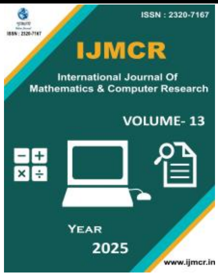
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 13 Issue 04 April 2025, Page no. – 5084-5086

Index Copernicus ICV: 57.55, Impact Factor: 8.615

DOI: 10.47191/ijmcr/v13i4.09



Relaxed Skolem Mean Labeling of 4 - Star Graph with Partition (3,1)

Dr. D. Angel Jovanna

Assistant Professor, Department of Mathematics, Wavoo Wajeeha Women's college of arts and science, kayalpatnam, Tuticorin, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, 627 012.

ARTICLE INFO	ABSTRACT
Published Online: 16 April 2025 **Corresponding Author:** Dr. D. Angel Jovanna	To prove that the 4 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta}$ where $\alpha_1 \leq \alpha_2 \leq \alpha_3$ is a relaxed skolem mean graph if $\beta - \alpha_1 - \alpha_2 - \alpha_3 = 6$ is the core objective of this article.

KEYWORDS: Relaxed skolem mean graphs, relaxed skolem mean labeling, 4-star graph.

1. INTRODUCTION

Relaxed skolem mean label for a graph was defined and coined by V. Balaji et. al. [5]. In the paper [5] he defined the relaxed skolem mean labeling for the first time. In the same paper we can find the basic properties for a graph to be relaxed skolem mean.

2. PRELIMINARIES

Definition 2.1: A graph G = (V, E) with p vertices and q edges is said to be a **skolem mean graph** if there exists a function $f : V \to \{1,2,3,\ldots,p = |V|\}$ such that the induced map $f^* : E \to \{2,3,\ldots,p = |V|\}$ given by

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } \big(f(u)+f(v)\big) \text{is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } \big(f(u)+f(v)+1\big) \text{is even} \end{cases}$$

then, the resulting distinct edge labels are from the set $\{2,3,\ldots,p = |V|\}$.

Definition 2.2 [5]: A graph $G = (V, E)$ with p vertices and q edges is said to be a relaxed skolem mean graph if there exists a function $f : V \to \{1,2,3,\ldots,p+1 = |V|+1\}$ such that induced edge map $\boldsymbol{f^*: E \to \{2,3,\ldots,p+1 = |V|+1\}}$ given by

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } \big(f(u)+f(v)\big) \text{is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } \big(f(u)+f(v)+1\big) \text{is even} \end{cases}$$

. The resulting distinct edge labels are from the set $\{2,3,\ldots,p+1 = |V|+1\}$.

Note 2.3: There are p vertices and available vertex labels are p + 1 and hence one number from the set $\{1,2,3,\ldots,p+1 = |V|+1\}$ is not used and we call that number as the relaxed label. When the relaxed label is p + 1, the relaxed mean labeling becomes a skolem mean labeling.

Result 2.4: In the relaxed skolem mean labeling p $\geq$ q.

Result 2.5: The three-star graph $K_{1,a} \cup K_{1,b} \cup K_{1,c}$ satisfies relaxed skolem mean labeling if $a + b \leq c \leq a + b + c$.

3. MAIN RESULT

Theorem 3.1: The 4-star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta}$ where $\alpha_1 \leq \alpha_2 \leq \alpha_3$ is a relaxed skolem mean graph if $\beta - \alpha_1 - \alpha_2 - \alpha_3 = 6$.

Proof: Let $\sigma_1 = \alpha_1; \sigma_2 = \alpha_1 + \alpha_2; \sigma_3 = \alpha_1 + \alpha_2 + \alpha_3$

Consider the 4 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta}$

The condition $\beta - \alpha_1 - \alpha_2 - \alpha_3 = 6$ gives rise to the case $\beta = \sigma_3 + 6$ In this case we will establish that the graph G is relaxed skolem mean. Let $V = V_1 \cup V_2 \cup V_3 \cup V_4$ be the vertex set of G where $V_k = \left\{v_{k,i} : 0 \leq i \leq \alpha_k\right\}$; for $1 \leq k \leq 3$ and $V_4 = \left\{v_{4,i} : 0 \leq i \leq \beta\right\}$. Let $E = \bigcup_{k=1}^{3}\{v_{k,0}v_{k,i} : 1 \leq i \leq \alpha_k\} \cup \{v_{4,0}v_{4,i} : 1 \leq i \leq \beta\}$ be the edge set of G.

Case 1: Let $\beta = \sigma_3 + 6$

G has $\sigma_3 + \beta + 4 = 2\sigma_3 + 10$ vertices and $\sigma_3 + \beta = 2\sigma_3 + 6$ edges.

We define the rsv function (relaxed skolem mean vertex function)

$f : V \to \{1, 2, ..., p+1 = \sigma_3 + \beta + 4 + 1 = 2\sigma_3 + 11\}$ as:

$f(v_{1,0}) = 1; \quad f(v_{2,0}) = 3; \quad f(v_{3,0}) = 5;$

$f(v_{4,0}) = \sigma_3 + \beta + 5 = 2\sigma_3 + 10$

$f(v_{1,\kappa}) = 2\kappa + 5 \qquad 1 \leq \kappa \leq \alpha_1$

$f(v_{2,\kappa}) = 2\sigma_1 + 2\kappa + 5 \qquad 1 \leq \kappa \leq \alpha_2$

$f(v_{3,\kappa}) = 2\sigma_2 + 2\kappa + 5 \qquad 1 \leq \kappa \leq \alpha_3$

$f(v_{4,\kappa}) = 2\kappa \qquad 1 \leq \kappa \leq \beta - 2 = \sigma_3 + 4$

$f(v_{4,\beta-1}) = \sigma_3 + \beta + 2 = 2\sigma_3 + 9$

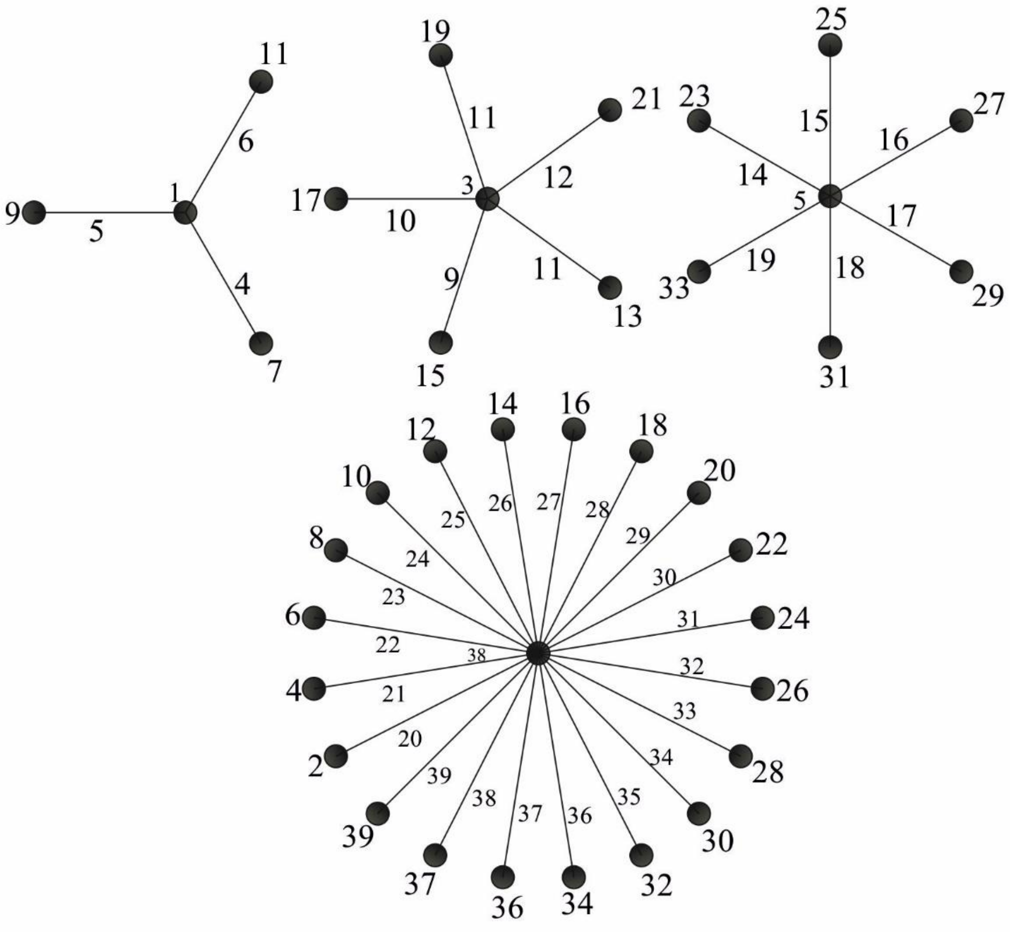
$f(v_{4,\beta}) = \sigma_3 + \beta + 4 = 2\sigma_3 + 11$

Here $2\sigma_3 + 7$ is the relaxed label.

We get the induced edge labels as follows:

The edge labels of $v_{1,0}v_{1,\kappa}$ is $\kappa + 3$ for $1 \leq \kappa \leq \alpha_1$ (edge labels are $4, 5, ..., \alpha_1 + 3 = \sigma_1 + 3$), $v_{2,0}v_{2,j}$ is $\sigma_1 + \kappa + 4$ for $1 \leq \kappa \leq \alpha_2$ ($\sigma_1 + 5$, $\sigma_1 + 6, ...,$ $\sigma_1 + \alpha_2 + 4 = \sigma_2 + 4$), $v_{3,0}v_{3,\kappa}$ is $\sigma_2 + \kappa + 5$ for $1 \leq \kappa \leq \alpha_3$ ($\sigma_2 + 6$, $\sigma_2 + 7, ...,$ $\sigma_2 + \alpha_3 + 5 = \sigma_3 + 5$), $v_{4,0}v_{4,\kappa}$ is $\sigma_3 + \kappa + 5$ for $1 \leq \kappa \leq \beta - 2 = \sigma_3 + 4$ ($\sigma_3 + 6$, $\sigma_3 + 7, ...,$ $\sigma_3 + (\sigma_3 + 4) + 5 = 2\sigma_3 + 9$), $v_{4,0}v_{\beta-1}$ is $2\sigma_3 + 10$ and $v_{4,0}v_{\beta}$ is $2\sigma_3 + 11$.

The edge labels are therefore $4, 5, ..., \sigma_1 + 3$, $\sigma_1 + 5, \sigma_1 + 6, ..., \sigma_2 + 4$, $\sigma_2 + 6, \sigma_2 + 7, ..., \sigma_3 + 5$, $\sigma_3 + 6, \sigma_3 + 7, ..., 2\sigma_3 + 9$, $2\sigma_3 + 10$ and $2\sigma_3 + 11$.



$K_{1,3} \cup K_{1,5} \cup K_{1,6} \cup K_{1,20}$

Example

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