, y_1) \text{ ————— } B(x_2, y_2)$$

1. Find the equation of line passing through two points (9, 7) and (5, 2).

Step – 1: put the difference of y co – ordinate – x co – ordinate and vice – versa.

Step – 2: The constant term (RHS) is obtained by substituting co – ordinates of either of the given points in LHS (From Step – 1).

$$x(7 - 2) - y(9 - 5) = 5x - 4y$$

The constant term RHS is obtained $5(9) - 4(7)$ or $5(5) - 4(2) = 17$

Hence, the required equation is $5x - 4y = 17$.

3. Conclusion

The beauty of Vedic mathematics is to solve the problems in easy and fast calculations without using tools in some situations. In this paper, a set of some Vedic techniques is considered to obtain the results. There are challenges as well as opportunities for teachers and researchers to implement it in order to make mathematics fun to all. It is hood to have regular logical debate among scholars on different aspects of mathematics relative to Vedic. This method helps to improve the mental development and solve the difficult problems with high speed and accuracy. Vedic method helps us to boost a positive attitude and drive away from phobia of mathematics from the students.

Finally, Vedic mathematics is applied with modern mathematics but modern mathematics has not touched it. It is fun and interesting. But the problem is that out our modern educational system is ignoring the importance of Vedic mathematics. Therefore, teachers should try to apply Vedic

techniques in the class room to make mathematics fun, easy, and interesting, also emphasis the importance of Vedic mathematics in the sense of retrospective to prospective study design by investigating the impact of Vedic mathematics. So, by incorporating principles from Vedic mathematics into modern mathematical education and information technology (IT) can enhance their computational skills, cultivate mathematical intuition, and appreciate the beauty of mathematical concepts and techniques. It helps us to boost at the attitude and away from phobia of mathematics from the students. Vedic mathematics in modern era to make more students' life easier in comparison complex calculations.

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