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Certain q-Kober fractional integral operator of generalized basic hypergeometric functions and q-polynomials

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Abstract

The object of this paper is to established the Kober fractional integral operator of the generalized basic hypergeometric function.Interestingly Kober fractional integral operator of various q-polynomials have been expressed in terms of the basic analogue of Kampė de Fėriet function.Some special cases have been deduced as an application of main result.

Keywords:

Generalized basic hypergeometric functions, Kober fractional integral operator, Kampė de Fėriet function.

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1 Introduction

For $q < 1$ and real or compleex a, the q-shifted factorial is defined as

$$(a,q)_n = \begin{cases} 1 & \text{if } n = 0; \\ (1-a)(1-aq)(1-aq^2)\dots(1-aq^{n-1}) & \text{if } n \in N. \end{cases} \tag{1.1}$$

The q-gamma function is given as

$$\Gamma_q(z) = \frac{(q;q)_\infty}{(q^z;q)_\infty}(1-q)^{1-z}. \tag{1.2}$$

In terms of gamma the equation (1) can be expressed as

$$(a;q)_n = \frac{\Gamma_q(a+n)}{\Gamma_q(a)}(1-q)^n. \tag{1.3}$$

Also

$$(x-y)_\nu = x^\nu \left(\frac{y}{x};q\right)_\nu \tag{1.4}$$

The genaralized basic hypergeometric Serie $[2,(1.2.22),p.4]$ is defined as

$${}_r\phi_s\left[\begin{array}{c} a_1,a_2,\ldots,a_r \\ ;q,z \\ b_1,b_2,\ldots,b_s \end{array}\right]$$

$$= \sum_{n=0}^{\infty} \frac{(a_1;q)_n(a_2;q)_n\ldots(a_r;q)_n}{(q;q)_n(b_1;q)_n(b_2;q)_n\ldots(b_s;q)_n}\left[(-1)^n q^{\binom{n}{2}}\right]^{1+s-r} z^n. \tag{1.5}$$

The Basic analogue of Kober fractional integral operator (cf.Agarwal [1]) is given by

$$I_q^{\eta,\alpha}\{f(x)\} = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)}\int_0^x (x-tq)_{\alpha-1}t^\eta f(t)d_qt. \tag{1.6}$$

where $\Re(\alpha)>0$,$|q|<1$ and $\eta \in R$

Also the basic integral $(cf.Gasper\&Rahman[3])$ is given by

$$\int_9^x f(x)d_qt = x(1-q)\sum_{k=0}^{\infty} q^k f(zq^k). \tag{1.7}$$

In view of (1.7) the equation (1.6) can be written as

$$I_q^{\eta,\alpha}\{f(x)\} = \frac{-q^{-\alpha}x^{1-\eta}z^{\eta-1}(1-q)\left(\frac{zq}{x};q\right)_\alpha}{\Gamma_q(\alpha)}\sum_{r=0}^{\infty}\frac{\left(\frac{z}{x}q^{1+\alpha};q\right)_r q^{\eta r}}{\left(\frac{zq}{x};q\right)_r(1-\frac{x}{z}q^{-(n+r)})}f(zq^r). \tag{1.8}$$

which on replacing x by z we have

$$I_q^{\eta,\alpha}\{f(z)\} = \frac{-q^{-\alpha}(1-q)(q;q)_\alpha}{\Gamma_q(\alpha)}\sum_{r=0}^{\infty}\frac{(q^{1+\alpha};q)_r q^{\eta r}}{(q;q)_r(1-q^{-(n+r)})}f(zq^r). \tag{1.9}$$

Heine's q-analogue of Gauss summation formula is given by

$${}_2\phi_1(a,b;c;q,c/ab) = \frac{(c/a,c/b;q)_\infty}{(c,c/ab;q)_\infty}. \tag{1.10}$$

the above formula is valid for $|c/ab| < 1$.For the terminating case when $a = q^{-n}$, (1.10) reduces to

$$ {}_2\phi_1(q^{-n}, b; c; q, cq^n/b) = \frac{(c/b;q)_\infty}{(c;q)_\infty}. \tag{1.11}$$

By changing the order of summation it follows from (1.11) that

$$ {}_2\phi_1(q^{-n}, b; c; q, q) = \frac{(c/b;q)_\infty}{(c;q)_\infty} b^n. \tag{1.12}$$

The q-binomial theorem due to Cauchy [1843], Heine[1847] is given by

$$ {}_1\phi_0(a; -; q, z) = \frac{(az;q)_\infty}{(z;q)_\infty} \quad |z| < 1, |q| < 1. \tag{1.13}$$

The basic analogue of Kampė de Fėriet function (cf.Srivastava and Karlson[3]) is defined as

$$\phi^{A:B;B'}_{C:D;D'}\left[\begin{matrix} (a):(b);(b') \\ \\ (c):(d);(d') \end{matrix} ;q,x,y\right]$$

$$= \sum_{m,n\geq 0} \frac{\prod_{p=1}^{A}(a_p;q)_{m+n}\prod_{p=1}^{B}(b_p;q)_m\prod_{p=1}^{B'}(b'_p;q)_n}{\prod_{p=1}^{C}(c_p;q)_{m+n}\prod_{p=1}^{D}(d_p;q)_m\prod_{p=1}^{D'}(d'_p;q)_n} \frac{x^m x^n}{(q;q)_m(q;q)_n} \tag{1.14}$$

where $|x| < 1, |y| < 1$ and $0 < |q| < 1$.

The abnormal type of genaralized basic hypergeometric Serie is defined as

$$ {}_r\phi_s\left[\begin{matrix} a_1, a_2, \ldots, a_r; q, z \\ \\ b_1, b_2, \ldots, b_s; q^\lambda \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \ldots, a_r; q)_n}{(q, b_1, b_2, \ldots b_s; q)_n} z^n q^{\lambda n(n-1)/2}. \tag{1.15}$$

where $\lambda \geq 0$.

In the proof of the main result, I will use the following definition of various q-polynomials in the proof of the main results: The q-Laguerre polynomials

$$L_n^{(\alpha)}(x;q) = \frac{(q^{\alpha+1};q)_n}{(q;q)_n}\ {}_1\phi_1\left[\begin{matrix} q^{-n} \\ \\ q^{\alpha+1} \end{matrix} ;q,-xq^n\right] \tag{1.16}$$

The Wall polynomials (or Little q-Laguerre polynomial)

$$W_n(x;b,q) = (-1)^n(b;q)_n q^{n(n+1)/2}\ {}_2\phi_1\left[\begin{matrix} q^{-n}, 0 \\ \\ b \end{matrix} ;q,x\right] \tag{1.17}$$

$$\Gamma_n^{\alpha,\beta}(z;q) = \frac{(q^{\alpha+1};q)_n}{(q;q)_n}\ {}_2\phi_1\left[\begin{matrix} q^{-n}, q^{\alpha+\beta+n+1} & \\ & ;q,-xq \\ q^{\alpha+1} & \end{matrix}\right] \tag{1.18}$$

The Stieltjes-Wigert polynomials

$$S_n(x;q) = {}_1\phi_0\left[\begin{matrix} q^{-n} & ;q,-x \\ - & ;q^1 \end{matrix}\right] \tag{1.19}$$

The q-Hahn polynomials

$$Q_n(x;\alpha,\beta,\nu;q) = \ {}_3\phi_2\left[\begin{matrix} q^{-n}, \alpha\beta q^{n+1}, x & \\ & ;q,q \\ \alpha q, \quad \nu q & \end{matrix}\right] \tag{1.20}$$

The q-Meixner polynomials

$$M_n(x;\beta,\gamma;q) = (\beta;q)_n\ {}_2\phi_1\left[\begin{matrix} q^{-n} \ ,x & \\ & ;q,\frac{q^{n+1}}{\gamma} \\ \beta & \end{matrix}\right] \tag{1.21}$$

The q-Charlier polynomials

$$C_n(x;\alpha;q) = \ {}_2\phi_1\left[\begin{matrix} q^{-n} \ ,x & \\ & ;q,-\frac{q^{n+1}}{\alpha} \\ 0 & \end{matrix}\right] \tag{1.22}$$

The q-Krawtchouk polynomials

$$K_n(x;a,N;q) = \ {}_3\phi_2\left[\begin{matrix} q^{-n}, -a^{-1}q^n, x & \\ & ;q,q \\ q^{-N}, \quad 0 & \end{matrix}\right] \tag{1.23}$$

The q-Konhauser polynomials of first Kind

$$Z_n^{\{\alpha\}}(z;k;q) = \frac{(q^{\alpha+1};q)_{nk}}{(q^k;q^k)_n}\sum_{i=0}^{n}\frac{(q^{-nk};q^k)_i}{(q^k;q^k)_i}\frac{q^{(1/2)ki(k-1)+ki(n+\alpha+1)}}{(q^{\alpha+1};q)_{ik}}x^{ik} \tag{1.24}$$

The q-Konhauser polynomials of second Kind

$$Y_n^{\{\alpha\}}(z;k;q) = \frac{1}{(q;q)_n}\sum_{i=0}^{n}\frac{q^{r(r-1)/2}x^r}{(q;q)_r}\sum_{i=0}^{r}\frac{(q^{-r};q)_i(q^{1+\alpha+i};q^k)_n q^i}{(q;q)_i} \tag{1.25}$$

The bibasic form of the q-analogue of the Bedient polynomials of first kind is given by:

$$R_n(\alpha,\beta;q,x)=\frac{(\alpha;q)_n(2x/\sqrt{\alpha})^n}{(q;q)_n}\,{}_3\phi_2\left[\begin{matrix} q^{-n},q^{-n+1}:\beta/\alpha \\ \\ -\quad :\beta,q^{1-n}/\alpha\end{matrix};q^2,q,(q/x)^n\right] \tag{1.26}$$

The bibasic form of the q-analogue of the Bedient polynomials of second kind is given by:

$$G_n(\alpha,\beta;q,x)=\frac{(\alpha;q)_n(2x)^n}{(q;q)_n(\alpha\beta;q)_n}\,{}_3\phi_2\left[\begin{matrix} q^{-n},q^{-n+1}:\quad q^{1-n}/\alpha\beta \\ \\ -\quad :q^{1-n}/\alpha,q^{1-n}/\beta\end{matrix};q^2,q,(1/x)^n\right] \tag{1.27}$$

and the Gegenbaur polynomials

$$R_n(\alpha,\beta;q,x)=\frac{(\alpha;q)_n(2x/\sqrt{\alpha})^n}{(q;q)_n}\,{}_2\phi_1\left[\begin{matrix} q^{-n},q^{-n+1}:- \\ \\ -\quad :q^{1-n}/\alpha\end{matrix};q^2,q,(q/\alpha x)^n\right] \tag{1.28}$$

The generalized multibasic hypergeometric function is defined as:

$$\phi\left[\begin{matrix} a_1,a_2,\ldots,a_r:c_{11},c_{12},\ldots,c_{1r_1};\ldots;c_{m1},c_{m2},\ldots,c_{mr_m} \\ \\ b_1,b_2,\ldots,b_s:d_{11},d_{12},\ldots,d_{1s_1};\ldots;d_{m1},d_{m2},\ldots,d_{ms_m}\end{matrix};q,q_1,q_2,\ldots,q_m;z\right]$$

$$=\sum_{n=0}^{\infty}\frac{(a_1,a_2,\ldots,a_r;q)_n}{(q,b_1,b_2,\ldots,b_s;q)_n}z^n\prod_{j=1}^{m}\frac{(c_{j1},c_{j2},\ldots,c_{jr_j};q_j)_n}{(d_{j1},d_{j2},\ldots,d_{js_j};q_j)_n}. \qquad (1.29$$

where $0<|q_j|,|q|<1$, $|z|<1$.

2 Main Results

$$I_q^{\eta,\alpha}\left\{(1-q^{-(\alpha+r)})_r\phi_s\left[\begin{matrix} a_1,a_2,\ldots,a_r \\ \\ b_1,b_2,\ldots,b_s\end{matrix};q,z\right]\right\}$$

$$= \frac{-q^{-\alpha}(1-q)(q;q)_\alpha (q^{1+\eta+\alpha};q)_\infty}{\Gamma_q(\alpha)(q^\eta;q)_\infty} \, {}_{r+1}\phi_{s+1} \left[\begin{matrix} a_1, a_2, \ldots, a_r, q^\eta \\ b_1, b_2, \ldots, b_s, q^{1+\eta+\alpha} \end{matrix} ; q, z \right] \tag{2.1}$$

$$I_q^{\eta,\alpha} \left\{ \frac{(1-q^{-(n+r)})(q^{1+\eta};q)_r}{(q^{2(1+\eta)+\alpha};q)_r} \right\} = \frac{-q^{-\alpha}(1-q)(q;q)_\alpha}{\Gamma_q(\alpha)} \prod \left[\begin{matrix} q^{1+2\eta}, q^{1+\alpha+\eta}; q \\ q^{2(1+\eta)+\alpha}, q^\eta \end{matrix} \right] \tag{2.2}$$

$$I_q^{\eta,\alpha} \left\{ \frac{(1-q^{-(n+r)})(q^{-r};q)_r q^{n(1-\eta)}}{(q^{1+\alpha+\eta};q)_r} \right\} = \frac{-q^{\alpha}(1-q)(q;q)_\alpha (q^\eta;q)_r}{\Gamma_q(\alpha)(q^{1+\alpha+\eta};q)_r} q^{r(1+\alpha)} \tag{2.3}$$

$$I_q^{\eta,\alpha} \left\{ (1-q^{-(r+\alpha)}) Q_n(z;\alpha,\beta,\nu;q) \right\}$$

$$= \frac{-(1-q)(q;q)_\alpha}{q^\alpha \Gamma_q(\alpha)} \, \phi_{0:1;2}^{1:1;2} \left[\begin{matrix} z : q^{1+\alpha}; q^{-n}, \alpha\beta q^{n+1} \\ - : z \quad ; \alpha q, \nu q \end{matrix} ; q, q^\eta, q \right] \tag{2.4}$$

$$I_q^{\eta,\alpha} \left\{ (1-q^{-(r+\alpha)}) M_n(z;\beta,\gamma;q) \right\}$$

$$= \frac{-(1-q)(q;q)_\alpha (\beta;q)_n}{q^\alpha \Gamma_q(\alpha)} \, \phi_{0:1;1}^{1:1;1} \left[\begin{matrix} z : q^{1+\alpha}; q^{-n} \\ - : z \quad ; \beta \end{matrix} ; q, q^\eta, \frac{q^{n+1}}{\gamma} \right] \tag{2.5}$$

$$I_q^{\eta,\alpha} \left\{ (1-q^{-(r+\alpha)}) C_n(z;\alpha;q) \right\}$$

$$= \frac{-(1-q)(q;q)_\alpha}{q^\alpha \Gamma_q(\alpha)} \, \phi_{0:1;0}^{1:1;1} \left[\begin{matrix} z : q^{1+\alpha}; q^{-n} \\ - : z \quad ; - \end{matrix} ; q, q^\eta, -\frac{q^{n+1}}{\alpha} \right] \tag{2.6}$$

$$I_q^{\eta,\alpha} \left\{ (1-q^{-(r+\alpha)}) K_n(z;a,N;q) \right\}$$

$$= \frac{-(1-q)(q;q)_\alpha}{q^\alpha \Gamma_q(\alpha)} \, \phi_{0:1;2}^{1:1;2} \left[\begin{matrix} z : q^{1+\alpha}; q^{-n}, -a^{-1}q^n \\ - : z \quad ; q^{-N}, \ 0 \end{matrix} ; q, q^\eta, q \right] \tag{2.7}$$

$$I_q^{\eta,\alpha} \left\{ (1-q^{-j-\alpha}) Z_n^{\{\alpha\}}(z;k;q) \right\}$$

$$= \frac{-(1-q)(q;q)_\alpha (q^{1+\alpha+\eta};q)_\infty}{q^\alpha \Gamma_q(\alpha)(q^\eta;q)_\infty} \frac{(q^{\alpha+1};q)_{nk}}{(q^k;q^k)_n} \sum_{i=0}^{n} \frac{(q^{-nk};q^k)_i (q^\eta;q)_{ik}}{(q^k;q^k)_i (q^{1+\alpha+\eta};q)_{ik}} \frac{q^{(1/2)ki(k-1)+ki(n+\alpha+1)}}{(q^{\alpha+1};q)_{ik}} z^{ik} \tag{2.8}$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-j-\alpha})Y_n^{\{\alpha\}}(z;k;q)\right\}$$
$$= \frac{-(1-q)(q;q)_\alpha (q^{1+\alpha+\eta};q)_\infty}{q^\alpha \Gamma_q(\alpha)(q^\eta;q)_\infty (q;q)_n} \sum_{i=0}^{n} \frac{(q^\eta;q)_r q^{r(r-1)/2} x^r}{(q;q)_r (q^{1+\alpha+\eta};q)_r} \sum_{i=0}^{r} \frac{(q^{-r};q)_i (q^{1+\alpha+i};q^k)_n q^i}{(q;q)_i} \tag{2.9}$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-j-\alpha})R_n(\alpha,\beta;q,x)\right\} = \frac{-(1-q)(q;q)_\alpha (q^{1+\alpha+\eta};q)_\infty (\alpha;q)_n (2z/\sqrt{\alpha})^n}{q^\alpha \Gamma_q(\alpha)(q^\eta;q)_\infty (q;q)_n}$$
$$\times {}_4\phi_3\left[\begin{matrix} q^{-n}, q^{-n+1} : \beta/\alpha, \quad q^{-\alpha-\eta} \\ \\ - \quad : \beta, q^{1-n}/\alpha, q^{1-\eta} \end{matrix} ; q^2, q, \frac{q^{n+\alpha+1}}{x^n}\right] \tag{2.10}$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-j-\alpha})G_n(\alpha,\beta;q,x)\right\} = \frac{-(1-q)(q;q)_\alpha (q^{1+\alpha+\eta};q)_\infty (\alpha;q)_n (2z)^n}{q^\alpha \Gamma_q(\alpha)(q^\eta;q)_\infty (q;q)_n (\alpha\beta;q)_n}$$
$$\times {}_4\phi_3\left[\begin{matrix} q^{-n}, q^{-n+1} : q^{1-n}/\alpha\beta, \quad q^{-\alpha-\eta} \\ \\ - \quad : q^{1-n}/\alpha, q^{1-n}/\beta, q^{1-\eta} \end{matrix} ; q^2, q, \frac{q^{\alpha+1}}{x^n}\right] \tag{2.11}$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-j-\alpha})C_n^{\{\alpha\}}(z;q)\right\} = \frac{-(1-q)(q;q)_\alpha (q^{1+\alpha+\eta};q)_\infty (\alpha;q)_n (2z/\sqrt{\alpha})^n}{q^\alpha \Gamma_q(\alpha)(q^\eta;q)_\infty (q;q)_n}$$
$$\times {}_3\phi_2\left[\begin{matrix} q^{-n}, q^{-n+1} : \quad q^{-\alpha-\eta} \\ \\ - \quad : q^{1-n}/\alpha, q^{1-\eta} \end{matrix} ; q^2, q, \frac{q^{n+\alpha+1}}{\alpha x^n}\right] \tag{2.12}$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-(\alpha+r)})\phi\left[\begin{matrix} a_1,\ldots,a_r : c_{11},\ldots,c_{1r_1};\ldots;c_{m1},\ldots,c_{mr_m} \\ \\ b_1,\ldots,b_s : d_{11},\ldots,d_{1s_1};\ldots;d_{m1},\ldots,d_{ms_m} \end{matrix} ; q, q_1, q_2, \ldots, q_m; z\right]\right\}$$

$$= \frac{-q^{-\alpha}(1-q)(q;q)_\alpha (q^{1+\eta+\alpha};q)_\infty}{\Gamma_q(\alpha)(q^\eta;q)_\infty}$$

$$\times\phi\left[\begin{matrix} a_1,\ldots,a_r,q^{\eta}\colon c_{11},\ldots,c_{1r_1};\ldots;c_{m1},\ldots,c_{mr_m} \\ ;q,q_1,q_2,\ldots,q_m;z \\ b_1,\ldots,b_s,q^{\alpha+\eta+1}\colon d_{11},\ldots,d_{1s_1};\ldots;d_{m1},\ldots,d_{ms_m} \end{matrix}\right] \tag{2.13}$$

Proof of the main results:

To prove the result (2.1), take

$$f(z) = (1-q^{-(\alpha+r)})_r\phi_s\left[\begin{matrix} a_1,a_2,\ldots,a_r \\ ;q,z \\ b_1,b_2,\ldots,b_s \end{matrix}\right]$$

in the equation(1.9). After interchanging the order of summation and using the formula(1.13) and after some simplification , we obtain

$$I_q^{\eta,\alpha}\left\{(1-q^{-(\alpha+r)})_r\phi_s\left[\begin{matrix} a_1,a_2,\ldots,a_r \\ ;q,z \\ b_1,b_2,\ldots,b_s \end{matrix}\right]\right\}$$

$$= \frac{-q^{-\alpha}(1-q)(q;q)_\alpha(q^{1+\eta+\alpha};q)_\infty}{\Gamma_q(\alpha)(q^{\eta};q)_\infty}\sum_{r=0}^{\infty}\frac{(a_1,a_2,\ldots,a_r,q^{\eta};q)_r}{(b_1,b_2,\ldots,b_s,q^{1+\alpha+\eta})_r}z^r\left[(-1)^r q^{\binom{r}{2}}\right]^{1+s-r}$$

On making use of equation(1.5), this reduces to result(2.1).
To prove the result (2.2), take

$$f(z) = (1-q^{-(\alpha+r)})\frac{(q^{1+\eta};q)_r}{(q^{2(1+\eta)+\alpha};q)_r}$$

in the equation (1.9), we qbtain

$$I_q^{\eta,\alpha}\left\{\frac{(1-q^{-(\alpha+r)})(q^{1+\eta};q)_r}{(q^{2(1+\eta)+\alpha};q)_r}\right\} = \frac{-q^{-\alpha}(1-q)(q;q)_\alpha}{\Gamma_q(\alpha)}{}_2\phi_1\left(q^{1+\alpha},q^{1+\eta};q^{2(1+\eta)+\alpha};q,q^{\eta}\right)$$

Making use of the equation(1.10) this reduces to the result (2.2).

To prove the result (2.3), take

$$f(z) = \frac{(1-q^{-(\alpha+r)})(q^{-n};q)_r q^{r(1-\eta)}}{(q^{1+\eta+\alpha};q)_r}$$

in the equation (1.9), we qbtain

$$I_q^{\eta,\alpha}\left\{\frac{(1-q^{-(n+r)})(q^{-r};q)_r q^{n(1-\eta)}}{(q^{1+\alpha+\eta};q)_r}\right\} = \frac{-q^{-\alpha}(1-q)(q;q)_\alpha}{\Gamma_q(\alpha)} {}_2\phi_1\left(q^{-n},q^{1+\alpha};q^{1+\eta+\alpha};q,q\right)$$

Making use of the equation (1.12) this reduces to the result (2.3).

To prove the result (2.4), take

$$f(z)=\left\{(1-q^{-(r+\alpha)})Q_n(z;\alpha,\beta,\nu;q)\right\}$$

in the equation (1.9), we obtain

$$I_q^{\eta,\alpha}\left\{(1-q^{-(r+\alpha)})Q_n(z;\alpha,\beta,\nu;q)\right\}$$
$$=\frac{-(1-q)(q;q)_\alpha}{q^\alpha\Gamma_q(\alpha)}\sum_{r=0}^{\infty}\frac{(q^{\alpha+1};q)_r q^{r\eta}}{(q;q)_r}\sum_{i=0}^{\infty}\frac{(q^{-n},\alpha\beta q^{n+1};q)_i(zq^r;q)_i}{(\alpha q,\nu q;q)_i(q;q)_i}q^i$$

On simplification this can be written as

$$=\frac{-(1-q)(q;q)_\alpha}{q^\alpha\Gamma_q(\alpha)}\sum_{r=0}^{\infty}\sum_{i=0}^{\infty}\frac{(z;q)_{r+i}(q^{\alpha+1};q)_r(q^{-n},\alpha\beta q^{n+1};q)_i}{(z;q)_r(\alpha q,\nu q;q)_i(q;q)_r(q;q)_i}q^{r\eta}q^i$$

In view of equation (1.14), this reduces to the result (2.4).

Proofs of the results (2.5), (2.6) and (2.7) can be seen similarly.

To prove the result (2.8), take

$$f(z)=\left\{(1-q^{-j-\alpha})Z_n^{\{\alpha\}}(z;k;q)\right\}$$

in the equation (1.9), we obtain:

$$I_q^{\eta,\alpha}\left\{(1-q^{-j-\alpha})Z_n^{\{\alpha\}}(z;k;q)\right\}$$
$$=\frac{-(1-q)(q;q)_\alpha}{q^\alpha\Gamma_q(\alpha)}\frac{(q^{\alpha+1};q)_{nk}}{(q^k;q^k)_n}\sum_{r=0}^{\infty}\frac{(q^{\alpha+1};q)_r q^{r\eta}}{(q;q)_r}\sum_{i=0}^{n}\frac{(q^{-nk};q^k)_i}{(q^k;q^k)_i}\frac{q^{(1/2)ki(k-1)+ki(n+\alpha+1)}}{(q^{\alpha+1};q)_{ik}}z^{ik}q^{ikr}$$

Interchanging the order of summation and using the formula(1.13) and after some simplification , we obtain the result the result (2.8).
Result (2.9) can be proved similarly.

To prove the result (2.10), take $f(z) = \{(1-q^{-j-\alpha})R_n(\alpha,\beta;q,x)\}$ in the equation (1.9). Interchanging the order of summation, we btain:

$$I_q^{\eta,\alpha}\left\{(1-q^{-j-\alpha})R_n(\alpha,\beta;q,x)\right\} = \frac{-(1-q)(q;q)_\alpha(\alpha;q)_n(2z/\sqrt{\alpha})^n}{q^\alpha\Gamma_q(\alpha)(q;q)_n}\times$$

$$\sum_{i=0}^{\infty}\frac{(q^{-n},q^{-n+1};q^2)_i(\beta/\alpha;)_i}{(q;q)_i(\beta,q^{1-n}/\alpha;q)_i}(\frac{q^n}{x^n})^i\sum_{r=0}^{\infty}\frac{(q^{\alpha+1};q)_r q^{r(\eta-i)}}{(q;q)_r}$$

Now using the formula (1.13) and simplifying, we obtain the result (2.10).
The results 2.11 and (2.12) can be proved similarly.
To Prove the result(2.133), take

$$f(z) = (1-q^{-(\alpha+r)})\phi\left[\begin{array}{l} a_1,\ldots,a_r\colon c_{11},\ldots,c_{1r_1};\ldots;c_{m1},\ldots,c_{mr_m} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ;q,q_1,q_2,\ldots,q_m;z \\ b_1,\ldots,b_s\colon d_{11},\ldots,d_{1s_1};\ldots;d_{m1},\ldots,d_{ms_m}\end{array}\right]$$

in the equation(1.9).Interchanging the order of summation and using the formula (1.133), we obtain the following:

$$I_q^{\eta,\alpha}\left\{(1-q^{-(\alpha+r)})\phi\left[\begin{array}{l} a_1,\ldots,a_r\colon c_{11},\ldots,c_{1r_1};\ldots;c_{m1},\ldots,c_{mr_m} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ;q,q_1,q_2,\ldots,q_m;z \\ b_1,\ldots,b_s\colon d_{11},\ldots,d_{1s_1};\ldots;d_{m1},\ldots,d_{ms_m}\end{array}\right]\right\}$$

$$= \frac{-q^{-\alpha}(1-q)(q;q)_\alpha(q^{1+\eta+\alpha};q)_\infty}{\Gamma_q(\alpha)(q^\eta;q)_\infty}\sum_{n=0}^{\infty}\frac{(a_1,\ldots,a_r,q^\eta;q)_n}{(q,b_1,\ldots,b_s,q^{\alpha+\eta+1};q)_n}z^n\prod_{j=1}^{m}\frac{(c_{j1},\ldots,c_{jr_j};q_j)_n}{(d_{j1},\ldots,d_{js_j};q_j)_n}.$$

Now using the definition (1.29), this reduces to the result(2.13).

3 Application of the result (2.1):

In this section, I will evaluate Kober fractional integral operator of some of the basic hypergeometric polynomials as an application of the main result(2.1).The results are listed below.

$$I_q^{\eta,\alpha}\left\{(1-q^{-(r+\alpha)})S_n(z;q)\right\}$$
$$= \frac{-(1-q)(q;q)_\alpha(q^{1+\alpha+\eta};q)_\infty}{q^\alpha\Gamma_q(\alpha)(q^\eta;q)_\infty}\,{}_2\phi_1\left[\begin{array}{c} q^{-n},\ \ q^\eta\ \ ;q,-z \\ q^{1+\alpha+\eta};q^1\end{array}\right] \tag{3.1}$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-(r+\alpha)})L_n^{(\alpha)}(z;q)\right\}$$
$$=\frac{-(1-q)(q;q)_\alpha(q^{\alpha+1};)_n(q^{1+\alpha+\eta};q)_\infty}{q^\alpha\Gamma_q(\alpha)(q;q)_n(q^\eta;q)_\infty}\ {}_2\phi_2\left[\begin{matrix}q^{-n}, & q^\eta & ;q,-zq^n\\ q^{\alpha+1}, q^{1+\alpha+\eta} & & \end{matrix}\right] \quad (3.2)$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-(r+\alpha)})W_n(z;b;q)\right\}$$
$$=\frac{(-1)^{n+1}(1-q)(q;q)_\alpha(b;q)_n(q^{1+\alpha+\eta};q)_\infty q^{n(n+1)/2}}{q^\alpha\Gamma_q(\alpha)(q^\eta;q)_\infty}\ {}_3\phi_2\left[\begin{matrix}q^{-n}, & 0, & q^\eta & \\ & & & ;q,z\\ b, & q^{1+\alpha+\eta} & & \end{matrix}\right] \quad (3.33)$$

$$I_q^{\eta,\alpha}\left\{(1-q^{-(r+\alpha)})\Gamma_n^{\alpha,\beta}(z;q)\right\}$$
$$=\frac{-(1-q)(q;q)_\alpha(q^{\alpha+l};q)_n(q^{1+\alpha+\eta};q)_\infty}{q^\alpha\Gamma_q(\alpha)((q;q)_nq^\eta;q)_\infty}\ {}_3\phi_2\left[\begin{matrix}q^{-n}, & q^{\alpha+\beta+n+1}, & q^\eta & \\ & & & ;q,zq\\ q^{n+1}, & q^{1+\alpha+\eta} & & \end{matrix}\right] \quad (3.4)$$

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