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Variance Gamma Model for Daily Stock Price Prediction (Case Study: Daily Stock Price of PT Industri Jamu dan Farmasi Sido Muncul Tbk)

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ARTICLE INFO	ABSTRACT

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Stocks have high volatility, with price fluctuations that are difficult to predict and often do not meet the assumption of a normal distribution. The Brownian Motion method is widely used in stock price modeling, but is unable to capture sharp spikes in leptokurtic data. The Variance Gamma model is applied by evaluating a modified Brownian Motion process using a random time following a Gamma process. This model has three parameters to control volatility, kurtosis, and skewness. This research analyzes the daily stock return data of PT Industri Jamu dan Farmasi Sido Muncul from December 2023 to December 2024. TheVariance Gamma parameter estimation using the Moment Method and Maximum Likelihood Method. Accuracy calculation uses Mean Absolute Percentage Error (MAPE). The results showed that the Variance Gamma model with the normal standard process approach of a gamma process produced a MAPE of 4.150345%. While the different approach of two independent Gamma processes has a MAPE of 4.515595%. Both methods are more accurate when compared to the Geometric Brownian Motion with Jump model, which has a MAPE of 6.866523%. Variance Gamma has a smaller MAPE value, so it is more suitable for modeling stock prices with jumps and not normally distributed.

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KEYWORDS: Stocks; *Brownian Motion*; Leptokurtic; *Variance Gamma*; Kurtosis; and Skewness.

I. INTRODUCTION

Investment refers to the allocation of funds or other resources with the objective of generating future returns. One of the most popular forms of investment is in financial assets, particularly securities, which represent claims on the assets of the issuing entities.

Stocks are a type of security that signify ownership or equity participation by individuals or institutions in a company, typically structured as a Limited Liability Company (Maruddani, 2019). However, stock investments are inherently risky due to their frequent and unpredictable price fluctuations. Stock price modeling aims to capture the stochastic nature of price movements by using historical data to forecast potential future values (Maruddani & Trimono, 2018).

Brownian Motion, or the Wiener process, is a continuous stochastic process where each increment follows a normal distribution. It plays a crucial role in probability theory and financial modeling (Dmouj, 2006). However, real-world stock prices often exhibit sharp jumps and heavy tails, causing deviations from the normality assumption. To address this limitation, the Variance Gamma (VG) process has been proposed. The VG model modifies Brownian Motion by incorporating a random time change governed by a gamma process, introducing flexibility through three key parameters: volatility (from Brownian motion), kurtosis (to control the distribution's peakedness), and skewness (to capture asymmetry in the return distribution) (Madan et al., 1998).

Previous studies have employed the Geometric Brownian Motion (GBM) model for stock price prediction. Abidin and Jafar (2014) demonstrated that GBM is particularly suitable for short-term forecasting, achieving a minimum Mean Absolute Percentage Error (MAPE) of 4.79% for two-week data intervals. In contrast, Hoyyi et al. (2021) applied the Variance Gamma model to daily stock prices of PT Bank BNI Tbk, which did not follow a normal distribution. Using Monte Carlo simulations, they explored two approaches: (1) the VG model where Brownian Motion with drift is subordinated by a gamma process (VG1), resulting in a MAPE of 5.8178%, and (2) the VG model using the difference of two gamma processes (VG2), yielding a MAPE of 5.7532%. Both VG models outperformed the GBM model, which produced a MAPE of 9.7083%, in modeling non-normally distributed stock data.

This study analyzes the daily stock return data of PT Industri Jamu dan Farmasi Sido Muncul Tbk from December 2023 to December 2024. The accuracy of the proposed model is evaluated using the Mean Absolute Percentage Error (MAPE) metric.

II. THEORETICAL FRAMEWORK

A. *Stock Price Return*

Stocks are defined as financial securities that represent ownership or equity participation by individuals or institutions in a company, typically structured as a Limited Liability Company (PT) (Maruddani, 2019). In financial analysis, the natural logarithm of returns (log returns or ln returns) is often preferred over simple returns because it possesses additive properties across different time intervals and provides more stable statistical characteristics.

To calculate the percentage change in stock prices between two time periods, the ln return can be expressed as follows:

$$r_t = ln\left(\frac{S_t}{S_{t-1}}\right)$$

Volatility refers to the degree of variation in stock prices over a specified period and is commonly used as a measure of investment risk. Higher volatility indicates a greater potential for substantial gains or losses. Assuming there are n return observations, the mean of the stock returns is calculated as follows:

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_t$$

The mean return is then used to estimate the return variance, and the square root of this variance provides an estimate of stock price volatility.

$$s^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2$$

B. *Skewness and Kurtosis*

Skewness is defined as a measure of the asymmetry of a dataset around its sample mean. Skewness quantifies the degree of symmetry in a distribution; a skewness value of zero indicates a perfectly symmetric distribution, which implies that the data follows a normal distribution (Maity & Saha, 2023). The formula for skewness is given as follows:

$$\theta = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{s^3}$$

Kurtosis measures the thickness of the tails in a distribution. A dataset is considered normally distributed if its kurtosis value is equal to 3 (Maity & Saha, 2023). A distribution with a kurtosis greater than 3 is referred to as leptokurtic, indicating a sharper peak and heavier tails. Conversely, if the kurtosis is less than 3, the distribution is called platykurtic, characterized by a flatter peak and lighter tails. The formula for kurtosis is expressed as follows:

$$v = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{s^4}$$

C. *Kolmogrov-Smirnov Normality Test*

The Kolmogorov–Smirnov (K–S) test is a formal statistical test used to determine whether a given dataset follows a specific distribution, particularly when the mean and variance are known. This test is considered more appropriate for large sample sizes (n > 40) (Biu et al., 2019). The following outlines the procedure for testing normality using the Kolmogorov–Smirnov test:

Hypotheses:
(data follows normal distribution)

$H_1: F(x_i) \neq F_0(x)$ (does not follow normal distribution)

Significance Level: α

Test Statistic:

$$D = max\ \{max\{|F(x_i) - F_0(x)|\ , |F(x_{i-1}) - F_0(x)|$$

where $F(x_i)$ is the cumulative probability of the empirical distribution and $F_0(x)$ is the cumulative probability of the normal distribution.

Decision Rule:

Reject H_0 if $D \geq d_{\left(1-\frac{\alpha}{2}\right)}$ or if the p-value is less than the significance level $P_{value} < \alpha$, where $d_{\left(1-\frac{\alpha}{2}\right)}$ is the critical value obtained from the Kolmogorov–Smirnov table.

D. *Brownian Motion*

Brownian Motion is a continuous stochastic process in which each increment is normally distributed. It is widely used in financial analysis to model the random behavior of asset prices (Dmouj, 2006).

1. Standard Brownian Motion

For $t > 0$, the random variable $W(t) = W(t) - W(0)$ represents the increment over the interval $[0, t]$, which follows a normal distribution with mean 0 and variance *t*. Thus, $W(t)$ is referred to as a standard Brownian motion, denoted as $W(t) \sim N(0, t)$.

2. Brownian Motion with Drift

According to Dmouj (2006), Brownian motion with a drift term can be expressed as follows:

$$B(t) = B(0) + \mu t + \sigma W(t)$$

with $B(t) \sim N(B(0)+\mu t, \sigma^2 t)$. Where $\{W(t), t>0\}$ denotes standard Brownian motion, $\mu(t)$ is the expected value, and σ is the standard deviation of the process at time t. The Brownian motion can also be written as $W(t) = Z\sqrt{t}$ where Z is a standard normal random variable.

3. Geometric Brownian Motion

The Brownian motion model with drift is defined as $B(t) = \mu^*(t) + \sigma W(t)\,; t \geq 0$ where parameter *drift* $\mu^* = \mu - \frac{1}{2}\sigma^2$, $\sigma^2$is the variance, and $W(t)$ is a standard Brownian motion starting from $W(0) = 0$. The Geometric Brownian Motion (GBM) model is then formulated as:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

That equation can be solved using *Itô's* Lemma, yielding the following closed-form solution:

$$S(t) = S(0)\,exp\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\,W(t)\right]$$

E. Variance Gamma Process

The Variance Gamma (VG) process is a pure jump process with an infinite rate of jump arrivals (Göncü et al., 2013). The Variance Gamma process is constructed by evaluating Brownian motion with drift and volatility under a random time change governed by a Gamma process (Madan et al., 1998). The Brownian motion with drift is given by:

$$b(t;\theta,\sigma) = \theta t + \sigma W(t)$$

Hence, the Variance Gamma process (X_{VG}) can be defined by subordinating Brownian motion with drift $b(t;\theta,\sigma)$ to a Gamma process $g \sim Gamma(1,v)$, such that:

$$X_{VG} = b(g;\theta,\sigma)$$
$$X_{VG} = \theta g + \sigma\sqrt{g}z$$

where $Z \sim N(0,1)$, and $g \sim Gamma(1,v)$. The VG process is characterized by three parameters: σ (volatility), v (variance of the Gamma process), and θ (drift).

F. Estimated Parameters of Variance Gamma

The Variance Gamma distribution is a subclass of the Generalized Hyperbolic distribution. Let $\mathbf{X} = (X_1, X_2, \ldots, X_d)^T$ be a random variable following a multivariate Variance Gamma distribution of dimension d. According to Weibel et al. (2011), for the univariate case $d=1$, the probability density function of the Variance Gamma distribution is given by:

$$f_X(x;\mu,\sigma,\theta,v) = C(\mu,\sigma,\theta,v) \times K_{\frac{1}{v}-\frac{1}{2}}\left(\frac{(x-\mu)\sqrt{\left(\frac{2\sigma^2}{v}+\theta^2\right)}}{\sigma^2}\right) exp\left(\frac{\theta(x-\mu)}{\sigma^2}\right)$$

where $K_v(\,)$ is the modified Bessel function of the second kind of order v, and:

$$C(\mu,\sigma,\theta,v) = \left((x-\mu)\right)^{\frac{1}{v}-\frac{1}{2}} \frac{2}{\sigma\sqrt{2\pi}\,v^{\frac{1}{v}}\,\Gamma\left(\frac{1}{v}\right)} \cdot \left(\frac{1}{\sqrt{\left(\frac{2\sigma^2}{v}+\theta^2\right)}}\right)^{\frac{1}{v}-\frac{1}{2}}$$

The parameters of the Variance Gamma process can be estimated using the Moments method or the Maximum Likelihood Estimation (MLE) approach. MLE involves estimating unknown parameters by maximizing the likelihood function over the parameter space Ω. According to Fragiadakis *et al.*, (2013), the log-likelihood function for the Variance Gamma model with a sample size *n* is defined as follows:

$$f_L = \ln L(\theta)$$

$$ln\,L(\theta) = -\frac{n}{2} ln\,\pi - n\,ln\,\sigma - n\,ln\,\Gamma\left(\frac{1}{v}\right) + \frac{\theta}{2\sigma}\sum_{j=1}^{n}(x_j-\mu)$$
$$-n\left(\frac{1}{v}-\frac{1}{2}\right) ln\left(\sigma\sqrt{\theta^2+4}\right) + \left(\frac{1}{v}-\frac{1}{2}\right)\sum_{j=1}^{n}|x_j-\mu|$$
$$+\sum_{j=1}^{n} ln\,K_{\frac{1}{v}-\frac{1}{2}}\left(\frac{(x_j-\mu)\sqrt{\theta^2+4}}{2\sigma}\right)$$

Parameter estimation for the Variance Gamma (VG) process can begin by determining the first four moments (m) of $X(t)$, yielding the following expressions:

$$E[X_{VG}] = \theta t$$
$$E[(X_{VG} - E[X_{VG}])^2] = (\theta^2 v + \sigma^2)t$$
$$E[(X_{VG} - E[X_{VG}])^3] = (2\theta^3 v^2 + 3\sigma^2\theta v)t$$
$$E[(X_{VG} - E[X_{VG}])^4] = (3\sigma^4 v + 12\sigma^2\theta^2 + 6\theta^4 v^3)t$$
$$+(3\sigma^4 + 6\sigma^2\theta^2 v + 3\theta^4 v^2)t^2$$

According to Seneta (2004), the higher-order moments satisfy $\theta^2 \approx \theta^3 \approx \theta^4 \approx 0$, which allows the VG parameters to be estimated accordingly:

$$\hat{\sigma} = \sqrt{Var(X_{VG})}$$

$$\hat{\theta} = \frac{\sigma\,Skewness(X_{VG})}{3v}$$

$$\hat{v} = \frac{Kurtosis(X_{VG})}{3} - 1$$

G. Variance Gamma Fit Test

The Chi-square test is a nonparametric statistical method used to evaluate the goodness-of-fit between observed data and an expected theoretical distribution. This test is more effective when applied to sufficiently large sample sizes, as smaller samples may lead to Type II errors —failing to reject a false null hypothesis (Rana & Singhal, 2015). The Chi-square test statistic is defined as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - Np_i)^2}{Np_i}$$

where o_i represents the observed frequency in the i-th subinterval, N is the sample size, and p_i is the theoretical probability for the i-th subinterval. The calculated Chi-square statistic is then compared with the critical value from the Chi-square distribution with $\chi_{\alpha-1-m}{}'$ degrees of freedom, where α is the significance level and m is the number of estimated parameters in the Variance Gamma model.

H. Variance Gamma Model on Stocks

In the Variance Gamma process, the deterministic time variable t in Brownian motion is replaced by a stochastic time variable from the Gamma process (Madan et al., 1998). This leads to the following stock price prediction model under the Variance Gamma framework:

$$S(t_i) = S(t_{i-1})\, exp[(\mu + \omega)t + X_{VG}]$$

with

$$\omega = \frac{1}{v} ln\left(1 - \theta v - \frac{1}{2}\sigma^2 v\right)$$

where μ is the mean of the log-returns of the stock price, σ is the volatility of the Brownian motion controlling the overall volatility, v is the variance of the Gamma time-change process which controls the kurtosis, and θ is the drift parameter in the Brownian motion which governs the skewness.

The Variance Gamma (VG) process, X_{VG}, can be defined using two approaches:

1. Variance Gamma process derived from a standard normal process time-changed by a Gamma process:

$$X_{VG1}(t_i) = \theta g + \sigma\sqrt{g}\, z$$

where $g \sim Gamma(1, v)$ and $Z \sim N(0,1)$ is a standard normal variable.

2. Variance Gamma process defined as the difference between two independent Gamma processes, g_r and g_s

$$X_{VG2}(t_i) = g_r(t) - g_s(t)$$

where g_r and g_s are independent Gamma processes with mean μ_r and μ_s, respectively, and a common variance parameter v (Madan et al., 1998).

I. Monte Carlo Simulation of the Variance Gamma

Monte Carlo Simulation is an algorithm that requires repetition or iteration and can predict error values based on the number of iterations (Priyantono et al., 2023). The Monte Carlo algorithm procedure with the Variance Gamma process is as follows:

(a) Simulation for $X_{VG1} = 0$, where the random time variable t in Brownian motion is replaced by a time variable in the Gamma process.

Input: Variance Gamma process parameters (θ, σ, v) and time intervals $(\Delta t_1, \dots, \Delta t_n$ with $\sum_{i=1}^{N} \Delta t_i = T$.

Initialization: Initial return value $S(t_0) = 0$

Loop: From i = 1 to N:

1. Generate $\Delta g \sim \Gamma\left(\frac{\Delta t_i}{v}, v\right), z \sim N(0,1)$, which are independent.
2. Calculate the return $S(t_i) = S(t_{i-1}) + \exp\left[(\mu + \omega)t + (\theta\Delta g + \sigma\sqrt{\Delta g}\, z)\right]$.

(b) Simulation for $X_{VG2} = 0$, where the difference between two Gamma processes is considered.

Input: Variance Gamma process parameters (θ, σ, v) and time intervals $(\Delta t_1, \dots, \Delta t_n$ with $\sum_{i=1}^{N} \Delta t_i = T$.

Initialization: Initial return value $S(t_0) = 0$

Loop: From i = 1 to N:

1. Generate $\Delta g_r \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_r\right), \Delta g_s \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_s\right)$ which are independent.
2. Calculate the return $S(t_i) = S(t_{i-1}) \exp[(\mu + \omega)t + (g_r - g_s)]$

J. Mean Absolute Percentage Error

Mean Absolute Percentage Error (MAPE) is a method that is well-suited for evaluating forecast accuracy based on the magnitude of the actual values. MAPE is considered effective for assessing prediction algorithms, with MAPE values below 10% indicating highly accurate predictions. The calculation of MAPE can be written as follows:

$$MAPE = \frac{\sum_{t=1}^{n}\left|\frac{X_t - F_t}{X_t}\right|}{n} \times 100\%$$

The criteria for MAPE in forecasting accuracy are explained in the following table:

Table I. Forecast Accuracy Levels with MAPE

MAPE Value	*Description*
MAPE ≤ 10%	Very good forecast accuracy
10% < MAPE ≤ 20%	Good forecast accuracy
20% < MAPE ≤ 50%	Forecast accuracy within acceptable limits
MAPE > 50%	Poor forecast accuracy

III. RESEARCH METHOD

This research uses secondary data from the closing prices of stocks obtained from the website https://finance.yahoo.com/. The data utilized consists of the daily closing prices of PT Industri Jamu dan Farmasi Sido Muncul Tbk (stock code: SIDO) from December 1, 2023, to December 2, 2024, with a total of 238 observations. The analysis method applied is the Variance Gamma (VG) Model. The steps for data analysis carried out in this study are as follows:

1. Collect historical closing price data.
2. Determine the in-sample and out-sample data.
3. Calculate the ln returns of the in-sample data.
4. Explore the data using descriptive analysis and create histograms of the data.
5. Test the normality of the ln returns of the stock using the Kolmogorov-Smirnov test.
6. Estimate parameters using the Maximum Likelihood method on the VG model.
7. Conduct a goodness-of-fit test for the Variance Gamma model using the Chi-square test.
8. Perform a Monte Carlo simulation to obtain the Variance Gamma process using the approach from the Gamma process and the Standard Normal distribution.
9. Model the data using the Variance Gamma model.
10. Make predictions using the out-sample data.
11. Calculate the prediction error using the Mean Absolute Percentage Error (MAPE) method.

IV. RESULT AND DISCUSSION

The in-sample data consists of 218 observations from December 1, 2023, to November 1, 2024, which were used for stock price modeling. The out-sample data consists of 20 observations from November 4, 2024 to December 2, 2024 and was used for model validation.

The distribution of the stock's ln returns can be examined through the values of skewness and kurtosis, as shown in the following descriptive statistics:

Table II. Descriptive Statistics of SIDO Ln Return

Statistic	Value
Minimum	-0,06203539
Maximum	0,07410797
Mean	0,000896905
Variance	0,0003452803
Standard Deviation	0,01858172
Skewness	0,5827563
Kurtosis	4,886596

Based on Table 2, the skewness value is 0.5827563, indicating that the data distribution is skewed to the right. The shape of the distribution's peakedness is observed in the kurtosis value, which is 4.886596 (kurtosis > 3), categorizing the data as leptokurtic, indicating a relatively high peak. With these kurtosis and skewness values, it can be assumed that the data does not follow a normal distribution and exhibits leptokurtic behavior.

The normality of the stock's ln returns was also visually assessed through a histogram, as shown below:

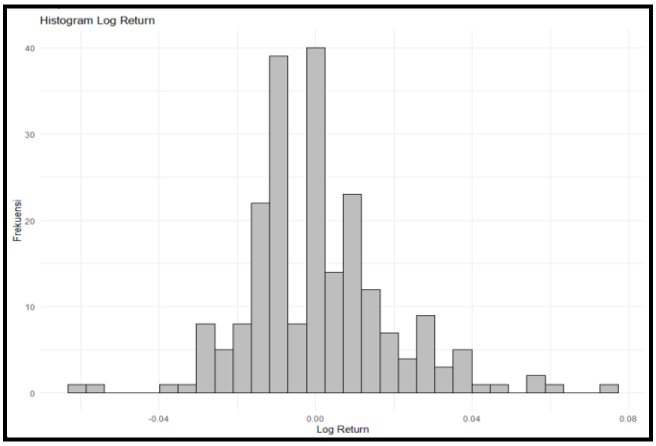

Figure 1. Histogram of Ln *Return* SIDO stock price

The histogram appears to resemble a symmetric shape (skewness = 0), and the kurtosis follows a leptokurtic distribution (kurtosis > 3), suggesting that the data does not follow a normal distribution. Next, the Kolmogorov-Smirnov test was conducted to formally test the normality assumption of the data.

Using a significance level of α = 5%, the test statistic obtained is D = 0.13676, which is greater than the critical value $D = 0,13676 > d_{(1-0,025)} = 0,08816$ and the P_{value} (0,0005967), which is less than α = 0.05. Thus, H_0 is rejected. This indicates that the distribution of the ln return of the daily SIDO stock price does not follow a normal distribution.

Using the method of moments, as detailed in Equations (14), (15), and (16), the parameter estimates for the Variance Gamma model are as follows:

The estimated value of $\hat{\sigma}$:

$$\hat{\sigma} = \sqrt{Var(X_{VG})} = 0{,}01858172$$

The estimated value of $\hat{v}$:

$$\hat{v} = \frac{Kurtosis(X_{VG})}{3} - 1 = 0{,}62886533$$

The estimated value of $\hat{\theta}$:

$$\hat{\theta} = \frac{\sigma\, Skewness(X_{VG})}{3v} = 0{,}00573976$$

Estimated parameter values were used as initial values for determining the parameters applied in the modeling process. With the assistance of RStudio software, the parameter estimates were obtained using the Maximum Likelihood Estimation (MLE) method as follows.

Table III. Estimated Parameters of VG

σ	θ	v
0,0176955	0,008385	0,928554

Manually, the goodness-of-fit of the data was evaluated using the Chi-square test with the following statistic test:

$$\chi^2 = \sum_{i=1}^{k} \frac{(o_i - Np_i)^2}{Np_i}$$

The test was performed at a significance level of α = 5%, with the decision rule: fail to reject H_0 if $\chi^2_{hit} \le \chi^2_{0,05;3}$ or if the $P_{value} \ge 0.05$. The test results yielded a calculated Chi-square value of $\chi^2_{hit} = 7{,}402525 < \chi^2_{0,05;3} = 7.81$ which is less than the critical value and a P_{value} of $0{,}06011662$, which is greater than α = 0.05. Therefore, the null hypothesis H_0 cannot be rejected, indicating that the daily ln return of SIDO stock prices follows the Variance Gamma distribution.

There are two modeling approaches to predict stock prices using the Variance Gamma process:

1. Approach Based on Brownian Motion with Drift under Gamma Time-Changed Framework

The Variance Gamma model X_{VG1} is derived from a standard normal process subordinated to a Gamma process and is expressed as:

$$X_{VG1} = \theta g + \sigma\sqrt{g}\, z$$

where g is generated from a Gamma process with shape parameter $\left(\frac{\Delta t_i}{v}\right) = \left(\frac{1}{0.92944}\right) = 1{,}075917$ and scale parameter $v = 0.92944$. The variable $z \sim N(0,1)$ is a standard normal random variable, independently generated from g. Using the Maximum Likelihood Estimation (MLE) method, the estimated parameters are θ = 0,0083863 and parameter σ = 0,0176947, dan parameter $v = 0.92944$. Thus, the stock price prediction model based on the Variance Gamma process is given by:

$$S(t_i) = S(t_{i-1}) \exp[(-0{,}00856 + 0{,}0008969)t + (0{,}0083863g + 0{,}0176947\sqrt{g}\, z)]$$

where

$$\omega = \frac{1}{0.92944} ln\left(1 - (0{,}0083863)(0.92944) - \frac{1}{2}(0{,}0176947)^2(0.92944)\right)$$

$$= -0{,}00856$$

2. Approach Based on Difference of Two Independent Gamma Processes

$$X_{VG2} = g_r - g_s$$

where g_r and g_s are two independent Gamma processes with means μ_r and μ_s, respectively, and share a common variance parameter v.

$$\mu_r = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{v}} + \frac{\theta}{2},$$

$$= \frac{1}{2}\sqrt{0{,}008385^2 + \frac{0{,}017696^2}{0{,}928097}} + \frac{0{,}008385}{2}$$

$$= 0{,}014289$$

$$\mu_s = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{v}} - \frac{\theta}{2}$$

$$= \frac{1}{2}\sqrt{0{,}008385^2 + \frac{0{,}017696^2}{0{,}928097}} - \frac{0{,}008385}{2}$$

$$= 0{,}005904$$

Next, Gamma processes are generated as follows: $\Delta g_r \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_r\right), \Delta g_s \sim \Gamma\left(\frac{\Delta t_i}{v}, v\mu_s\right)$, with $v = 0{,}62886533$ and $\Delta t_i = 1$. These are then substituted into the following model equation:

$$S(t_i) = S(t_{i-1}) \exp[(-0{,}00856 + 0{,}0008969)t + (g_r - g_s)]$$

Mean Absolute Percentage Error (MAPE) is employed as a method to evaluate and interpret the prediction accuracy. Using the RStudio software, the MAPE results are as follows:

Table IV. MAPE Results of VG and GBM with Jumps

$VG1$	$VG2$	*GBM with Jump*
4,153509%	4,51862%	6,866523%

In addition to the Variance Gamma model, predictions were also carried out using the Geometric Brownian Motion (GBM) with Jump model as a benchmark, which is capable of handling non-normally distributed data and potential outliers. Based on Table 4, all three MAPE values fall within the interval MAPE ≤ 10%, indicating that the forecasting accuracy of each model can be categorized as very good.

V. CONCLUSION

The Variance Gamma process is suitable for modeling stock price data that exhibit sudden jumps and do not satisfy the assumption of normal distribution, as indicated by a kurtosis value greater than 3. Two predictive models based on Variance Gamma were constructed using Monte Carlo simulation with two different approaches.

The first approach, X_{VG1}, is derived from a standard normal process subordinated by a Gamma process, yielding the following prediction model:

$$S(t_i) = S(t_{i-1})\exp\left[(-0{,}00856 + 0{,}0008969)t + (0{,}0083863g + 0{,}0176947\sqrt{g}\ z)\right]$$

The second approach is based on the difference between two independent Gamma processes with parameters $\mu_r = 0{,}014289$; $\mu_s = 0{,}005904$, and Gamma random variables. The resulting predictive model is:

$$S(t_i) = S(t_{i-1})\exp[(-0{,}00856 + 0{,}0008969)t + (g_r - g_s)]$$

The study showed that the Variance Gamma model using the standard normal–Gamma process approach produced a MAPE of 4.150345%, while the approach based on the difference of two independent Gamma processes yielded a MAPE of 4.515595%. Both models outperformed the Geometric Brownian Motion with Jump model, which had a MAPE of 6.866523%. Therefore, the Variance Gamma model is more accurate and appropriate for modeling stock prices with sharp jumps and non-normal distribution characteristics.

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