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Triple-Diffusive Incompressible Couple-Stress Fluid Through Porous Medium

Mahinder Singh
Department of Mathematics, Govt College Seema (Rohru) Distt Shimla (HP), India,

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Corresponding Author:
Mahinder Singh

ABSTRACT

The present study investigates the onset of triple-diffusive convection in a couple-stress fluid layer subjected to a porous medium, with heat and solute gradients imposed from below. The system is further influenced by a uniform vertical magnetic field and uniform rotation. The governing equations are formulated, incorporating the effects of couple-stress, rotation, magnetic field, and medium permeability. A linear stability analysis is performed, and the corresponding dispersion relation is derived. For the case of stationary convection, the presence of a stable solute gradient and rotation exerts a stabilizing influence on the system. In contrast, the effect of medium permeability on stability is dependent on the presence of rotation; it can be either destabilizing or stabilizing. The magnetic field and couple-stress parameter similarly exhibit dual effects, acting as either stabilizing or destabilizing agents under different conditions. However, in the absence of rotation, medium permeability destabilizes the system, while both the magnetic field and couple-stress parameter contribute to stabilization. The dispersion relation is further examined numerically to understand the system's behaviour in detail. It is observed that the presence of a stable solute gradient, rotation, and magnetic field introduces oscillatory modes into the system, which are absent when these factors are not considered. These findings highlight the complex interplay between multiple diffusive effects, couple-stress, and external forces, providing insights into the fundamental nature of convective instability in porous media.

KEYWORDS: Triple-diffusive convection, Couple-stress fluid, Porous medium, Magnetic field, Rotation, Stability analysis.

1. INTRODUCTION

The study of **thermal convection in a fluid layer**, in both the presence and absence of **rotation** and a **magnetic field**, has been extensively analyzed, with significant contributions made by **Chandrasekhar** [1]. The phenomenon of **thermohaline convection**, wherein a fluid layer is heated from below while subjected to a stable salinity gradient, was first examined by **Veronis** [2]. In such systems, **buoyancy forces** arise not only from temperature-induced density differences but also from variations in **solute concentration**. These **double-diffusive convection** problems have diverse applications in fields such as **oceanography, limnology, and engineering**.

Particular examples of interest include **solar ponds** built to trap heat (**Tabor and Matz** [3]) and the study of **Antarctic lakes** (**Shirtcliffe** [4]). A similar physical mechanism operates in **stellar atmospheres**, where helium mimics the role of salt, increasing density and diffusing more slowly than heat. In stellar convection studies, considering a **solute gradient** and **free boundaries** is essential, as the classical single-component fluid models do not adequately describe astrophysical conditions.

The problem of **double-diffusive convection in porous media** is of considerable importance in **geophysics, soil sciences, groundwater hydrology, and astrophysics**. The growing interest in **geothermal energy resources** has further emphasized the need to understand **convective processes in porous media**. **Hydrothermal circulation**, which dominates heat transfer in young oceanic crust, is a crucial component of this study (**Lister** [5]). Additionally, **comets, meteorites, and interplanetary dust** exhibit highly porous structures, emphasizing the significance of **porosity in astrophysical contexts** (**McDonnel** [6]).

The **influence of a magnetic field** on convection stability is particularly relevant to **geophysics**, especially in the study of **Earth's core dynamics**, where the **mantle**, composed of a conducting fluid, behaves like a **porous medium** and can become convectively unstable due to differential diffusion. Moreover, understanding **convective flow through porous media under a magnetic field** is essential for geothermal research. **Stommel and Fedorov** [7] and **Linden** [8] have observed that in oceanic convection layers, the characteristic length scales may be sufficiently large for **Earth's rotation** to influence their formation. The **rotation of the Earth** also distorts hexagonal convection cells in porous media, significantly impacting **geothermal energy extraction**. **Brakke** [9] described a **double-diffusive instability** occurring when a solution of a slowly diffusing protein is layered over a denser sucrose solution. **Nason et al.** [10] found that such instability, detrimental to biochemical separations, can be suppressed using **ultracentrifuge rotation**.

The **theory of couple-stress fluids**, formulated by **Stokes** [11], has applications in biomechanics, particularly in understanding the **lubrication mechanisms of synovial joints**. These **dynamically loaded bearings**, such as the **shoulder, ankle, knee, and hip joints**, experience **squeeze-film action**, which provides surface protection. **Walicki and Walicka** [12] modeled **synovial fluid** as a **couple-stress fluid**, given its composition of **hyaluronic acid molecules**, which impart a high viscosity, near-gel-like behavior. **Goel et al.** [13] investigated the **hydromagnetic stability** of an unbounded **couple-stress binary fluid mixture** under **rotation with vertical temperature and concentration gradients**. **Sharma et al.** [14] studied a **couple-stress fluid with suspended particles heated from below**, while **Sunil et al.** [15] examined a **couple-stress fluid in a porous medium under the influence of a magnetic field and rotation**. **Kumar et al.** [16] analyzed the **thermal instability** of a **couple-stress fluid layer under uniform rotation**, concluding that **rotation stabilizes the system**, whereas **couple-stress effects** can be both **stabilizing and destabilizing**.

Given the fundamental and applied significance of these mechanisms in **geophysics, astrophysics, hydrology, and engineering**, the present study investigates **Triple-diffusive convection in a couple-stress fluid within a porous medium** under the simultaneous influence of **uniform rotation and a uniform magnetic field**. The effects of various physical parameters on the **stability of the system** are analyzed, with special attention to **oscillatory modes**, which emerge due to the combined effects of **stable solute gradients, rotation, and magnetic fields**.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Here we consider an infinite, horizontal, layer of thickness d, of an electrically non – conducting incompressible Couple – Stress fluid heated and soluted from below. The temperature T and solute concentration $C^1 and\ C^2$ respectively, and a uniform temperature gradient β $\left(=\left|\frac{dT}{dZ}\right|\right)$ and a uniform solute gradient are β' $\left(=\left|\frac{dC^1}{dz}\right|\right)$ and $\beta''\left(=\left|\frac{dC^2}{dz}\right|\right)$ are maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$ and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ∈ and medium permeability k_1.

Let p, ρ, T, C, α, α′, g, η, μ_e and $\vec{q}(u,v,w)$ denote respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of couple-stress fluid are

$$\frac{1}{\in}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)$$

$$-\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H}+\frac{2}{\in}\left(\vec{q}\times\vec{\Omega}\right), \qquad (1)$$

$$\nabla.\vec{q}=0 \quad ,$$

(2) $$E\frac{\partial T}{\partial t}+\left(\vec{q}.\nabla\right)T=\chi\nabla^2 T,$$

(3)

$$E'\frac{\partial C}{\partial t}+\left(\vec{q}.\nabla\right)T=\chi'\nabla^2 C^1, \qquad (4)$$

$$E''\frac{\partial C}{\partial t}+\left(\vec{q}.\nabla\right)T=\chi''\nabla^2 C^2, \qquad (5)$$

In terms of temperature T and the concentrations C^1 *and* C^2, we suppose the density of the mixture is given by (known as density equation of state)

$$\rho = \rho_0 \,[1-\alpha\,(T - T_0)+\alpha^{/}(C^1 - C_0{}^1) + \alpha''\left(C^2 - C^2{}_0\right)]\,, \qquad (6)$$

where the suffix zero refers to values at the reference level z = 0 and in writing Eq.(1), use has been made of Boussinesq approximation. The magnetic permeability μ_e, the kinematic viscosity ν, couple-stress viscosity $\mu^{/}$, the thermal diffusivity χ and the solute diffusivity $\chi^{/}$ are all assumed to be constants.

The Maxwell's equations yield

$$\in \frac{d\vec{H}}{dt} = \left(\vec{H}\cdot\nabla\right)\vec{q} + \in \eta\nabla^2\vec{H}, \qquad (7)$$

and

$$\nabla\cdot\vec{H} = 0, \qquad (8)$$

where $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \in^{-1} \vec{q}\cdot\nabla$ stands for the convective derivative. Here E = $\in + (1-\in)\left(\frac{\rho_s C_s}{\rho_0 C_i}\right)$ is a constant and $E^{/}$ is a constant analogous to E but corresponding to solute rather that heat; ρ_s, C_s and ρ_o, C_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The steady state solution is

$$\vec{q} = (0,\, 0\,,\, 0\,),\ T = T_0 - \beta z,\ C^1 = -\beta' z + C^1 a,\ C^2 = -\beta'' z + C^2 a\ ,\ \beta' = \left(C_0{}^1 - C^1\right)/\,d\,,\ \beta'' = \left(C_1^2 - C^2\right)/\,d$$

and $\rho = \rho_0\,(1 + \alpha\beta z - \alpha^{/}\beta^{/} z - \alpha''\beta'' z\ )$. (9)

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution, and let δp, $\delta\rho$, θ, γ, $\vec{h}(h_x, h_y, h_z)$ and $\vec{q}$ (u, v, w) denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\vec{H}(0,0,0)$ and velocity $\vec{q}$ (0, 0, 0). The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = -\,\rho_0\,(\,\alpha\theta - \alpha^{/}\gamma - \alpha''\gamma'\,). \qquad (10)$$

Then the linearized perturbation equations become

$$\frac{1}{\in}\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}(\alpha\theta - \alpha^{/}\gamma) - \frac{1}{k_1}\left(\nu - \frac{\mu^{/}}{\rho_0}\nabla^2\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H} + \frac{2}{\in}\left(\vec{q}\times\vec{\Omega}\right), \qquad (11)$$

$$\nabla.\vec{q} = 0, \qquad (12)$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \chi\nabla^2\theta\ , \qquad (13)$$

$$E^{/}\frac{\partial\gamma}{\partial t} = \beta^{/} w + \chi^{/}\nabla^2\gamma\ , \qquad (14)$$

$$E''\frac{\partial\gamma'}{\partial t} = \beta'' w + \chi''\nabla^2\gamma'\,, \qquad (15)$$

$$\in\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}\cdot\nabla\right)\vec{q} + \in\eta\nabla^2\vec{h}\,, \qquad (16)$$

and

$$\nabla.\vec{h} = 0. \qquad (17)$$

3. THE DISPERSION RELATION

In the analysis of disturbances, we assume that the perturbation quantities can be expressed in terms of normal modes, where each mode corresponds to a specific characteristic frequency or spatial configuration.

$[w, h_z, \theta, \gamma, \gamma', \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), \psi(z), X(z)]\exp(ik_x x + ik_y y + nt)$, (18) where k_x, k_y are the wave numbers along the x- and y- directions respectively, and k= ($\sqrt{k_x^2+k_y^2}$) is the resultant wave number, n is the growth rate which is in general a complex constant. $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density.

To simplify the problem, we express the coordinates x, y, and z in terms of a new unit of length d, such that the length scale is normalized, for this a = kd, $\sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\chi}$ (Prandtl number), $p_2 = \frac{\nu}{\eta}$, $q = \frac{\nu}{\chi'}$, $P_\ell = \frac{\kappa_1}{d^2}$ and $F = \frac{\mu'}{\rho_0 d^2 \nu}$. Equations (11)- (17), using (18), yield

$$\left[\frac{\sigma}{\in}+\frac{1}{P_\ell}-\frac{F}{P_\ell}\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W+\frac{ga^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma-\alpha''\psi)-\frac{\mu_e Hd}{4\pi\rho_o\nu}\left(D^2-a^2\right)DK+\frac{2\Omega d^3}{\in\nu}DZ=0, \quad (19)$$

$$\left[\frac{\sigma}{\in}+\frac{1}{P_\ell}-\frac{F}{P_\ell}\left(D^2-a^2\right)\right]Z=\left(\frac{\mu_e Hd}{4\pi\rho_o\nu}\right)DX+\left(\frac{2\Omega d}{\in\nu}\right)DW, \quad (20)$$

$$(D^2-a^2-p_2\sigma)K=-\left(\frac{Hd}{\in\eta}\right)DW, \quad (21)$$

$$(D^2-a^2-p_2\sigma)X=-\left(\frac{Hd}{\in\eta}\right)DZ, \quad (22)$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-\left(\frac{\beta d^2}{\chi}\right)W, \quad (23)$$

$$(D^2-a^2-E'q_1\sigma)\Gamma=-\left(\frac{\beta' d^2}{\chi'}\right)W. \quad (24)$$

$$(D^2-a^2-E''q_2\sigma)\psi=-\left(\frac{\beta'' d^2}{\chi''}\right)W. \quad (25)$$

Consider the scenario where both boundaries are free and serve as perfect conductors of both heat and solute concentration, while the adjoining medium is assumed to be perfectly conducting. Although the case of two free boundaries may appear somewhat artificial, it allows for the derivation of analytical solutions and facilitates the formulation of qualitative insights. The boundary conditions relevant to this setup, with respect to which the equations (19)–(25) must be solved, are as follows:

$W = D^2W = X = DZ = 0$, $\Theta = 0$, $\Gamma = 0$, at z = 0 and z = 1 ,

DX = 0, K = 0 on a perfectly conducting boundary and X = 0 and h_x, h_y, h_z are continuous with an external vacuum field on a non-conducting boundary. (26)

Although the case of two free boundaries may be somewhat artificial, it is the most suitable approximation for stellar atmospheres (Spiegel [17]). By applying the aforementioned boundary conditions, it can be demonstrated that all even-order derivatives of W must vanish at z = 0 and z = 1. Therefore, the correct solution for W, corresponding to the lowest mode, is as follows: $W = W_0 \sin\pi z$, (27)

where

W_0 is a constant.

Eliminating Θ, Γ, K, Z and X between Eqs. (19) – (25) and substituting the proper solution $W = W_0 \sin \pi z$, in the resultant equation, we obtain the dispersion relation

$$R_1=\left(\frac{1+x}{x}\right)(1+x+iEp_1\sigma_1)\left[\left\{\frac{i\sigma_1}{\in}+\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x+ip_2\sigma_1)+Q_1\right]+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q_1\sigma_1)}$$

$$S_2\frac{(1+x+iEp_2\sigma_1)}{(1+x+iE''q_2\sigma_1)}+T_1\frac{(1+x+iEp_1\sigma_1)(1+x+ip_2\sigma_1)}{x\left[\left\{\frac{i\sigma_1}{\in}+\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x+ip_2\sigma_1)+Q_1\right]}\quad , \quad (28)$$

where

$$R_1=\frac{g\alpha\beta d^4}{\upsilon\chi\pi^4},\ S_1=\frac{g\alpha^{/}\beta^{/}d^4}{\upsilon\chi^{/}\pi^4},\ Q_1=\frac{\mu_e H^2 d^2}{4\pi\rho_0\upsilon\eta\in\pi^2},\ T_1=\left(\frac{2\Omega d^2}{\in\nu\pi^2}\right)^2,\ x=\frac{a^2}{\pi^2},\ i\sigma_1=\frac{\sigma}{\pi^2},\ F_1=\pi^2 F \text{ and. } P=\pi^2 P_l$$

4. THE STATIONARY CONVECTION

When instability manifests as stationary convection, the marginal state is characterized by $\sigma = 0$, substituting $\sigma = 0$ into the dispersion relation (28) simplifies it to:

$$R_1=\frac{(1+x)^2}{x}\left[\left\{\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x)+Q_1\right]+S_1+S_2+T_1\frac{(1+x)^2}{x\left[\left\{\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x)+Q_1\right]}$$

. (29) To analyze the effects of the stable solute gradient, rotation, magnetic field, couple-stress parameter, and medium permeability, we systematically examine the behavior of $\frac{dR_1}{dS_1}, \frac{dR_1}{dS_2}, \frac{dR_1}{dT_1}, \frac{dR_1}{dQ_1}, \frac{dR_1}{dF_1}$ and $\frac{dR_1}{dP}$ through analytical methods.

Equation (26) yields

$$\frac{dR_1}{dS_1}=+1=\frac{dR_1}{dS_2}, \tag{30}$$

$$\frac{dR_1}{dT_1}=\frac{(1+x)^2}{x\left[\left\{\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x)+Q_1\right]}, \tag{31}$$

$$\frac{dR_1}{dQ_1}=\frac{(1+x)^2}{x}\left[1-\frac{T_1}{\left[\left\{\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x)+Q_1\right]^2}\right], \tag{32}$$

$$\frac{dR_1}{dF_1}=\frac{(1+x)^4}{xP}\left[1-\frac{T_1}{\left[\left\{\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x)+Q_1\right]^2}\right], \tag{33}$$

$$\frac{dR_1}{dP}=-\frac{(1+x)^3}{xP^2}\left[1-\frac{T_1}{\left[\left\{\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x)+Q_1\right]^2}\right]. \tag{34}$$

Equations (30) and (31) indicate that, for stationary convection, both the stable solute gradient and rotation exert stabilizing effects on the system. Equations (32)–(34) show that, in the absence of rotation ($T_1 = 0$), $\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dF_1}$ is always positive while $\frac{dR_1}{dP}$ is always negative. This suggests that the magnetic field and the couple-stress parameter have stabilizing effects, whereas the medium permeability has a destabilizing effect on the system when rotation is absent. In a rotating system, the magnetic field and couple-stress parameter may either stabilize or destabilize the system, depending on the conditions, while the effect of medium permeability can be either destabilizing or stabilizing, influencing the instability of the couple-stress rotating fluid in a porous medium under hydromagnetic conditions if

$$T_1 < (\text{ or } >) \left[\left\{\frac{1}{P}+\frac{F_1}{P}(1+x)\right\}(1+x)+Q_1\right]^2. \tag{35}$$

The dispersion relation (29) is also examined numerically for different values of S_1, T_1, P, F_1 and Q_1. As shown in Figures 1–5, it is evident that the stable solute gradient and rotation have stabilizing effects on the system, while the magnetic field, couple-stress parameter, and medium permeability exhibit both stabilizing and destabilizing effects, depending on the specific conditions.

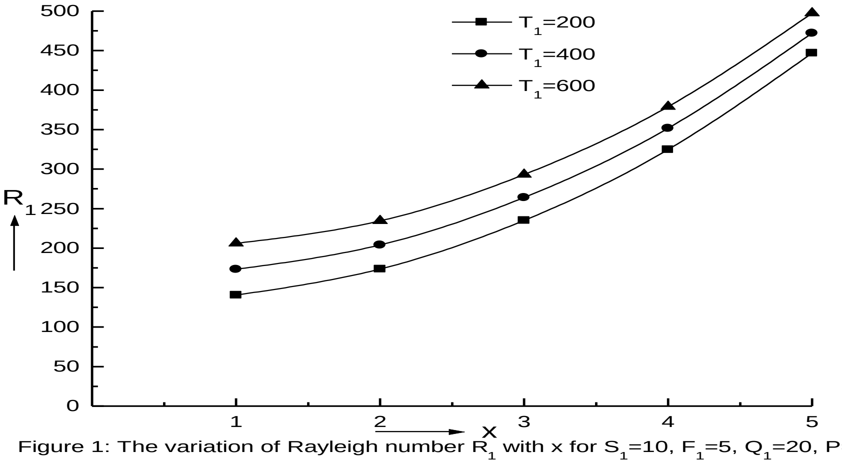

Figure 1: The variation of Rayleigh number R_1 with x for S_1=10, F_1=5, Q_1=20, P=5 and T_1=200, 400 and 600.

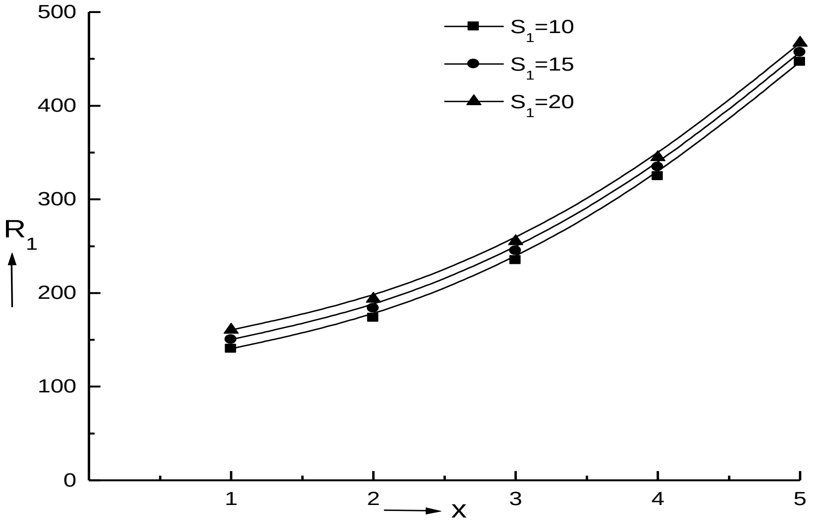

Figure 2: The variation of Rayleigh number R_1 with x for F_1=5, Q_1=20, P=5, T_1=200 and S_1= 10, 15 and 20.

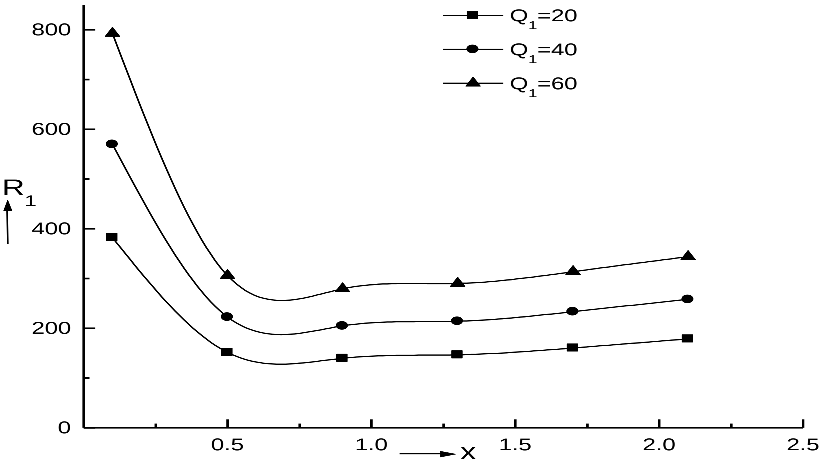

Figure 3: The variation of Rayleigh number R_1 with x for S_1=10, F_1=5, T_1=200, P=5 and Q_1 =20, 40 and 60.

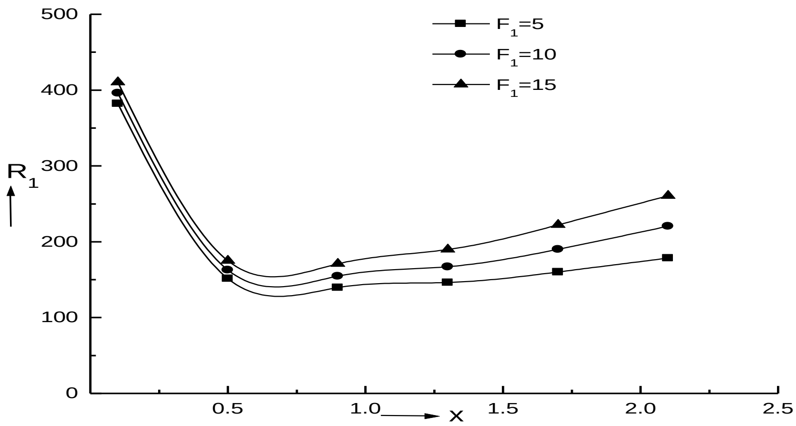

Figure 4: The variation of Rayleigh number R_1 with x for S_1=10, T_1=200, Q_1= 20, P=5 and F_1= 5, 10 and 15.

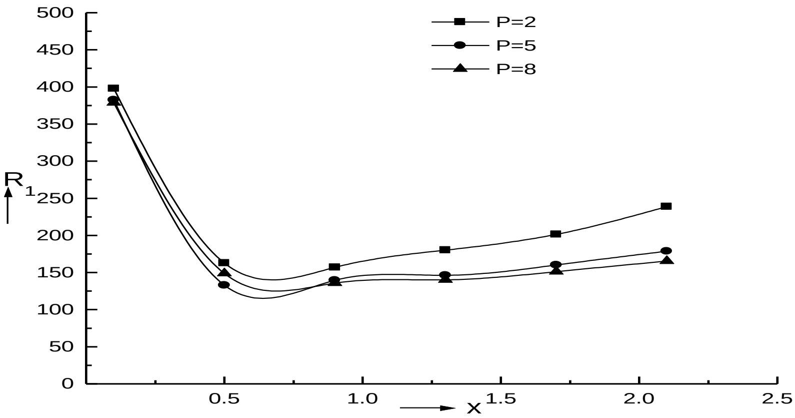

Figure 5: The variation of Rayleigh number R_1 with x for S_1=10, T_1=200, Q_1=20, F_1=5 and P=2, 5 and 8.

5. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

In this section, we investigate the potential for oscillatory modes, if any, in the stability problem arising from the presence of a stable solute gradient, magnetic field, and rotation, by multiplying Eq. (19) by W^* the complex conjugate of W, and integrating over the range of z, while utilizing Eqs. (20)–(25) along with the boundary conditions (26), we obtain the following:

$$-\left[\frac{\sigma}{\in}+\frac{1}{P_\ell}\right]I_1-\frac{F}{P_\ell}I_2+\frac{ga^2\alpha\chi}{\beta\nu}\left[I_3+Ep_1\sigma^*I_4\right]-\frac{ga^2\alpha'\chi'}{\beta'\nu}\left[I_5+E'q\sigma^*I_6\right]-\frac{ga^2\alpha''\chi''}{\nu\beta''}\left[I_6+E''q_2\sigma*I_7\right]$$

$$-\frac{\mu_e\in\eta}{4\pi\rho_0\nu}\left[I_7+p_2\sigma^*I_8\right]-\frac{d^2F}{P_\ell}I_9-d^2\left[\frac{\sigma^*}{\in}+\frac{1}{P_\ell}\right]I_{10}-\frac{\mu_e d^2\in\eta}{4\pi\rho_0\nu}I_{11}-\frac{\mu_e d^2\in\eta p_2\sigma}{4\pi\rho_0\nu}I_{12}=0, \tag{36}$$

where

$$I_1=\int_0^1(|DW|^2+a^2|W|^2)dz;\qquad I_2=\int_0^1(|D^2W|^2+2a^2|DW|^2+a^4|W|^2)dz\ ,$$

$$I_3=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)dz\ ;\qquad I_4=\int_0^1(|\Theta|^2)dz\ ,$$

$$I_5=\int_0^1(|D\Gamma|^2+a^2|\Gamma|^2)dz\ ;\qquad I_6=\int_0^1(|\Gamma|^2)dz\ ,\ I_7=\int_0^1|\psi|^2dz,$$

$$I_8=\int_0^1(|D^2K|^2+2a^2|DK|^2+a^4|K|^2)dz\ ;\ I_9=\int_0^1(|DK|^2+a^2|K|^2)dz\ ,$$

$$I_{10} = \int_0^1 (|DZ|^2 + a^2|Z|^2)dz \quad ; \quad I_{11} = \int_0^1 (|Z|^2)dz\ ,$$

$$I_{12} = \int_0^1 (|DX|^2 + a^2|X|^2)dz \quad ; \quad I_{13} = \int_0^1 (|X|^2)dz\ , \qquad (37)$$

Here, σ∗ represents the complex conjugate of σ. The integrals $I_1 - I_{13}$ are all positive definite. By expressing $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of Eq. (36), we obtain the following:

$$\left[-\frac{I_1}{\in} + \frac{ga^2\alpha\chi}{\beta\nu}Ep_1I_4 - \frac{ga^2\alpha'\chi'}{\beta'\nu}E'q_1I_6 + \frac{ga^2\alpha''\chi''}{\beta''\nu}E''q_2I_8 - \frac{\mu_e \in \eta}{4\pi\rho_0\nu}p_2I_8 - \frac{d^2}{\in}I_{10} - \frac{\mu_e d^2 \in \eta}{4\pi\rho_0\nu}p_2I_{12}\right]\sigma_r$$

$$= \frac{I_1}{P_\ell} + \frac{F}{P_\ell}I_2 - \frac{ga^2\alpha\chi}{\beta\nu}I_3 + \frac{ga^2\alpha'\chi'}{\beta'\nu}I_5 + \frac{ga^2\alpha''\chi''}{\beta''\nu}I_7\,\frac{\mu_e \in \eta}{4\pi\rho_0\nu}I_7 + \frac{d^2F}{P_\ell}I_9 + \frac{d^2}{P_\ell}I_{10} + \frac{\mu_e d^2 \in \eta}{4\pi\rho_0\nu}I_{11}, \qquad (38)$$

and

$$\left[\frac{I_1}{\in} + \frac{ga^2\alpha\chi}{\beta\nu}Ep_1I_4 - \frac{ga^2\alpha'\chi'}{\beta'\nu}E'q_1I_6 - \frac{ga^2\alpha''\chi'}{\beta''\nu}E'q_2I_8 - \frac{\mu_e \in \eta}{4\pi\rho_0\nu}p_2I_8 - \frac{d^2}{\in}I_{10} + \frac{\mu_e d^2 \in \eta}{4\pi\rho_0\nu}p_2I_{12}\right]\sigma_i = 0$$

. (39)

It is evident from Eq. (38) that σ_r can be either positive or negative, indicating that the system is stable or unstable, respectively. From Eq. (39), it is apparent that σi\sigma_iσi may be either zero or non-zero, implying that the modes could be non-oscillatory or oscillatory.

Specifically, Eq. (39) shows that σ_i is zero when the term multiplying it is non-zero, and arbitrary when this term equals zero. If $\sigma_i \neq 0$, then Eq. (39) yields:

$$-\frac{I_1}{\in} - \frac{\mu_e d^2 \in \eta}{4\pi\rho_0\nu}p_2I_{12} = \frac{ga^2\alpha\chi}{\beta\nu}Ep_1I_4 - \frac{ga^2\alpha'\chi'}{\beta'\nu}E'q_1I_6 - \frac{ga^2\alpha''\chi''}{\beta''\nu}E'q_2I_8 - \frac{\mu_e \in \eta}{4\pi\rho_0\nu}p_2I_8 - \frac{d^2}{\in}I_{10}. \qquad (40)$$

Substituting in Eq. (38), we have

$$\frac{2\sigma_r I_1}{\in} + \frac{1}{P_\ell}I_1 + \frac{F}{P_\ell}I_2 + \frac{ga^2\alpha'\chi'}{\beta'\nu}I_5 + \frac{ga^2\alpha''\chi''}{\beta''\nu}I_7 + \frac{\mu_e \in \eta}{4\pi\rho_0\nu}I_7 + \frac{d^2F}{P_\ell}I_9$$

$$+ \frac{d^2}{P_\ell}I_{10} + \frac{\mu_e d^2 \in \eta}{4\pi\rho_0\nu}I_{11} + \frac{\sigma_r\mu_e d^2 \in \eta}{2\pi\rho_0\nu}p_2I_{12} = \frac{ga^2\alpha\chi}{\beta\nu}I_3 \qquad (41)$$

Equation (41) after using Rayleigh –Ritz inequality becomes

$$\frac{(\pi^2 + a^2)^3}{a^2}\int_0^1 |W|^2dz + \frac{(\pi^2 + a^2)}{a^2}\frac{P_\ell}{F}\left\{\frac{\sigma_r\mu_e d^2 \in \eta}{2\pi\rho_0\nu}p_2I_{12} + \frac{\mu_e d^2 \in \eta}{4\pi\rho_0\nu}I_{11} + \frac{d^2}{P_\ell}I_{10} + \frac{d^2F}{P_\ell}I_9 + \right.$$

$$\left.\frac{\mu_e \in \eta}{4\pi\rho_0\nu}I_7 + \frac{ga^2\alpha'\chi'}{\beta'\nu}I_5 + \frac{ga^2\alpha''\chi''}{\beta''\nu}I_7 + \frac{1}{P_\ell}I_1 + \frac{2\sigma_r}{\in}I_1\right\} \leq \frac{g\alpha\chi}{\nu\beta}\frac{P_\ell}{F}\int_0^1 |W|^2dz\ . \qquad (42)$$

Therefore, It follows from Eq. (42) that

$$\left[\frac{27\pi^4}{4} - \frac{g\alpha\chi}{\nu\beta}\frac{P_\ell}{F}\right]\int_0^1 |W|^2dz + \frac{(\pi^2 + a^2)}{a^2}\frac{P_\ell}{F}\left\{\frac{\sigma_r\mu_e d^2 \in \eta}{2\pi\rho_0\nu}p_2I_{12} + \frac{\mu_e d^2 \in \eta}{4\pi\rho_0\nu}I_{11} + \frac{d^2}{P_\ell}I_{10} + \frac{d^2F}{P_\ell}I_9\right.$$

$$\left. + \frac{\mu_e \in \eta}{4\pi\rho_0\nu}I_7 + \frac{ga^2\alpha'\chi'}{\beta'\nu}I_5 + \frac{ga^2\alpha''\chi''}{\beta''\nu}I_7 + \frac{1}{P_\ell}I_1 + \frac{2\sigma_r I_1}{\in}\right\} \leq 0\ , \qquad (43)$$

since minimum value of $\frac{(\pi^2 + a^2)^3}{a^2}$ with respect to a^2 is $\frac{27\pi^4}{4}$.

Now, let $\sigma_r \geq 0$, we necessary have from (43) that

$$\frac{g\alpha\chi}{\nu\beta}\frac{P_\ell}{F} > \frac{27\pi^4}{4}\ . \qquad (44)$$

Hence, if

$$\frac{g\alpha\chi}{\nu\beta}\frac{P_\ell}{F}\le\frac{27\pi^4}{4}\,, \qquad (45)$$

If $\sigma_r < 0$, the system is stable. Therefore, under the condition given by Eq. (45), the system remains stable, whereas under the condition in Eq. (44), the system becomes unstable.

In the absence of a stable solute gradient, rotation, and magnetic field, Eq. (38) simplifies to:

$$\sigma_i\left(\frac{I_1}{\in}+\frac{ga^2\alpha\chi}{\nu\beta}Ep_1I_4\right)=0 \qquad (46)$$

In this case, the terms within the brackets are positive definite, leading to $\sigma_i = 0$. This implies that oscillatory modes are not permitted, and the principle of exchange of stabilities holds for the couple-stress fluid in a porous medium, provided there is no stable solute gradient, rotation, or magnetic field. The introduction of oscillatory modes occurs due to the presence of these factors—stable solute gradient, rotation, and magnetic field—which were not present in their absence.

6. CONCLUSIONS

This study investigates triple-diffusive convection in a couple-stress fluid heated and soluted from below in a porous medium, with a focus on the effects of a uniform vertical magnetic field and uniform rotation. Triple-diffusive convection is a complex phenomenon involving heat, mass, and solute transfer, with significant implications in fields such as oceanography, limnology, geophysics, and astrophysics. This behaviour is particularly relevant in systems like solar heat-trapping ponds and Antarctic lakes. The main conclusions from the analysis of this paper are as follows:

(a) Stationary Convection:

(i) Both the stable solute gradient and rotation have a stabilizing effect on the system, promoting a more stable convection state.

(ii) In the presence of rotation, the medium permeability exhibits a dual effect, either destabilizing or stabilizing the system, while the magnetic field and couple-stress parameter also show complex stabilizing or destabilizing behaviour.

(iii) In the absence of rotation, the medium permeability destabilizes the system, whereas the magnetic field and couple-stress parameter act to stabilize the fluid motion.

(b) Effect of Parameters:

From the analysis presented in Figures 1-5, it is evident that stable solute gradient and rotation consistently have stabilizing effects. Conversely, the magnetic field, couple-stress parameter, and medium permeability can both stabilize and destabilize the system under varying conditions.

(c) Oscillatory Modes: Finally, it is noteworthy that the stable solute gradient, rotation, and magnetic field introduce oscillatory modes in the system that was previously non-existent without these factors. This observation is crucial in understanding the dynamic behaviour of triple-diffusive convection under varying physical conditions.

(d) Stability Conditions: It is found that if $\frac{g\alpha\chi}{\nu\beta}\frac{P_l}{F}\le\frac{27\pi^4}{4}$, the system is stable and under the condition $\frac{g\alpha\chi}{\nu\beta}\frac{P_l}{F}>\frac{27\pi^4}{4}$, the system becomes unstable.

(e) Absence of Solute Gradient and Rotation: In the absence of stable solute gradient, rotation, and magnetic field, oscillatory modes are prohibited, and the principle of exchange of stabilities holds true, ensuring system stability.

These results highlight the intricate interplay of physical parameters in governing the stability and dynamics of triple-diffusive convection in porous media.

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