



APOS Theory-Based Learning: Impact on Mathematical Problem-Solving Skills

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ARTICLE INFO

Published Online:
11 July 2025

ABSTRACT

Learning is a fundamental process aimed at enhancing students' cognitive abilities, with particular emphasis on the development of problem-solving skills in the context of 21st century education. One instructional approach that supports this objective is the APOS theory-based learning model. The APOS theory, consisting of the components Action, Process, Object, and Schema, provides a framework for understanding how students construct mathematical knowledge. The present study seeks to investigate the effect of APOS theory-based learning on students' problem-solving abilities in the domain of probability. Employing a quantitative research design with a pretest-posttest approach, the study involved 31 eighth-grade students from Class VIII A at SMPN 4 Yogyakarta. The learning material focused on probability, and a set of test items was utilized to assess the students' problem-solving skills. Data were analyzed using the N-Gain statistical hypothesis test to evaluate the extent of improvement in the students' performance. The results of the study indicate a statistically significant positive effect of APOS theory-based learning on the students' problem-solving capabilities. Post-intervention, students exhibited a deeper conceptual understanding of probability and demonstrated a more structured approach to problem-solving. These findings suggest that APOS theory-based learning can be an effective pedagogical strategy for enhancing students' problem-solving skills, particularly in mathematics. Consequently, the integration of APOS theory in teaching probability offers a viable instructional model that not only promotes mathematical comprehension but also fosters critical thinking skills essential for academic and real-life problem-solving.

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KEYWORDS: APOS Theory, Problem Solving, Probability

I. INTRODUCTION

In the 21st century, technological developments have triggered significant changes in various areas of life, where the role of information and communication technology has become increasingly important and is even able to replace many functions of human resources. This massive use of technology affects various sectors, including education, the economy, and industry, ultimately demanding adjustments in the skills needed in the world of work [1]. This advancement has an impact on the competencies expected of individuals entering the workforce, so 21st century skills are essential for students to master from an early age. Important skills that students must develop include critical thinking, problem-solving skills, creativity, innovation, collaboration, and effective communication [2]. All of these skills are relevant because the development of technology requires individuals

to think more adaptively, work together in teams, and innovate to find solutions to complex problems.

Problem-solving skills are a very important skill for students' future. In this dynamic era, it is not enough for students to have only theoretical knowledge; students need to have the ability to identify, analyze, and solve problems effectively. This is important because every job or challenge in daily life often requires good problem-solving skills, from solving technical problems and dealing with interpersonal conflicts to making strategic decisions in uncertain situations. Suharsono stated that education experts agree on the importance of these skills and that problem-solving skills can be developed through a variety of subjects and disciplines taught in schools [3]. Every subject, be it science, math, language, or social sciences, has the potential to be a means

for students to practice these skills in a structured and sustainable way.

Problem-solving skills are essential skills that include the ability to think analytically to make decisions or choose solutions from a variety of alternatives encountered in everyday life. In addition, this skill involves critical thinking in new situations, including the process of searching for information, conducting analysis, and identifying the core of the problem with the goal of producing effective alternative solutions that allow one to take the right action to achieve the desired target [4]. Problem-solving is at the core of mathematics teaching and learning. Problem-solving is defined as a process that involves applying mathematical knowledge and skills to solve problems that do not have a clear or immediate solution [5].

Highlighting that mathematical problem-solving requires not only cognitive skills but also metacognitive [6]. Students must be able to manage resources such as time, energy, and prior knowledge when facing a problem. mathematical problem solving as a complex thinking process that requires the application of mathematical knowledge to solve problems for which the solution is unknown [7]. With this foundation, students are required to have good problem-solving skills, as these skills are not only useful in completing academic tasks but are also important for overcoming real-life challenges. Students who are able to solve problems well will be better prepared to face complex situations, both in school and outside of school, because they are used to thinking critically, seeking relevant information, and devising systematic steps to achieve solutions. These skills will also support them in making appropriate and effective decisions in the future, forming an independent, critical, and adaptive mindset in facing various challenges.

Problem-solving is a process that involves trying to overcome difficulties in order to achieve a specific goal [8]. This process includes not only the ability to find answers but also analytical and critical thinking skills in dealing with challenges that may arise along the way through the problem-solving process. Teaching problem-solving skills is a process in which teachers play a role in helping students understand the problems they are facing and guiding them through the steps needed to find a solution [9]. This means that teachers not only provide answers or formulas directly but also encourage students to develop logical and independent ways of thinking.

However, based on findings in the field, student's ability to solve mathematical problems is still relatively low [10]. This can be seen from the difficulties faced by students when given questions that require problem-solving, especially non-routine questions that require deeper thinking and the application of special strategies. Midawati noted that many students experience confusion or difficulty in solving problem-solving problems, especially when teachers give non-routine tasks [11]. Non-routine questions, different from routine questions that usually follow certain patterns or steps,

require students to apply a creative and in-depth understanding of concepts, as well as analyze situations they were not familiar with before. When students are faced with these non-routine questions, they tend to feel less confident or confused because they are used to routine questions whose answers are relatively easy to find. This situation highlights the importance for teachers to strengthen problem-solving skills through a more contextual and interactive approach, which can help students improve their critical thinking skills, as well as prepare them for different types of challenges outside the classroom.

Some of the factors that cause students' low ability to solve mathematical problems include the lack of student habits in working on problem-solving problems, students' difficulties in understanding problems, obstacles in doing calculations, and the lack of students' habits to recheck the answers they have given. Given the importance of problemsolving skills, improving the quality of learning is a must. One solution to overcome this problem is to choose the right learning strategy. Teachers must be able to innovate in determining effective and efficient learning to improve students' ability to face problems. One of the learning innovations that can be used is learning with APOS theory.

The APOS theory was first introduced by Dubinsky in 1985. APOS theory is a development of Piaget's idea to see the development of mathematical knowledge in individuals by looking at stages that include actions, processes, objects, and schemas [12]. APOS theory is a constructivist theory that discusses how to learn mathematical concepts [13]. The APOS theory affirms that an individual deals with mathematical conditions by using certain mental mechanisms to build cognitive structures applied to situations in order to form and develop cognitive structures applied to those situations [14].

Action (Action) talks about the construction of understanding of mathematical concepts, namely the change of an object as a result of external stimuli (physical activity). The process (Process) is a mental structure that performs the same Actions as the Actions internalized in the mind of each individual. At the same time, an object is defined as the understanding of a mathematical concept that is the result of actions and processes [15]. The last schema is defined as the collection of the results of actions, processes, and objects that are interrelated with each other[16]. This process can enhance self-regulated learning. According to Alyani & Ramadhina (2022), there is a relationship between SRL (Self-Regulated Learning) and mathematical problemsolving. Based on this, one of the important aspects that need attention is the APOS theory adopted in the learning process. Proper learning can affect the way students understand the material and improve their skills. This study aims to see the effectiveness of learning with APOS theory on the mathematical problem-solving ability of junior high school students. It is hoped that with the application of learning with APOS theory, students will be more actively involved in the

learning process, which can help them develop problemsolving as well as understanding mathematical concepts more deeply and applicatively.

II. METHODS

This study uses a quantitative approach with a preexperimental research type, which aims to measure the extent to which APOS (Action, Process, Object, and Schema) theory-based learning methods can improve students' problem-solving skills. In this case, the quantitative approach allows researchers to obtain numerical data that can be statistically analyzed to assess the effects of the given learning methods. The research design used is a one-group pretest-posttest design, where the problemsolving ability test is carried out twice, namely before the treatment (pretest) and after the treatment (posttest). The pretest test serves to measure the initial level of students' problem-solving skills, while the posttest test is used to assess the development of these skills after the application of APOS theory-based learning methods. By comparing the results of the pretest and posttest, researchers can identify changes or improvements in students' abilities, which can be attributed to the applied learning methods. This design also allows researchers to assess the effectiveness of direct treatment and provide a concrete picture of the influence of APOS theory on the mastery of problem-solving skills in students. The design of the research that will be carried out in this study can be seen in the following table:

Before Treatment (Pretest)	Treatment (Treatment)	After Treatment (Posttest)
O1	X	O2

The population in this study is grade VIII students of SMP Negeri 4 Yogyakarta for the 2023/2024 academic year, totaling 30 students, consisting of 1 class. The sampling technique used is *convenient sampling*.

The data analysis used is N-Gain analysis. Gain is the difference in score between *pretest* and *posttest*. The gain reflects the improvement of students' ability or mastery of concepts after learning; the normalized gain normality test (N-Gain) can be calculated using the hake equation.

$$N - gain = \frac{\text{nilai posttest} - \text{nilai pretest}}{\text{nilai maksimum} - \text{nilai pretest}}$$

It was explained that the normalized gain (N-Gain) is g , and the maximum score (ideal) is the result of the initial and final trials. N-gain can be classified as follows:

Great N-Gain	Interpretation
$g \geq 0.7$	Tall
$0.7 > g \geq 0.3$	Keep
$g < 0.3$	Low

[18]

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III. RESULT AND DISCUSSION

Results

The student's ability to solve the problems that are the focus of this research is assessed based on the results of working on questions related to the opportunities provided. The problem-solving process is evaluated based on four stages of problem-solving, namely understanding the problem, designing a solution, implementing a solution plan, and conducting a final evaluation.

The problem comprehension stage means that students can identify the information they know as well as what is being asked for in the question. The stage of designing a solution includes the ability of students to determine the relationship between known information and the question, choose an appropriate strategy to solve the problem and determine the relevant formula. At the stage of implementing the plan, students are expected to be able to carry out the steps that have been designed, apply relevant formulas, and complete the problem to completion. Finally, the final evaluation stage requires students to draw conclusions from the completion that has been made.

In this study, learning with APOS theory is used. This method emphasizes the student's activeness during the learning process, while the teacher only acts as a facilitator. The research lasted for five meetings, including giving a pretest at the first meeting, the implementation of learning activities at the second and third meetings, and learning about opportunity material. In the fifth meeting, the students were given a posttest of 3 description questions to measure problem-solving skills.

The stages of implementing learning activities include several steps, namely actions, processes, objects, and schemes through discussion activities. Each group of students is given problems listed in the Student Worksheet (LKPD). The LKPD contains issues related to the opportunity material that has been agreed upon with the APOS learning steps. After all groups completed the LKPD, each group was asked to present the results of their discussion in front of the class. The researcher, along with other students, gave responses to complete the information presented by the group. The group discussion activity went smoothly. Most students actively participated in the process. If students had difficulties or needed guidance, the researcher provided assistance so that they could understand the problem discussed.

Data was collected using a problem-solving ability test sheet. Based on the results of the research, the pretest and posttest scores of students in learning with APOS theory are obtained in the following table. Table 3.

Account	Pretest	Post-test
Maximum value	70	90
Minimum value	50	75

Average	60,3	83,2121
Standard deviation	6,237	5,518

The results of the trial from 33 grade VIII students showed that the problem-solving pretest score had a maximum score of 70, a minimum score of 50, and an average of 60.3 with a standard deviation of 6.237. In the table, it can be seen that the average result of the student problem-solving pretest is still below the KKTP determined by the school, which is 70.

Referring to the table, the problem-solving pretest has a maximum score of 90, a minimum score of 75, and an average of 83.2121, with a standard deviation of 5.5269 so that the average score of the problem-solving pretest is above the KKTP standard, which is 70.

Table 4. Minimum	Maksimum	Mean	Std. Deviasi	
N-gain	0,2647	0,80	0,5747	0,13440
N-gain(%)	26,47	80,00	57,47	13,44

N-gain reflects the improvement of students' problem-solving abilities after being given treatment. The results of the problem-solving N-gain test had an average of 0.5747, so it increased with the medium category with an increase of 57.4654%. The following are the results of the N-gain test for problem-solving skills. From the table, it can be seen that there is an increase in problem-solving ability by 57.4654%. So it can be said that it is quite influential on students' problem-solving skills.

DISCUSSION

The results of the trial on 33 grade VIII students showed that their problem-solving skills before being given intervention or additional learning were still below the expected standard. In the pretest stage, students' problemsolving scores have a maximum score of 70 and a minimum score of 50, with an average of 60.3 and a standard deviation of 6.237. This average is below the High Completeness Criteria (KKTP) set by the school, which is 70. This shows that, overall, most students have not reached the level of understanding or problem-solving skills that the school considers adequate. In addition, a standard deviation of 6.237 indicates a variation in students' initial abilities. Some students are close to being at the grade standard, while others still have lower grades. Overall, the results of this pretest show the need for more effective interventions or learning methods so that students can achieve KKTP standards and improve their ability to solve problems.

The posttest results on 33 grade VIII students showed a significant improvement in their problem-solving abilities after the intervention or additional learning. The posttest score has a maximum score of 90 and a minimum score of 75, with an average score of 83.21 and a standard deviation of 5.5269. This average shows that, overall, the student's posttest score is already well above the standard of

the High Completeness Criterion (KKTP) set by the school, which is 70. This indicates that the majority of students have managed to achieve or exceed the expected level of understanding and skills in problem-solving. The standard deviation lower than the pretest, which is 5.5269, indicates that the variation in the posttest scores is smaller, which means that the students' results are more consistent and within a narrower range around the mean. This consistency shows that the learning methods or interventions applied are not only effective in improving the average student's problem-solving ability but also succeed in making the improvement more evenly distributed among all students. Overall, the results of this posttest show that the interventions provided are able to significantly improve students' problem-solving abilities and bring the majority of students above the set standards.

Based on the data from the N-gain test for students' problem-solving skills, the average score was 0.5747. This value falls into the "moderate" category, which indicates that the treatment provided has a positive impact on improving students' problem-solving skills, although not optimally. In percentage terms, this increase is equivalent to 57.47%, which means that there is an increase of about half of the student's initial ability. The N-gain score is calculated by comparing the pre-and post-treatment scores, measuring how much change occurs proportionately to the maximum potential improvement. These results indicate that the methods or interventions applied are quite effective in helping students improve their abilities, but there is still room for further improvement. Although the improvement is in the moderate category, the positive impact achieved shows the potential for students to develop better problemsolving skills if provided with support or further learning.

The results obtained are in line with the research. Eliza Found that M-APOS learning was proven to improve students' problem-solving skills in statistical materials [19]. In line with this, Suske stated that the use of APOS theory in learning is considered effective in improving problemsolving. Syam, in his research, divided students into two, namely those who were taught with APOS theory and in a conventional way, then found that students who were taught using APOS theory had higher problem-solving skills than those who were not treated[20].

IV. CONCLUSION

Based on the data analysis and discussion that has been presented in the previous chapter, it can be concluded that the practice of opportunity learning innovation in grade VIII of junior high school with APOS theory is quite influential on students' problem-solving skills. This can be seen from the average posttest score of the problem-solving ability of the KKTP with a moderate level of influence and in the category of quite influential. So, learning with APOS theory can be an innovation in learning opportunity material at school.

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