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A New Weighted Fuyi Distribution and Its Application to the Cancer Data

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ABSTRACT
In this study, a one-parameter fuyi distribution was found to extend this further and continue my new study; a weighted model is given for this one-parameter fuyi distribution. This is called a new weighted fuyi distribution. Statistical properties of the weighted fuyi distribution (WD), including probability density function (PDF) and cumulative distribution function (CDF), Reliability analysis, Order statistics, Moment generating function and Entropies. The goodness-of-fit of the distribution on the new version shows its better fit to the data.

KEYWORDS: Weighted distribution, Fuyi distribution, Goodness of fit, Maximum likelihood estimation, Reliability analysis, Order Statistics, Moment generating function, Renyi entropy.

I. INTRODUCTION

In this study, a single parameter fuyi distribution has been observed. Now I have created a weighted model into a single-parameter move on a two-parameter fuyi distribution named as a new weighted fuyi distribution. Fuyi distribution is introduced in 2024. In this, weighted distributions are used in many fields, including branching processes, reliability, and biomedicine. The proposed distribution includes its shape, moments, and kurtosis. The single parameter fuyi distribution plays a significant role in fields such as medical science, engineering, economics, and other science-related areas. The single-parameter fuyi distribution was developed, for example; Ghitany M.E., Atieh B., & Nadarajan, S. (2008) proposed a new distribution by exploring a two-component distribution to obtain a one-parameter distribution called the Lindley distribution with an increasing hazard rate function using the exponential distribution with a scale parameter and a Gamma distribution having a shape parameter of two and a scale parameter θ with a mixing proportion $P=\frac{\theta}{\theta+1}$, Padgett W.J. (2011). In this paper, a new one-parameter continuous distribution having its probability density function (pdf) is

$$f_{FD}(x,\theta)=\frac{\theta}{720(\theta^7+\theta^3+1)}(\theta^{13}x^6+360\theta^5x^2+720)e^{-\theta x}dx$$

(1)

It is proposed this distribution, fuyi distribution.

The PDF (1) is a mixture of three distributions: an exponential distribution with a scale parameter θ, a gamma distribution with shape and scale parameters 7 and θ respectively, and a gamma distribution with shape and scale parameters 3 and θ respectively.

$$f_{FD}(x,\theta)=p_1g_1(x;\theta)+p_2g_2(x;7,\theta)+p_3g_3f_{FD}(x;3,\theta)$$

Where,

$P_1=\frac{1}{\theta^7+\theta^3+1}$, $P_2=\frac{\theta^7}{\theta^7+\theta^3+1}$, and $P_3=\frac{\theta^3}{\theta^7+\theta^3+1}$

are the mixing proportion such that, $P_1+P_2+P_3=1$.

The cumulative distribution function is given as

$$F_{FD}(x,\theta)=1-\left[1+\frac{\theta^4x(\theta^9x^5+6\theta^8x^4+30\theta^7x^3+120\theta^6x^2+360\theta^5x+720\theta^4+720)}{720(\theta^7+\theta^3+1)}\right]$$

(2)

The weighted fuyi distribution is given by,

The Weighted Fuyi Distribution (WFD)

The probability density function of new weighted fuyi Distribution is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))};\lambda>0,$$

(3)

$$f_w(x)=\frac{x^\lambda f(x)}{E(x^\lambda)};\lambda>0$$

Where $w(x)$ be a non-negative weight function

and $E(w(x)) = \int w(x)f(x)dx < \infty.$

$$f_w(x) = \frac{x^\lambda f(x)}{E(x^\lambda)}; \lambda > 0$$

Where,

$$E(x^\lambda) = \int_0^\infty x^\lambda\, f(x;\theta,\lambda)\, dx$$

$$= \int_0^\infty x^\lambda \frac{\theta}{720(\theta^7+\theta^3+1)} (\theta^{13}x^6 + 360\theta^5 x^2 + 720) e^{-\theta x} dx$$

$$= \frac{\theta}{720(\theta^7+\theta^3+1)} \int_0^\infty x^\lambda (\theta^{13}x^6 + 360\theta^5 x^2 + 720) e^{-\theta x} dx$$

$$= \frac{\theta}{720(\theta^7+\theta^3+1)} \theta^{13} \int_0^\infty x^{\lambda+6}\, e^{-\theta x} dx + 360\theta^5 \int_0^\infty x^{\lambda+2}\, e^{-\theta x} dx + 720 \int_0^\infty x^\lambda\, e^{-\theta x} dx$$

$$= \frac{\theta}{720(\theta^7+\theta^3+1)} \left(\frac{\theta^{13}\,\Gamma(\lambda+7) + 360\theta^9\,\Gamma(\lambda+3) + 720\theta^6\,\Gamma(\lambda+1)}{\theta^{\lambda+7}} \right)$$

$$E(x^\lambda) = \left(\frac{\theta^7\,\Gamma(\lambda+7) + 360\theta^3\,\Gamma(\lambda+3) + 720\,\Gamma(\lambda+1)}{\theta^\lambda\left(720\left(\theta^7+\theta^3+1\right)\right)} \right)$$

(4)

Substitute (1) and (4) in equation (3), we will get the required probability density function of the weighted fuyi distribution as

$$f_w(x,\theta,\lambda) = \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7) + 360\theta^3\,\Gamma(\lambda+3) + 720\,\Gamma(\lambda+1))} x^\lambda (\theta^{13}x^6 + 360\theta^5 x^2 + 720) e^{-\theta x} dx$$

Now, the cumulative distribution function (cdf) of the weighted fuyi distribution is obtained as

$$F_w(x;\theta,\lambda) = \int_0^x f_w(x;\theta,\lambda) dx$$

(5)

$$= \int_0^x \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7) + 360\theta^3\,\Gamma(\lambda+3) + 720\,\Gamma(\lambda+1))} x^\lambda (\theta^{13}x^6 + 360\theta^5 x^2 + 720) e^{-\theta x} dx$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7) + 360\theta^3\,\Gamma(\lambda+3) + 720\,\Gamma(\lambda+1))} \int_0^x x^\lambda (\theta^{13}x^6 + 360\theta^5 x^2 + 720) e^{-\theta x} dx$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7) + 360\theta^3\,\Gamma(\lambda+3) + 720\,\Gamma(\lambda+1))} \theta^{13} \int_0^x x^{6+\lambda} e^{-\theta x} dx + 360\theta^5 \int_0^x x^{2+\lambda} e^{-\theta x} dx + 720 \int_0^x x^\lambda e^{-\theta x} dx$$

After simplification, the cumulative distribution function of the weighted version of the fuyi distribution is derived.

$$F_w(x;\theta,\lambda) = \frac{\theta^7\gamma(\lambda+7,\theta x) + 360\theta^3\gamma(\lambda+3,\theta x) + 720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7) + 360\theta^3\,\Gamma(\lambda+3) + 720\,\Gamma(\lambda+1))}$$

(6)

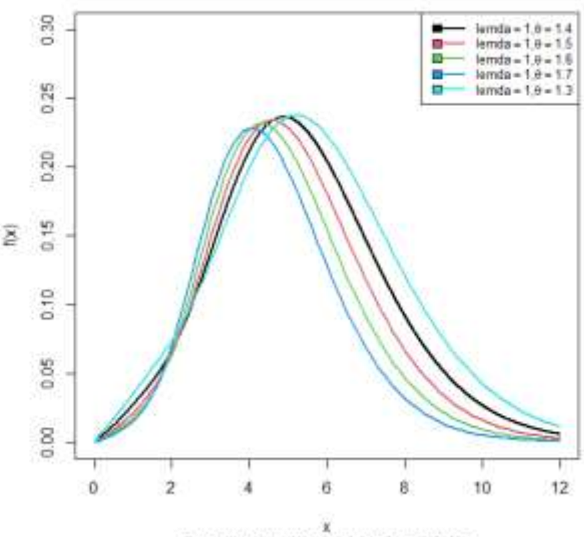

Fig 1: pdf plot of Weighted fuyi distribution

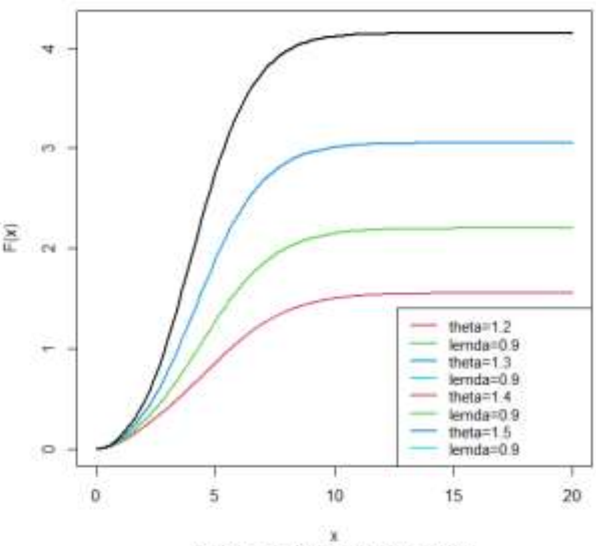

Fig 2 Cdf plot of weighted fuyi distribution

RELIABILITY ANALYSIS

We will discuss the survival function, failure rate, reverse hazard rate and the Mills ratio of the weighted fuyi distribution.

Survival Function

The survival function of the weighted fuyi distribution is given by

$$S(x;\theta,\lambda) = 1 - F_w(x;\theta,\lambda)$$

$$S(x;\theta,\lambda) = 1 - \left(\frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)$$

Hazard Function

Weighted fuyi distribution in hazard function is given by

$$h(x;\theta,\lambda) = \frac{f_w(x;\theta,\lambda)}{1-F_w(x;\theta,\lambda)}$$

$$h(x;\theta,\lambda) = \frac{\frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}x^\lambda(\theta^{13}x^6+360\theta^5x^2+720)e^{-\theta x}dx}{1-\left(\frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)}$$

$$h(x;\theta,\lambda) = \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1)-\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x))}x^\lambda(\theta^{13}x^6 + 360\theta^5x^2+720)e^{-\theta x}dx \quad (8)$$

Reverse Hazard Rate Function

$$h_r(x;\theta,\lambda) = \frac{f_w(x;\theta,\lambda)}{F_w(x;\theta,\lambda)}$$

$$h_r(x;\theta,\lambda) = \frac{\theta^{\lambda+1}}{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}x^\lambda(\theta^{13}x^6 + 360\theta^5x^2+720)e^{-\theta x}dx$$

(9)

Odds Rates Function

$$o(x;\theta,\lambda) = \frac{F_w(x;\theta,\lambda)}{1-F_w(x;\theta,\lambda)}$$

$$o(x;\theta,\lambda) = \frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{1-\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}$$

Cumulative hazard Rate Function

$$H(x;\theta,\lambda) = -ln\,(1 - F_w(x;\theta,\lambda))$$

$$H(x;\theta,\lambda) = -ln\left(1 - \frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)$$

Mills Ratio

Mills Ratio = $\frac{1}{h_r(x;\theta,\lambda)}$

Mills Ratio= $\frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{\theta^{\lambda+1}x^\lambda(\theta^{13}x^6+360\theta^5x^2+720)}$

(12)

Statistical Properties Moments

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_w(x;\theta,\lambda)dx$$

$$= \int_0^\infty x^r\left(\frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}x^\lambda(\theta^{13}x^6 + 360\theta^5x^2+720)\right)e^{-\theta x}dx \quad (7$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\int_0^x x^r x^\lambda(\theta^{13}x^6 + 360\theta^5x^2+720)e^{-\theta x}dx$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\theta^{13}\int_0^x x^{r+\lambda+6}e^{-\theta x}dx + 360\theta^5\int_0^x x^{r+\lambda+2}e^{-\theta x}dx + 720\int_0^x x^{r+\lambda}e^{-\theta x}dx$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\theta^{13}\int_0^x x^{(r+\lambda+7)-1}e^{-\theta x}dx + 360\theta^5\int_0^x x^{(r+\lambda+3)-1}e^{-\theta x}dx + 720\int_0^x x^{(r+\lambda+1)-1}e^{-\theta x}dx$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\left(\theta^{13}\left(\frac{\Gamma r+\lambda+7}{\theta^{r+\lambda+7}}\right) + 360\theta^5\left(\frac{\Gamma r+\lambda+3}{\theta^{r+\lambda+3}}\right) + 720\left(\frac{\Gamma r+\lambda+1}{\theta^{r+\lambda+1}}\right)\right)$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\left(\frac{\theta^{13}\,\Gamma r+\lambda+7+360\theta^9\,\Gamma r+\lambda+3+720\theta^6\,\Gamma r+\lambda+1}{\theta^{r+\lambda+7}}\right)$$

$$\mu_r' = \frac{\theta^7\,\Gamma r+\lambda+7+360\theta^3\,\Gamma r+\lambda+3+720\,\Gamma r+\lambda+1}{\theta^r(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \quad (10)$$

Put $r = 1$ and $r = 2$ we get the moments and variance of newly weighted fuyi distribution is,

$$\mu_1' = \frac{\theta^7\,\Gamma\lambda+8+360\theta^3\,\Gamma\lambda+4+720\,\Gamma\lambda+2}{\theta(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}$$

$$\mu_2' = \frac{\theta^7\,\Gamma\lambda+9+360\theta^3\,\Gamma\lambda+5+720\,\Gamma\lambda+3}{\theta^2(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \quad (11)$$

Variance

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_2 = \left(\frac{\theta^7\,\Gamma\lambda+9+360\theta^3\,\Gamma\lambda+5+720\,\Gamma\lambda+3}{\theta^2\left(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1)\right)} - \left(\frac{\theta^7\,\Gamma\lambda+8+360\theta^3\,\Gamma\lambda+4+720\,\Gamma\lambda+2}{\theta\left(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1)\right)}\right)^2\right)$$

$$\mu_2 = \left(\frac{\theta^7\,\Gamma\lambda+9+360\theta^3\,\Gamma\lambda+5+720\,\Gamma\lambda+3}{\theta^2\left(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1)\right)} - \frac{(\theta^7\,\Gamma\lambda+8+360\theta^3\,\Gamma\lambda+4+720\,\Gamma\lambda+2)^2}{\left(\theta\left(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1)\right)\right)^2}\right)$$

S.D = Variance

Moment Generating Function

$$M_x(t) = E(e^{tx})$$

$$M_x(t) = \int_0^\infty e^{tx} f_w(x;\theta,\lambda)\,dx$$

Using Taylor's series expansion

$$M_X(t) = \int_0^\infty \left[1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots\right]$$

$$M_X(t) = \sum_{r=0}^\infty \left(\frac{t^r}{r!}\right) \int_0^\infty f_w(x;\theta,\lambda)\,dx$$

$$M_X(t) = \sum_{r=0}^\infty \left(\frac{t^r}{r!}\right) \mu_r{}'$$

$$M_X(t) = \sum_{r=0}^\infty \left(\frac{t^r}{r!}\right)\left(\frac{\theta^7\,\Gamma r+\lambda+7+360\theta^3\,\Gamma r+\lambda+3+720\,\Gamma r+\lambda+1}{\theta^r(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)$$

Similarly, the Characteristics function of the new weighted fuyi Distribution can be obtained by

$$\phi_x(it) = E(e^{itx})$$

$$\phi_x(it) = \int_0^\infty e^{itx} f_w(x;\theta,\lambda)\,dx$$

$$\phi_x(it) = \sum_{r=0}^\infty \left(\frac{it^r}{r!}\right)\left(\frac{\theta^7\,\Gamma r+\lambda+7+360\theta^3\,\Gamma r+\lambda+3+720\,\Gamma r+\lambda+1}{\theta^r(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)$$

Order Statistics

Let $X_{(1)}, X_{(2)}, \ldots X_{(n)}$ be the order statistics of the continuous population with probability density function $f_w(x)$ and cumulative distribution function with $F_w(x)$. Then the probability density function of r^{th} order statistics $X_{(r)}$ is given by,

$$f_{w(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_w(x)[F_w(x)]^{r-1}[1 - F_w(x)]^{n-r}$$

The probability density function of r^{th} order statistics $X_{(r)}$ of the weighted fuyi distribution is given by

$$f_{w(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{\theta^{\lambda+1}x^\lambda(\theta^{13}x^6+360\theta^5x^2+720)e^{-\theta x}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)$$

$$\times\left(\frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)^{r-1}$$

$$\times\left(1 - \frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)^{n-r} \quad (14)$$

Therefore, the Probability density function of first Order Statistics X_n of weighted fuyi distribution is can be obtained as

$$f_{w(1)}(x) = \frac{n(n-1)!}{(1-1)!(n-1)!}\left(\frac{\theta^{\lambda+1}x^\lambda(\theta^{13}x^6+360\theta^5x^2+720)e^{-\theta x}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)$$

$$\times\left(1 - \frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)^{n-r}$$

$$f_{w(n)}(x) = \frac{n!}{(n-1)!(1-1)!}\left(\frac{\theta^{\lambda+1}x^\lambda(\theta^{13}x^6+360\theta^5x^2+720)e^{-\theta x}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right) \quad (15)$$

$$\times\left(\frac{\theta^7\gamma(\lambda+7,\theta x)+360\theta^3\gamma(\lambda+3,\theta x)+720\gamma(\lambda+1,\theta x)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)^{n-1}$$

Likelihood Ratio Test

Let $X_1, X2_{,\ldots,} X_n$ be a random sample from the new fuyi distribution. To test the hypothesis

$H_0: f_w(x) = f_w(x;\theta,\lambda)$ against

$H_1: f_w(x) = f_w(x;\theta,\lambda)$

In order to test whether the random sample of size n comes from the weighted fuyi distribution, the following test statistics is used by

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^n \frac{f_w(x;\theta,\lambda)}{f(x;\theta,\lambda)}$$

$$= \prod_{i=0}^n \left(\frac{\frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}x^\lambda(\theta^{13}x^6+360\theta^5x^2+720)}{\frac{\theta}{720(\theta^7+\theta^3+1)}(\theta^{13}x^6+360\theta^5x^2+720)}\right)$$

$$= \prod_{i=0}^{n} \left(\frac{\theta^{\lambda}\left(720(\theta^7+\theta^3+1)\right)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \right) {x_i}^{\lambda}$$

$$= \left(\frac{\theta^{\lambda}\left(720(\theta^7+\theta^3+1)\right)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \right)^{n} \prod_{i=0}^{n} {x_i}^{\lambda}$$

We should reject the null hypothesis, if

$$\Delta = \left(\frac{\theta^{\lambda}\left(720(\theta^7+\theta^3+1)\right)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \right)^{n} \prod_{i=0}^{n} {x_i}^{\lambda} > k$$

(or)

Equivalently, We shall reject the null hypothesis, if

$$\Delta^* = \prod_{i=0}^{n} {x_i}^{\lambda} > k \left(\frac{\theta^{\lambda}\left(720(\theta^7+\theta^3+1)\right)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \right)^{n}$$

$$\Delta^* = \prod_{i=0}^{n} {x_i}^{\lambda} > k^* \text{ where,}$$

$$k^* = k \left(\frac{\theta^{\lambda}\left(720(\theta^7+\theta^3+1)\right)}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \right)^{n}$$

Then

$p(\Delta^* > \lambda^*)$, where, $\lambda^* = \prod_{i=0}^{n} {x_i}^{\lambda}$ is less than a specified level of significance, and $\prod_{i=0}^{n} {x_i}^{\lambda}$ is the observed value of Δ^*

MAXIMUM LIKELIHOOD ESTIMATE AND FISHER INFORMATION MEASURE

$$L(x) = \prod_{i=1}^{n} f_w(x)$$

$$L(x) = \prod_{i=1}^{n} \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} {x_i}^{\lambda}(\theta^{13}x^6 + 360\theta^5x^2 + 720)e^{-\theta xi}$$

$$L(x) = \left(\frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \right)^{n} \prod_{i=1}^{n} {x_i}^{\lambda}(\theta^{13}x^6 + 360\theta^5x^2 + 720)e^{-\theta xi}$$

The log likelihood function is given by

$$= n(\lambda+1)\,log(\theta) - n\log(\theta^7\,\Gamma(\lambda+7) + 360\theta^3\,\Gamma(\lambda+3) + 720\,\Gamma(\lambda+1)) + + \sum_{i=1}^{n} {x_i}^{\lambda} log\,(\theta^{13}x^6 + 360\theta^5x^2 + 720) - \theta \sum_{i=1}^{n} x_i$$

Now, differentiating the log likelihood equation (20)

$$\frac{\partial logL}{\partial\theta} = \frac{n(\lambda+1)}{\theta} - n\left(\frac{7\theta^6\,\Gamma\lambda+7+1080\theta^2\,\Gamma\lambda+3}{\theta^7\,\Gamma\lambda+7+360\theta^3\,\Gamma\lambda+3+720\,\Gamma\lambda+1} \right) + \sum_{i=1}^{n} \left(\frac{13\theta^{12}{x_i}^6+1800\theta^4{x_i}^2}{\theta^{13}x^6+360\theta^5x^2+720} \right) - \sum_{i=1}^{n} x_i = 0$$

$$\frac{\partial logL}{\partial\lambda} = nlog(\theta) - n\left(\frac{\theta^7\psi(\lambda+7)+360\theta^3\psi(\lambda+3)+720\psi(\lambda+1)}{\theta^7\,\Gamma\lambda+7+360\theta^3\,\Gamma\lambda+3+720\,\Gamma\lambda+1} \right) + \sum_{i=1}^{n} \log(x_i) = 0$$

For the purpose of obtaining the confidence interval we use the asymptotic normaling results. We have that the MLE of $\beta = (\theta, \lambda)$ We can state the results as follows

$$\sqrt{n}\,(\hat{\beta} - \beta) \rightarrow N(0, I^{-1}(\beta))$$

Where, $I(\lambda)$ is Fisher's Information Matrix. i.e.,

$$I(\beta) = -\frac{1}{n}\begin{bmatrix} E\left[\frac{\partial^2 logL}{\partial\theta^2}\right] & E\left[\frac{\partial^2 logL}{\partial\theta\partial\lambda}\right] \\ E\left[\frac{\partial^2 logL}{\partial\lambda\partial\theta}\right] & E\left[\frac{\partial^2 logL}{\partial\lambda^2}\right] \end{bmatrix}$$

BONFERRONI AND LORENZ CURVES

The Bonferroni and Lorenz curves are used in economics in order to study income, etc., but they are used in other fields like demography, insurance, medicine, and reliability. The Bonferroni and Lorenz curves are given by

$$B(p) = \frac{1}{p\mu_1} \int_0^q xf(x)dx$$

$$B(p) = \frac{1}{p\mu_1} \int_0^q xf(x;\theta,\lambda)dx$$

and

$$L(p) = \frac{1}{\mu_1} \int_0^q xf(x;\theta,\lambda)dx$$

Where, $q = F^{-1}(p);\ q\epsilon[0,1]$ and $\mu = (x)$

Hence, the Bonferroni and Lorenz curves of our distribution are given by,

$$\mu = \frac{\theta^7\,\Gamma\lambda+9+360\theta^3\,\Gamma\lambda+5+720\,\Gamma\lambda+3}{\theta^2(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}$$

$$B(p) = \frac{\theta^2(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}{\theta^7\,\Gamma\lambda+9+360\theta^3\,\Gamma\lambda+5+720\,\Gamma\lambda+3} \int_0^q \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} x^{\lambda}(\theta^{13}x^6 + 360\theta^5x^2 + 720)e^{-\theta x}dx \quad (17)$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))} \int_0^q x^{\lambda+1}(\theta^{13}x^6 + 360\theta^5x^2 + 720)e^{-\theta x}dx$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))} \int_0^q (\theta^{13} x^{7+\lambda} + 360\theta^5 x^{\lambda+3} + 720 x^{\lambda+1}) e^{-\theta x} dx$$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))} \theta^{13} \int_0^q x^{7+\lambda} e^{-\theta x} dx + 360\theta^5 \int_0^q x^{\lambda+3} e^{-\theta x} dx + 720 \int_0^q x^{\lambda+1} e^{-\theta x} dx$$

Put, $\theta x = t,\quad x = \frac{t}{\theta},\quad dx = \frac{dt}{\theta}$

when $x \to 0, t \to 0$, and $x \to x, t \to \theta x$

$$= \frac{\theta^{\lambda+1}}{(\theta^7\ \gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))} \int_0^q \left(\frac{t}{\theta}\right)^{7+\lambda} e^{-t} \frac{dt}{\theta} + 360\theta^5 \int_0^q \left(\frac{t}{\theta}\right)^{3+\lambda} e^{-t} \frac{dt}{\theta} + 720 \int_0^q \left(\frac{t}{\theta}\right)^{1+\lambda} e^{-t} \frac{dt}{\theta}$$

After the simplification, we get

$$B(p) = \frac{\left(\theta^7 \gamma(8+\lambda,\theta q)+360\theta^3 \gamma(4+\lambda,\theta q)+720\gamma(\lambda+2,\theta q)\right)}{\theta(\theta^7\ \Gamma\lambda+7+360\theta^3\ \Gamma\lambda+3+720\ \Gamma\lambda+1)}$$

Where,

$$L(p) = B(p)$$

$$L(p) = \frac{\left(\theta^7 \gamma(8+\lambda,\theta q)+360\theta^3 \gamma(4+\lambda,\theta q)+720\gamma(\lambda+2,\theta q)\right)}{\theta(\theta^7\ \Gamma\lambda+7+360\theta^3\ \Gamma\lambda+3+720\ \Gamma\lambda+1)}$$

(18)

ENTROPIES

Entropy is important in different areas such as probability and economics, communication theory, physics, and statistics. Entropies are applied to quantify a system's diversity, uncertainty, or randomness. An indicator of the uncertainty's variation is the entropy of a random X.

Renyi Entropy

The Renyi entropy is significant as a diversity index. The Renyi entropy is also important in quantum information. It can be used as a measure of entanglement for a given probability distribution. Renyi entropy is given by

$$R_\beta = \frac{1}{1-\beta} log \int_0^\infty (f_w(x))^\beta dx\ ;\ \beta > 0, \beta \neq 1$$

$$R_\beta = \frac{1}{1-\beta} log \int_0^\infty (f_w(x;\theta,\lambda))^\beta dx$$

$$R_\beta = \frac{1}{1-\beta} log \int_0^\infty \left(\frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))}\right) x^\lambda (\theta^{13} x^6 + 360\theta^5 x^2 + 720) e^{-\theta x} dx$$

$$R_\beta = \frac{1}{1-\beta} log \left(\frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))}\right)^\beta \int_0^\infty x^{\lambda\beta} (\theta^{13} x^6 + 360\theta^5 x^2 + 720)^\beta e^{-\beta\theta x} dx$$

(19)

Using binomial expansion

$$= \sum_{i=0}^\beta \binom{\beta}{i} 720^{\beta-i} (\theta^{13} x^6 + 360\theta^5 x^2)^i$$

$$R_\beta = \frac{1}{1-\beta} log \left(\frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))}\right)^\beta \sum_{i=1}^\beta \sum_{j=0}^i \binom{\beta}{i} \binom{i}{j} \theta^{13(i-j)} 360^{5j} \int_0^\infty x^{(\beta\lambda+6(i-j)+2j+1)-1} e^{-\beta\theta x} dx$$

$$R_\beta = \frac{1}{1-\beta} log \left(\frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))}\right)^\beta \sum_{i=1}^\beta \sum_{j=0}^i \binom{\beta}{i} \binom{i}{j} \theta^{13(i-j)} 360^{5j} \frac{(\Gamma\beta\lambda+6(i-j)+2j+1)-1}{(\theta\lambda)^{\beta\lambda+6(i-j)+2j+1}}$$

Tsallis Entropy.

The Boltzmann-Gibbs (B-G) statistical property generalization initiated by Tsallis has received a great deal of attention. This B-G statistic was first introduced as the mathematical expansion of Tsallis entropy (Tsallis, 1988) for continuous random variables; this generalization of B-G was introduced in order to suggest. Which is defined as

$$T_\beta = \frac{1}{\beta-1}\left(1 - \int_0^\infty (f_w(x;\theta,\lambda))^\beta dx\right) \beta > 0, \beta \neq 1$$

$$T_\beta = \frac{1}{\beta-1}\left(1 - \int_0^\infty \left(\frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))}\right) x^\lambda (\theta^{13} x^6 + 360\theta^5 x^2 + 720)\ e^{-\theta x}\right)^\beta dx$$

$$T_\beta = \frac{1}{\beta-1}\left(1 - \left(\frac{\theta^{\lambda+1}}{(\theta^7\ \Gamma(\lambda+7)+360\theta^3\ \Gamma(\lambda+3)+720\ \Gamma(\lambda+1))}\right)^\beta \int_0^\infty x^{\beta\lambda} (\theta^{13} x^6 + 360\theta^5 x^2 + 720)^\beta e^{-\beta\theta x} dx\right)$$

(20)

Using binomial expansion

$$= \sum_{i=0}^\beta \binom{\lambda}{i} 720^{\beta-i} (\theta^{13} x^6 + 360\theta^5 x^2)^i$$

$$T_\beta = \frac{1}{1-\beta}\left(1 - \left(\frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)^\beta \sum_{i=1}^{\beta}\sum_{j=0}^{i}\binom{\beta}{i}\binom{i}{j}\theta^{13(i-j)}360^{5j}\int_0^\infty x^{(\beta\lambda+6(i-j)+2j+1)-1}e^{-\beta\theta x}dx\right)$$

$$T_\beta = \frac{1}{1-\beta}\left(1 - \left(\frac{\theta^{\lambda+1}}{(\theta^7\,\Gamma(\lambda+7)+360\theta^3\,\Gamma(\lambda+3)+720\,\Gamma(\lambda+1))}\right)^\beta \sum_{i=1}^{\beta}\sum_{j=0}^{i}\binom{\beta}{i}\binom{i}{j}\theta^{13(i-j)}360^{5j}\,\frac{(\Gamma\beta\lambda+6(i-j)+2j+1)-1}{(\theta\lambda)^{\beta\lambda+6(i-j)+2j+1}}\right)$$

Data Analysis

The data under consideration are we demonstrate the applicability of the lifetime’s data relating to show that New weighted fuyi distribution can be better than New fuyi distribution.

We consider a data set of 48 patients suffering from leukaemia blood cancer (non-censored data) are the data sets.

The survival times (in years) of a group of patients given by chemotherapy treatment alone.

0.047,0.115,0.121,0.132,0.164,0.197,0.203,0.260,0.282,0.296,0.334,0.395,0.458,0.466,0.501,0.507,0.529,0.534,0.534,0.540,0.570,0.641,0.644,0.696,0.841,0.863,1.099,1.219,1.271,1.326,1.447,1.485,1.553,1.581,1.581,1.589,2.178,2.343,2.461,2.444,2.825,2.830,3.578,3.658,3.743,3.978,4.003,4.033

In order to compare the performance of New weighted fuyi distribution with New fuyi distribution.

We are using the criteria values, like AIC (Akaike information criterion), $AICC$ (corrected Akaike information criterion) and BIC (Bayesian information criterion). The better distribution corresponds to lesser values of $AIC, AICC, BIC$ and $-2\log L$ can be evaluated by using the formulas as follows:

$AIC = 2K - 2\log L \quad AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$ and

$BIC = k\log n - 2\log L$

Where, $K =$ number of parameters, n samplesize and $-2\log L$ is the maximized value of loglikelihood function.

Table1; $MLEs\ AIC, BIC, AICC,$ and $-2\log L$ of the fitted distribution for given data set

Sr. No.	*Distribution*	*ML Estimates*	*-2log l*	*AIC*	*BIC*	*AICC*
1	Weighted Fuyi	θ=0.49433574 (0.08658692)	25.42137	30.39867	34.14107	30.61178
2	Fuyi	λ=0.48450000 (0.16227604) θ=0.59437435	66.34607	137.6198	137.4910	137.3843

From the table, it can be observed that the result is an weighted Fuyi distribution have AIC, BIC, AICC, -2logL, and compared to the values of Fuyi distributions. Our conclusion is the weighted Fuyi distribution, given the better fits over the above Fuyi distributions.

CONCLUTION

The present study examined a new one-parameter fuyi distribution as a weighted fuyi distribution and it has been found that the weighted fuyi distribution. The new distribution using the weighting technique and the single parameter has been obtained by also using some mathematical properties. The maximum likelihood technique along with the reliability measures is discussed. The real-life data has been applied for the new weighted distribution. The results are compared with a fuyi distribution. and it has been found that the weighted fuyi distribution. The new weighted fuyi distribution provides a better fit than fuyi distribution.\

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