

Neighborhood Kepler Bahatti and Modified Neighborhood Kepler Bahatti Indices of Certain Dendrimers

V.R. Kulli

Department of Mathematics, Gulbarga University, Gulbarga 585106, India

ARTICLE INFO	ABSTRACT
<p>Published Online: 21 March 2025</p> <p>Corresponding Author: V.R. Kulli</p>	<p>In this paper, we introduce the neighborhood Kepler Bahatti index, modified neighborhood Kepler Bahatti index and their corresponding exponentials of a graph. Also we compute these neighborhood Kepler Bahatti indices of certain dendrimers. Furthermore, we establish some properties of newly defined the neighborhood Kepler Bahatti index.</p>
<p>KEYWORDS: neighborhood Kepler Bahatti index, modified neighborhood Kepler Bahatti index, dendrimer.</p>	

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u . Let $S_G(u)$ denote the sum of the degrees of all vertices adjacent to a vertex u . We refer [1] for undefined notations and terminologies.

A graph index is a numerical parameter mathematically derived from the graph structure. Many graph indices have been considered in Theoretical Chemistry and several graph indices were defined by using vertex degree concept [2]. The Zagreb, Gourava, Nirmala, Sombor, Revan, E-Bahatti indices are the most degree based graph indices in Chemical Graph Theory, see [3-39]. Graph indices have their applications in various disciplines in Science and Technology [40-41].

The Kepler Bahatti index [42] of a graph G is defined as

$$KB(G) = \sum_{uv \in E(G)} [(d_G(u) + d_G(v)) + \sqrt{d_G(u)^2 + d_G(v)^2}].$$

Recently, some Kepler Bahatti indices were studied in [43-46].

The neighborhood Kepler Bahatti index of a graph G is defined as

$$NKB(G) = \sum_{uv \in E(G)} ((S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}).$$

Considering the neighborhood Kepler Bahatti index, we introduce the neighborhood Kepler Bahatti exponential of a graph G and defined it as

$$NKB(G, x) = \sum_{uv \in E(G)} x^{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}}.$$

We define the modified neighborhood Kepler Bahatti index of a graph G as

$${}^m NKB(G) = \sum_{uv \in E(G)} \frac{1}{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}}.$$

Considering the modified neighborhood Kepler Bahatti index, we introduce the modified neighborhood Kepler Bahatti exponential of a graph G and defined it as

$${}^m NKB(G, x) = \sum_{uv \in E(G)} \frac{1}{x^{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}}}.$$

Recently, some neighborhood indices were studied in [47-53].

In 2011 [54], Graovac et al. introduced the fifth M -Zagreb indices (now we call the first and second neighborhood indices) defined as

$$NM_1(G) = \sum_{uv \in E(G)} [S_G(u) + S_G(v)],$$

$$NM_2(G) = \sum_{uv \in E(G)} S_G(u)S_G(v)$$

The neighborhood Sombor index [55] of a molecular graph G is defined as

$$NSO(G) = \sum_{uv \in E(G)} \sqrt{S_G(u)^2 + S_G(v)^2}$$

In this paper, we compute the neighborhood Kepler Bahatti index, modified neighborhood Kepler Bahatti index and their corresponding exponentials of certain families of dendrimers.

2. RESULTYS FOR PAMAM DENDRIMER $PD_1[n]$

We consider the PAMAM dendrimers with n growth stages, denoted by $PD_1[n]$ for every $n \geq 0$, see Figure 1.

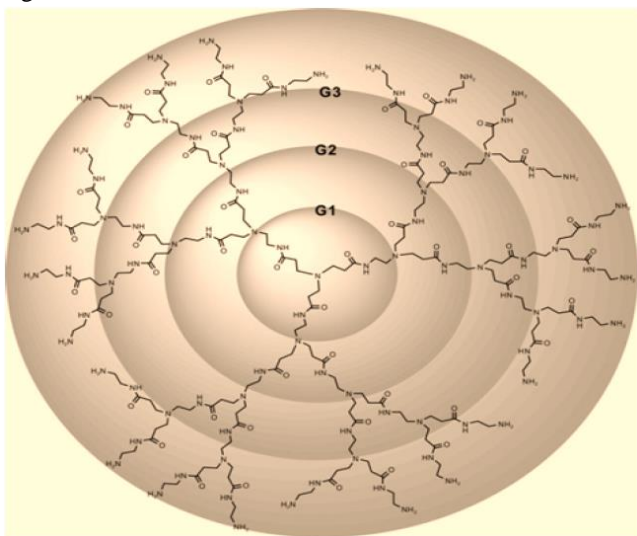


Figure 1. PAMAM dendrimer $PD_1[n]$

Let G be the graph of PAMAM dendrimer $PD_1[n]$. By calculation, we see that G has $12 \times 2^{n+2} - 23$ vertices and $12 \times 2^{n+2} - 24$ edges. Also the edge partition of the form (2,3), (3,4), (3,5), (4,5), (5,5), (5,6) for PAMAM dendrimer $PD_1[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained, as given in Table 1.

Table 1

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(2, 3)	3×2^n
(3, 4)	3×2^n
(3, 5)	$6 \times 2^n - 3$
(4, 5)	$9 \times 2^n - 6$
(5, 5)	$18 \times 2^n - 9$
(5, 6)	$9 \times 2^n - 6$

Theorem 1. The neighborhood Kepler Bahatti index of a PAMAM dendrimer $PD_1[n]$ is

$$NKB(G) = (459 + 3\sqrt{13} + 6\sqrt{34} + 9\sqrt{41} + 90\sqrt{2} + 6\sqrt{61})2^n - 235 - 3\sqrt{34} - 6\sqrt{41} - 45\sqrt{2} - 6\sqrt{61}$$

Proof: Let $G = PD_1[n]$. We have

$$\begin{aligned} NKB(G) &= \sum_{uv \in E(G)} ((S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}) \\ &= 3 \times 2^n ((2+3) + \sqrt{2^2 + 3^2}) + 3 \times 2^n ((3+4) + \sqrt{3^2 + 4^2}) \\ &\quad + (6 \times 2^n - 3)((3+5) + \sqrt{3^2 + 5^2}) \\ &\quad + (9 \times 2^n - 6)((4+5) + \sqrt{4^2 + 5^2}) \\ &\quad + (18 \times 2^n - 9)((5+5) + \sqrt{5^2 + 5^2}) \\ &\quad + (9 \times 2^n - 6)((5+6) + \sqrt{5^2 + 6^2}) \\ &= (459 + 3\sqrt{13} + 6\sqrt{34} + 9\sqrt{41} + 90\sqrt{2} + 6\sqrt{61})2^n - 235 - 3\sqrt{34} - 6\sqrt{41} - 45\sqrt{2} - 6\sqrt{61} \end{aligned}$$

Theorem 2. The neighborhood Kepler Bahatti exponential of a PAMAM dendrimer $PD_1[n]$ is

$$\begin{aligned} NKB(G, x) &= 3 \times 2^n x^{5+\sqrt{13}} + 3 \times 2^n x^{12} \\ &\quad + (6 \times 2^n - 3)x^{8+\sqrt{34}} + (9 \times 2^n - 6)x^{9+\sqrt{41}} \\ &\quad + (18 \times 2^n - 9)x^{10+5\sqrt{2}} + (9 \times 2^n - 6)x^{11+\sqrt{61}} \end{aligned}$$

Proof: Let $G = PD_1[n]$. We have

$$\begin{aligned} NKB(G, x) &= \sum_{uv \in E(G)} x^{(S_G(u)+S_G(v))+\sqrt{S_G(u)^2+S_G(v)^2}} \\ &= 3 \times 2^n x^{(2+3)+\sqrt{2^2+3^2}} + 3 \times 2^n x^{(3+4)+\sqrt{3^2+4^2}} \\ &\quad + (6 \times 2^n - 3)x^{(3+5)+\sqrt{3^2+5^2}} + (9 \times 2^n - 6)x^{(4+5)+\sqrt{4^2+5^2}} \\ &\quad + (18 \times 2^n - 9)x^{(5+5)+\sqrt{5^2+5^2}} + (9 \times 2^n - 6)x^{(5+6)+\sqrt{5^2+6^2}} \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 3. The modified neighborhood Kepler Bahatti index of a PAMAM dendrimer $PD_1[n]$ is

$$\begin{aligned} {}^m NKB(G) &= \left(\frac{3}{5+\sqrt{13}} + \frac{3}{12} + \frac{6}{8+\sqrt{34}} + \frac{9}{9+\sqrt{41}} + \frac{18}{10+5\sqrt{2}} \right) 2^n \\ &\quad + \left(\frac{9}{11+\sqrt{61}} \right) 2^n - \frac{3}{8+\sqrt{34}} - \frac{6}{9+\sqrt{41}} - \frac{9}{10+5\sqrt{2}} \\ &\quad - \frac{6}{11+\sqrt{61}} \end{aligned}$$

Proof: Let $G = PD_1[n]$. We have

$$\begin{aligned}
 & {}^m NKB(G) \\
 &= \sum_{uv \in E(G)} \frac{1}{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}} \\
 &= \frac{3 \times 2^n}{(2+3) + \sqrt{2^2 + 3^2}} + \frac{3 \times 2^n}{(3+4) + \sqrt{3^2 + 4^2}} \\
 &+ \frac{6 \times 2^n - 3}{(3+5) + \sqrt{3^2 + 5^2}} + \frac{9 \times 2^n - 6}{(4+5) + \sqrt{4^2 + 5^2}} \\
 &+ \frac{18 \times 2^n - 9}{(5+5) + \sqrt{5^2 + 5^2}} + \frac{9 \times 2^n - 6}{(5+6) + \sqrt{5^2 + 6^2}}.
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 4. The modified neighborhood Kepler Banhatti exponential of a PAMAM dendrimer $PD_1[n]$ is

$$\begin{aligned}
 & {}^m NKB(G) \\
 &= 3 \times 2^n x^{\frac{1}{5+\sqrt{13}}} + 3 \times 2^n x^{\frac{1}{12}} + (6 \times 2^n - 3) x^{\frac{1}{8+\sqrt{34}}} \\
 &+ (9 \times 2^n - 6) x^{\frac{1}{9+\sqrt{41}}} + (18 \times 2^n - 9) x^{\frac{1}{10+5\sqrt{2}}} \\
 &+ (9 \times 2^n - 6) x^{\frac{1}{11+\sqrt{61}}}.
 \end{aligned}$$

Proof: Let $G = PD_1[n]$. We have

$$\begin{aligned}
 & {}^m NKB(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}}} \\
 &= 3 \times 2^n x^{\frac{1}{(2+3) + \sqrt{2^2 + 3^2}}} + 3 \times 2^n x^{\frac{1}{(3+4) + \sqrt{3^2 + 4^2}}} \\
 &+ (6 \times 2^n - 3) x^{\frac{1}{(3+5) + \sqrt{3^2 + 5^2}}} + (9 \times 2^n - 6) x^{\frac{1}{(4+5) + \sqrt{4^2 + 5^2}}} \\
 &+ (18 \times 2^n - 9) x^{\frac{1}{(5+5) + \sqrt{5^2 + 5^2}}} + (9 \times 2^n - 6) x^{\frac{1}{(5+6) + \sqrt{5^2 + 6^2}}}.
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

3. RESULTLYS FOR POPAM DENDRIMERS $POD_2[n]$

In this section, we focus on the molecular graph of POPAM dendrimers. This family of dendrimers is denoted by $POD_2[n]$, where n is the steps of growth in this type of dendrimers. The molecular graph of $POD_2[2]$ is shown in Figure 2.

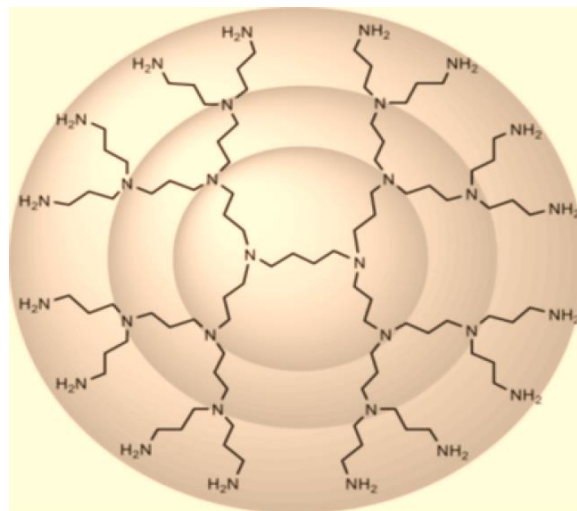


Figure 2. The molecular graph of $POD_2[n]$

Let G be the molecular graph of POPAM dendrimers $POD_2[n]$. By algebraic method, we obtain that $|V(POD_2[n])| = 2^{n+5} - 10$ and $|E(POD_2[n])| = 2^{n+5} - 11$. The edge partition of $POD_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 2.

Table 2. Edge partition of $POD_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(2, 3)	2^{n+2}
(3, 4)	2^{n+2}
(4, 4)	1
(4, 5)	$3 \times 2^n - 6$
(5, 6)	$3 \times 2^n - 6$

Theorem 5. The neighborhood Kepler Banhatti index of a POPAM dendrimer $POD_2[n]$ is

$$\begin{aligned}
 & NKB(G) \\
 &= (12 + \sqrt{13}) \times 2^{n+2} + 60 \times 2^n + 8 + 4\sqrt{2} - 6\sqrt{41} - 6\sqrt{61}.
 \end{aligned}$$

Proof: Let $G = POD_2[n]$. We have

$$\begin{aligned}
 & NKB(G) \\
 &= \sum_{uv \in E(G)} \left((S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2} \right) \\
 &= 2^{n+2} \left((2+3) + \sqrt{2^2 + 3^2} \right) + 2^{n+2} \left((3+4) + \sqrt{3^2 + 4^2} \right) \\
 &+ 1 \left((4+4) + \sqrt{4^2 + 4^2} \right) \\
 &+ (3 \times 2^n - 6) \left((4+5) + \sqrt{4^2 + 5^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 &+(3 \times 2^n - 6)((5 + 6) + \sqrt{5^2 + 6^2}) \\
 &= (12 + \sqrt{13})2^{n+2} + 60 \times 2^n + 8 + 4\sqrt{2} - 6\sqrt{41} - 6\sqrt{61}.
 \end{aligned}$$

Theorem 6. The neighborhood Kepler Bhanthi exponential of a POPAM dendrimer $POD_2[n]$ is

$$\begin{aligned}
 NKB(G, x) &= 2^{n+2} x^{5+\sqrt{13}} + 2^{n+2} x^{12} \\
 &+ 1x^{8+4\sqrt{2}} + (3 \times 2^n - 6)x^{9+\sqrt{41}} + (3 \times 2^n - 6)x^{11+\sqrt{61}}.
 \end{aligned}$$

Proof: Let $G = POD_2[n]$. We have

$$\begin{aligned}
 NKB(G, x) &= \sum_{uv \in E(G)} x^{(S_G(u)+S_G(v))+\sqrt{S_G(u)^2+S_G(v)^2}} \\
 &= 2^{n+2} x^{(2+3)+\sqrt{2^2+3^2}} + 2^{n+2} x^{(3+4)+\sqrt{3^2+4^2}} \\
 &+ 1x^{(4+4)+\sqrt{4^2+4^2}} + (3 \times 2^n - 6)x^{(4+5)+\sqrt{4^2+5^2}} \\
 &+ (3 \times 2^n - 6)x^{(5+6)+\sqrt{5^2+6^2}}.
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 7. The modified neighborhood Kepler Bhanthi index of a POPAM dendrimer $POD_2[n]$ is

$$\begin{aligned}
 {}^m NKB(G) &= \left(\frac{1}{5+\sqrt{13}} + \frac{1}{12} \right) 2^{n+2} + \left(\frac{3}{9+\sqrt{41}} + \frac{3}{11+\sqrt{61}} \right) 2^n \\
 &+ \frac{1}{8+4\sqrt{2}} - \frac{6}{9+\sqrt{41}} - \frac{6}{11+\sqrt{61}}.
 \end{aligned}$$

Proof: Let $G = PD_1[n]$. We have

$$\begin{aligned}
 {}^m NKB(G) &= \sum_{uv \in E(G)} \frac{1}{(S_G(u)+S_G(v))+\sqrt{S_G(u)^2+S_G(v)^2}} \\
 &= \frac{2^{n+2}}{(2+3)+\sqrt{2^2+3^2}} + \frac{2^{n+2}}{(3+4)+\sqrt{3^2+4^2}} \\
 &+ \frac{1}{(4+4)+\sqrt{4^2+4^2}} + \frac{3 \times 2^n - 6}{(4+5)+\sqrt{4^2+5^2}} \\
 &+ \frac{3 \times 2^n - 6}{(5+6)+\sqrt{5^2+6^2}}.
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 8. The modified neighborhood Kepler Bhanthi exponential of a POPAM dendrimer $POD_2[n]$ is

$${}^m NKB(G)$$

$$\begin{aligned}
 &= 2^{n+2} x^{\frac{1}{5+\sqrt{13}}} + 2^{n+2} x^{\frac{1}{12}} + 1x^{\frac{1}{8+4\sqrt{2}}} \\
 &+ (3 \times 2^n - 6)x^{\frac{1}{9+\sqrt{41}}} + (3 \times 2^n - 6)x^{\frac{1}{11+\sqrt{61}}}.
 \end{aligned}$$

Proof: Let $G = PD_1[n]$. We have

$$\begin{aligned}
 {}^m NKB(G, x) &= \sum_{uv \in E(G)} x^{\frac{1}{(S_G(u)+S_G(v))+\sqrt{S_G(u)^2+S_G(v)^2}}} \\
 &= 2^{n+2} x^{\frac{1}{(2+3)+\sqrt{2^2+3^2}}} + 2^{n+2} x^{\frac{1}{(3+4)+\sqrt{3^2+4^2}}} \\
 &+ 1x^{\frac{1}{(4+4)+\sqrt{4^2+4^2}}} + (3 \times 2^n - 6)x^{\frac{1}{(4+5)+\sqrt{4^2+5^2}}} \\
 &+ (3 \times 2^n - 6)x^{\frac{1}{(5+6)+\sqrt{5^2+6^2}}}.
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

4. RESULTS FOR TETRATHIAFULVALENE DENDRIMERS $TD_2[n]$

In this section, we focus on the molecular graph of a tetrathiafulvalene dendrimer. This family of tetrathiafulvalene dendrimers is denoted by $TD_2[n]$, where n is the steps of growth in this type of dendrimers for $n \geq 0$. The molecular graph of $TD_2[2]$ is shown in Figure 3.

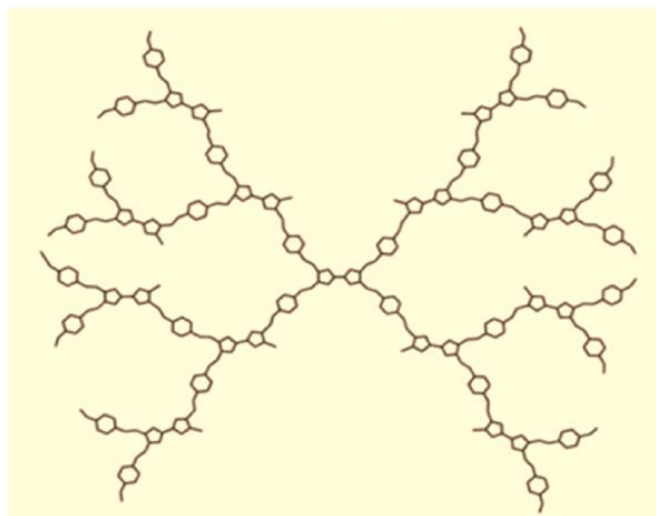


Figure 3. The molecular graph of $TD_2[2]$

Let G be the molecular graph of tetrathiafulvalene dendrimers $TD_2[n]$. By algebraic method, we obtain that $|V(G)|=31 \times 2^{n+2} - 74$ and $|E(G)|=35 \times 2^{n+2} - 85$. Also the edge partition of $TD_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 3

Table 3. Edge partition of $TD_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(2, 4)	2^{n+2}

(3, 6)	$2^{n+2} - 4$
(4, 6)	2^{n+2}
(5, 5)	$7 \times 2^{n+2} - 16$
(5, 6)	$11 \times 2^{n+2} - 24$
(5, 7)	$3 \times 2^{n+2} - 8$
(6, 6)	$2^{n+2} - 4$
(6, 7)	$8 \times 2^{n+2} - 24$
(7, 7)	$2 \times 2^{n+2} - 5$

Theorem 9. The neighborhood Kepler Bhanhatti index of a dendrimer $TD_2[n]$ is

$$NKB(G) = (132 + 5\sqrt{5} + 55\sqrt{2} + 2\sqrt{13} + 11\sqrt{61} + 3\sqrt{74} + 8\sqrt{85})2^{n+2} - 986 - 12\sqrt{5} - 139\sqrt{2} - 24\sqrt{61} - 8\sqrt{74} - 24\sqrt{85}.$$

Proof: Let $G = TD_2[n]$. We have

$$\begin{aligned} NKB(G) &= \sum_{uv \in E(G)} \left((S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2} \right) \\ &= 2^{n+2} \left((2+4) + \sqrt{2^2 + 4^2} \right) \\ &\quad + (2^{n+2} - 4) \left((3+6) + \sqrt{3^2 + 6^2} \right) \\ &\quad + 2^{n+2} \left((4+6) + \sqrt{4^2 + 6^2} \right) \\ &\quad + (7 \times 2^{n+2} - 16) \left((5+5) + \sqrt{5^2 + 5^2} \right) \\ &\quad + (11 \times 2^{n+2} - 24) \left((5+6) + \sqrt{5^2 + 6^2} \right) \\ &\quad + (3 \times 2^{n+2} - 8) \left((5+7) + \sqrt{5^2 + 7^2} \right) \\ &\quad + (2^{n+2} - 4) \left((6+6) + \sqrt{6^2 + 6^2} \right) \\ &\quad + (8 \times 2^{n+2} - 24) \left((6+7) + \sqrt{6^2 + 7^2} \right) \\ &\quad + (2 \times 2^{n+2} - 5) \left((7+7) + \sqrt{7^2 + 7^2} \right) \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 10. The neighborhood Kepler Bhanhatti exponential of a dendrimer $TD_2[n]$ is

$$\begin{aligned} NKB(G, x) &= 2^{n+2} x^{6+2\sqrt{5}} + (2^{n+2} - 4) x^{9+3\sqrt{5}} \\ &\quad + 2^{n+2} x^{10+2\sqrt{13}} + (7 \times 2^{n+2} - 16) x^{10+5\sqrt{2}} \\ &\quad + (11 \times 2^{n+2} - 24) x^{11+\sqrt{61}} + (3 \times 2^{n+2} - 8) x^{12+\sqrt{74}} \\ &\quad + (2^{n+2} - 4) x^{12+6\sqrt{2}} + (8 \times 2^{n+2} - 24) x^{13+\sqrt{85}} \\ &\quad + (2 \times 2^{n+2} - 5) x^{14+7\sqrt{2}}. \end{aligned}$$

Proof: Let $G = TD_2[n]$. We have

$$\begin{aligned} NKB(G, x) &= \sum_{uv \in E(G)} x^{(S_G(u)+S_G(v))+\sqrt{S_G(u)^2+S_G(v)^2}} \\ &= 2^{n+2} x^{(2+4)+\sqrt{2^2+4^2}} + (2^{n+2} - 4) x^{(3+6)+\sqrt{3^2+6^2}} \\ &\quad + 2^{n+2} x^{(4+6)+\sqrt{4^2+6^2}} + (7 \times 2^{n+2} - 16) x^{(5+5)+\sqrt{5^2+5^2}} \\ &\quad + (11 \times 2^{n+2} - 24) x^{(5+6)+\sqrt{5^2+6^2}} \\ &\quad + (3 \times 2^{n+2} - 8) x^{(5+7)+\sqrt{5^2+7^2}} + (2^{n+2} - 4) x^{(6+6)+\sqrt{6^2+6^2}} \\ &\quad + (8 \times 2^{n+2} - 24) x^{(6+7)+\sqrt{6^2+7^2}} \\ &\quad + (2 \times 2^{n+2} - 5) x^{(7+7)+\sqrt{7^2+7^2}}. \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 11. The modified neighborhood Kepler Bhanhatti index of a dendrimer $TD_2[n]$ is

$$\begin{aligned} {}^m NKB(G) &= \left(\frac{1}{6+2\sqrt{5}} + \frac{1}{9+3\sqrt{5}} + \frac{1}{10+2\sqrt{13}} + \frac{7}{10+5\sqrt{2}} \right) 2^{n+2} \\ &\quad + \left(\frac{11}{11+\sqrt{61}} + \frac{3}{12+\sqrt{74}} + \frac{1}{12+6\sqrt{2}} + \frac{8}{13+\sqrt{85}} \right) 2^{n+2} \\ &\quad + \left(\frac{2}{14+7\sqrt{2}} \right) 2^{n+2} - \frac{4}{9+3\sqrt{5}} - \frac{16}{10+5\sqrt{2}} - \frac{24}{11+\sqrt{61}} \\ &\quad - \frac{8}{12+\sqrt{74}} - \frac{4}{12+6\sqrt{2}} - \frac{24}{13+\sqrt{85}} - \frac{5}{14+7\sqrt{2}}. \end{aligned}$$

Proof: Let $G = TD_2[n]$. We have

$$\begin{aligned} {}^m NKB(G) &= \sum_{uv \in E(G)} \frac{1}{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}} \\ &= \frac{2^{n+2}}{(2+4) + \sqrt{2^2 + 4^2}} + \frac{2^{n+2} - 4}{(3+6) + \sqrt{3^2 + 6^2}} \\ &\quad + \frac{2^{n+2}}{(4+6) + \sqrt{4^2 + 6^2}} + \frac{7 \times 2^{n+2} - 16}{(5+5) + \sqrt{5^2 + 5^2}} \\ &\quad + \frac{11 \times 2^{n+2} - 24}{(5+6) + \sqrt{5^2 + 6^2}} + \frac{3 \times 2^{n+2} - 8}{(5+7) + \sqrt{5^2 + 7^2}} \\ &\quad + \frac{2^{n+2} - 4}{(6+6) + \sqrt{6^2 + 6^2}} + \frac{8 \times 2^{n+2} - 24}{(6+7) + \sqrt{6^2 + 7^2}} \\ &\quad + \frac{2 \times 2^{n+2} - 5}{(7+7) + \sqrt{7^2 + 7^2}} \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 12. The modified neighborhood Kepler Bhanthi exponential of a dendrimer $TD_2[n]$ is

$$\begin{aligned}
 {}^m NKB(G) &= 2^{n+2} x^{\frac{1}{6+2\sqrt{5}}} + (2^{n+2} - 4) 2^{n+2} x^{\frac{1}{9+3\sqrt{5}}} + 2^{n+2} x^{\frac{1}{10+2\sqrt{13}}} \\
 &+ (7 \times 2^{n+2} - 16) x^{\frac{1}{10+5\sqrt{2}}} + (11 \times 2^{n+2} - 24) x^{\frac{1}{11+\sqrt{61}}} \\
 &+ (3 \times 2^{n+2} - 8) x^{\frac{1}{12+\sqrt{74}}} + (2^{n+2} - 4) x^{\frac{1}{12+6\sqrt{2}}} \\
 &+ (8 \times 2^{n+2} - 24) x^{\frac{1}{13+\sqrt{85}}} + (2 \times 2^{n+2} - 5) x^{\frac{1}{14+7\sqrt{2}}}.
 \end{aligned}$$

Proof: Let $G = TD_2[n]$. We have

$$\begin{aligned}
 {}^m NKB(G, x) &= \sum_{uv \in E(G)} x^{\frac{1}{(S_G(u)+S_G(v))+\sqrt{S_G(u)^2+S_G(v)^2}}} \\
 &= 2^{n+2} x^{\frac{1}{(2+4)+\sqrt{2^2+4^2}}} + (2^{n+2} - 4) 2^{n+2} x^{\frac{1}{(3+6)+\sqrt{3^2+6^2}}} \\
 &+ 2^{n+2} x^{\frac{1}{(4+6)+\sqrt{4^2+6^2}}} + (7 \times 2^{n+2} - 16) x^{\frac{1}{(5+5)+\sqrt{5^2+5^2}}} \\
 &+ (11 \times 2^{n+2} - 24) x^{\frac{1}{(5+6)+\sqrt{5^2+6^2}}} + (3 \times 2^{n+2} - 8) x^{\frac{1}{(5+7)+\sqrt{5^2+7^2}}} \\
 &+ (2^{n+2} - 4) x^{\frac{1}{(6+6)+\sqrt{6^2+6^2}}} + (8 \times 2^{n+2} - 24) x^{\frac{1}{(6+7)+\sqrt{6^2+7^2}}} \\
 &+ (2 \times 2^{n+2} - 5) x^{\frac{1}{(7+7)+\sqrt{7^2+7^2}}}.
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

5. RESULTS FOR $NS_2[n]$ DENDRIMERS

In this section, we focus on the class of $NS_2[n]$ dendrimers with $n \geq 1$. The graph of $NS_2[2]$ is shown in Figure 4.

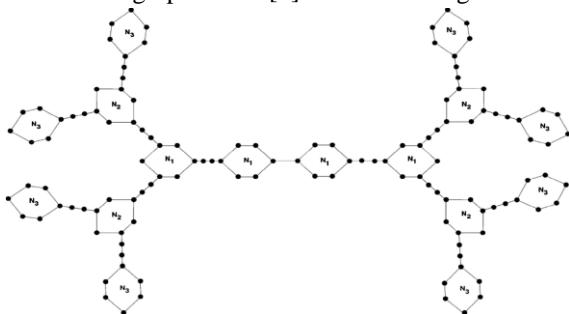


Figure 4. The graph of $NS_2[2]$

Let G be the graph of $NS_2[n]$ dendrimers. By algebraic method, we obtain that $|V(G)|=16 \times 2^n - 4$ and $|E(G)|=18 \times 2^n - 5$. Also the edge partition of $NS_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 4.

Table 4. Edge partition of $NS_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v) \setminus uv \in E(G)$	Number of edges
(4, 4)	2×2^n
(5, 4)	2×2^n
(5, 5)	$2 \times 2^n + 2$
(5, 6)	6×2^n
(7, 7)	1
(5, 7)	4
(6, 6)	$6 \times 2^n - 12$

Theorem 13. The neighborhood Kepler Bhanthi index of a dendrimer $NS_2[n]$ is

$$NKB(G) = (192 + 54\sqrt{2} + 2\sqrt{41} + 6\sqrt{61}) 2^n - 69 - 55\sqrt{2} + 4\sqrt{74}.$$

Proof: Let $G = NS_2[n]$. We have

$$\begin{aligned}
 NKB(G) &= \sum_{uv \in E(G)} \left((S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2} \right) \\
 &= 2 \times 2^n \left((4+4) + \sqrt{4^2+4^2} \right) + 2 \times 2^n \left((5+4) + \sqrt{5^2+4^2} \right) \\
 &+ (2 \times 2^n + 2) \left((5+5) + \sqrt{5^2+5^2} \right) \\
 &+ 6 \times 2^n \left((5+6) + \sqrt{5^2+6^2} \right) \\
 &+ 1 \left((7+7) + \sqrt{7^2+7^2} \right) + 4 \left((5+7) + \sqrt{5^2+7^2} \right) \\
 &+ (6 \times 2^n - 12) \left((6+6) + \sqrt{6^2+6^2} \right)
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result

Theorem 14. The neighborhood Kepler Bhanthi exponential of a dendrimer $NS_2[n]$ is

$$\begin{aligned}
 NKB(G, x) &= 2 \times 2^n x^{8+4\sqrt{2}} + 2 \times 2^n x^{9+\sqrt{41}} \\
 &+ (2 \times 2^n + 2) x^{10+5\sqrt{2}} + 6 \times 2^n x^{11+\sqrt{61}} \\
 &+ 1 x^{14+7\sqrt{2}} + 4 x^{12+\sqrt{74}} + (6 \times 2^n - 12) x^{12+6\sqrt{2}}
 \end{aligned}$$

Proof: Let $G = NS_2[n]$. We have

$$\begin{aligned}
 NKB(G, x) &= \sum_{uv \in E(G)} x^{\frac{1}{(S_G(u)+S_G(v))+\sqrt{S_G(u)^2+S_G(v)^2}}} \\
 &= 2 \times 2^n x^{\frac{1}{(4+4)+\sqrt{4^2+4^2}}} + 2 \times 2^n x^{\frac{1}{(5+4)+\sqrt{5^2+4^2}}} \\
 &+ (2 \times 2^n + 2) x^{\frac{1}{(5+5)+\sqrt{5^2+5^2}}} + 6 \times 2^n x^{\frac{1}{(5+6)+\sqrt{5^2+6^2}}} \\
 &+ 1 x^{\frac{1}{(7+7)+\sqrt{7^2+7^2}}} + 4 x^{\frac{1}{(5+7)+\sqrt{5^2+7^2}}} \\
 &+ (6 \times 2^n - 12) x^{\frac{1}{(6+6)+\sqrt{6^2+6^2}}}
 \end{aligned}$$

By simplifying the above equation, we obtain the desired result

Theorem 15. The modified neighborhood Kepler Bahatti index of a POPAM dendrimer $POD_2[n]$ is

$${}^m NKB(G) = \left(\frac{1}{4+2\sqrt{2}} + \frac{2}{9+\sqrt{41}} + \frac{2}{10+5\sqrt{2}} + \frac{6}{11+\sqrt{61}} \right) 2^n + \left(\frac{1}{2+\sqrt{2}} \right) 2^n + \frac{1}{7+7\sqrt{2}} + \frac{4}{12+\sqrt{74}} - \frac{2}{2+\sqrt{2}}.$$

Proof: Let $G = NS_2[n]$. We have

$${}^m NKB(G) = \sum_{uv \in E(G)} \frac{1}{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}} = \frac{2 \times 2^n}{(4+4) + \sqrt{4^2 + 4^2}} + \frac{2 \times 2^n}{(5+4) + \sqrt{5^2 + 4^2}} + \frac{2 \times 2^n + 2}{(5+5) + \sqrt{5^2 + 5^2}} + \frac{6 \times 2^n}{(5+6) + \sqrt{5^2 + 6^2}} + \frac{1}{(7+7) + \sqrt{7^2 + 7^2}} + \frac{4}{(5+7) + \sqrt{5^2 + 7^2}} + \frac{6 \times 2^n - 12}{(6+6) + \sqrt{6^2 + 6^2}}.$$

By simplifying the above equation, we obtain the desired result.

Theorem 16. The modified neighborhood Kepler Bahatti exponential of a POPAM dendrimer $POD_2[n]$ is

$${}^m NKB(G) = 2 \times 2^n x^{\frac{1}{8+4\sqrt{2}}} + 2 \times 2^n x^{\frac{1}{9+\sqrt{41}}} + (2 \times 2^n + 2) x^{\frac{1}{10+5\sqrt{2}}} + (2 \times 2^n + 2) x^{\frac{1}{10+5\sqrt{2}}} + 6 \times 2^n x^{\frac{1}{11+\sqrt{61}}} + 1 x^{\frac{1}{14+7\sqrt{2}}} + 4 x^{\frac{1}{12+\sqrt{74}}} + (6 \times 2^n - 12) x^{\frac{1}{12+6\sqrt{2}}}.$$

Proof: Let $G = NS_2[n]$. We have

$${}^m NKB(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{(S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2}}} = 2 \times 2^n x^{\frac{1}{(4+4) + \sqrt{4^2 + 4^2}}} + 2 \times 2^n x^{\frac{1}{(5+4) + \sqrt{5^2 + 4^2}}} + (2 \times 2^n + 2) x^{\frac{1}{(5+5) + \sqrt{5^2 + 5^2}}} + 6 \times 2^n x^{\frac{1}{(5+6) + \sqrt{5^2 + 6^2}}} + 1 x^{\frac{1}{(7+7) + \sqrt{7^2 + 7^2}}} + 4 x^{\frac{1}{(5+7) + \sqrt{5^2 + 7^2}}}$$

$$+ (6 \times 2^n - 12) x^{\frac{1}{(6+6) + \sqrt{6^2 + 6^2}}}.$$

By simplifying the above equation, we obtain the desired result.

6. PROPERTIES OF THE TEMPERATURE ELLIPTIC SOMBOR INDEX

Theorem 17. Let G be a connected graph. Then

$$\sqrt{2} NM_1(G) \leq TNKB(G) < 2 NM_1(G).$$

Proof: For any two positive numbers a and b ,

$$\frac{1}{\sqrt{2}}(a+b) \leq \sqrt{a^2 + b^2} < a+b.$$

$$\sqrt{2}(a+b) \leq (a+b) + \sqrt{a^2 + b^2} < 2(a+b)$$

For $a = S_G(u)$ and $b = S_G(v)$, the above inequality becomes

$$\sqrt{2}(S_G(u) + S_G(v)) \leq (S_G(u) + S_G(v)) + \sqrt{(S_G(u)^2 + S_G(v)^2)} < 2(S_G(u) + S_G(v))$$

By the definitions, we have

$$\sqrt{2} \sum_{uv \in E(G)} (S_G(u) + S_G(v)) \leq \sum_{uv \in E(G)} \left((S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2} \right) < 2 \sum_{uv \in E(G)} (S_G(u) + S_G(v))$$

Thus we get the desired result.

Theorem 18. Let G be a connected graph. Then

$$NKB(G) = NM_1(G) + NSO(G).$$

Proof: We have

$$\sum_{uv \in E(G)} \left((S_G(u) + S_G(v)) + \sqrt{S_G(u)^2 + S_G(v)^2} \right) = \sum_{uv \in E(G)} (S_G(u) + S_G(v)) + \sum_{uv \in E(G)} \sqrt{S_G(u)^2 + S_G(v)^2} = NM_1(G) + NSO(G)$$

7. CONCLUSION

In this study, we have introduced the neighborhood Kepler Bahatti index, modified neighborhood Kepler Bahatti index of a graph. We have computed these indices and their corresponding exponentials of certain families of dendrimers.

REFERENCES

1. V.R.Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application*

- Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds) IGI Global, USA (2019) 66-91.
3. I.Gutman and N.Trinajstic, Graph Theory and Molecular Orbitals. Total phi-electron energy of alternant hydrocarbons, *Chemical Physics Letters*, 17(4) (1972) 535-538.
 4. M.Aruvi, J.M.Joseph and E.Ramganes, The second Gourava index of some graph products, *Advances in Mathematics: Scientific Journal*, 9(12) (2020) 10241-10249.
 5. V.R.Kulli, Gourava domination indices of graphs, *International Journal of Mathematics and Computer Research*, 11(8) (2023) 3680-3684. DOI:10.47191/ijmcr/v11i8.08.
 6. V.R.Kulli, On hyper Gourava domination indices, *International Journal of Engineering Sciences & Research Technology*, 12(10) (2023) 12-20. DOI: 10.5281/zenodo.10036777.
 7. I.Gutman and V.R.Kulli, Estimating the Gourava Sombor index, *Ser. A: Appl. Math. Inform. And Mech.* (2024).
 8. V.R.Kulli, On alpha and gamma Gourava indices, *International Journal of Mathematics and Computer Research*, 12(4) (2024) 4139-4144. DOI:10.47191/ijmcr/v12i4.04.
 9. V.R.Kulli, Nirmala alpha Gourava and modified Nirmala alpha Gourava indices of certain dendrimers, *International Journal of Mathematics and Computer Research*, 12(5) (2024) 4256-4263.
 10. R.P.Somani and V.Jethwani, Introducing new exponential Gourava indices for graphs, *International Journal of Mathematics and Statistics Invention*, 11(3) (2023) 1-5.
 11. V.R.Kulli, Reverse Nirmala index, *International Journal of Engineering Sciences & Research Technology*, 11(8) (2022) 12-19. DOI: 10.5281/zenodo.7079293.
 12. V.R.Kulli, Multiplicative Nirmala and Bahatti-Nirmala indices of certain nanostar dendrimers, *International Journal of Mathematical Archive*, 13(10) (2022) 8-15.
 13. B.K.Majhi, V.R.Kulli ad I.N.Cangul, Nirmala leap indices of some chemical drugs against COVID-19, *Journal of Discrete Mathematics and its Applications*, 10(1) (2023) 13-21. DOI: 10.22061/JDMA.2023.10093.1058.
 14. S.Raghu, V.R.Kulli, K.M.Niranjana and V.M.Goudar, Nirmala indices of certain antiviral drugs, *International Journal of Mathematics and Computer Research*, 11(11) (2023) 3862-3866.
 15. V.R.Kulli, I.Gutman, B.Furtula and I.Redzepovic, Sombor, Nirmala Dhaewad and F-Sombor indices: A Comparative Study, *SSRG International Journal of Applied Chemistry*, 10(2) (2023) 7-10.
 16. V.R.Kulli, Domination Nirmala indices of graphs, *International Journal of Mathematics and Computer Research*, 11(6) (2023) 3497--3502.
 17. V.R.Kulli, ve-Degree Sombor indices of certain networks, *International Journal of Mathematics Trends and Technology*, 68(9) (2022) 5-10. <https://doi.org/10.14445/22315373/IJMTT-V68I9P502>.
 18. V.R.Kulli, Multiplicative KG Sombor indices of some networks, *International Journal of Mathematics Trends and Technology*, 68(10) (2022) 1-7.
 19. I.Gutman, V.R.Kulli and I.Redzepovic, Irregularity Sombor, index, *Bull. Acad. Serbe. Sci. Arts (Cl. Sci. Math. Natur.)* 156 (2023) 31-37.
 20. V.R.Kulli, Modified elliptic Sombor index and its exponential of a graph, *International Journal of Mathematics and Computer Research*, 12(1)(2024)3949-3954.
 21. V.R.Kulli, Reverse elliptic Sombor and modified reverse elliptic Sombor indices , *International Journal of Mathematical Archive*, 15(1) (2024) 1-7.
 22. I.Gutman, Note on the temperature Sombor index, *Vijnotehancki Glasnik/ Military Technical courier*, 71(3) (2023) 507-515..
 23. I.Gutman, B.Furtula and M.S.Oz, Geometric approach to vertex degree based topological indices-Elliptic Sombor index theory and application, *International Journal of Quantum Chemistry*, 124(2) (2024) e27151.
 24. V.R.Kulli, Multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices, *Annals of Pure and Applied Mathematics*, 29(1) (2024) 19-23.
 25. V.R.Kulli, Status elliptic Sombor and modified status elliptic Sombor indices of graphs, *Journal of Mathematics and Informatics*, 27 (2024) 49-54.
 26. R.Aguilar-Sanchez,I.F.Herrera-Gonzalez, J.A.Mendez-Bermudez and J.M.Sigaareta, Revan degree indices on random graphs, arXiv: 2210.04749v1 [math. CO] 10 Oct. 2022.
 27. M.Kamran, S.Delen, R.H.Khan, N.Salamat, A.Q.Baig, I.N.Cangul and Md.A.alam, Physico-Chemical Characterization of amylase and amylopectin using Revan topological indices, *Journal of Mathematics*, vol.2022, 12 pages, Article ID 2840217, 2022.
 28. V.R.Kulli, Elliptic Revan index and its exponential of certain networks, *International Journal of Mathematics and Computer Research*, 12(2) (2024) 4055-4061.
 29. V.R.Kulli, Modified elliptic Revan index of certain two families of nanotubes, *Annals of Pure and Applied Mathematics*, 29(2) (2024) 103-107.

30. B.K.Majhi, V.R.Kulli and I.N.Cangul, Revan indices and their polynomials of square snake graphs, *Montes Taurus J. Pure Appl. Math.* 6(1) (2024) 12-17.
31. V.R.Kulli, Revan Kepler Banhatti and modified Revan Kepler Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 30(2), (2024) 129-136.
32. A.Mahboob, G.Muhiuddin, I.Siddique and S.M.Alam, A view of Banhatti and Revan indices in chemical graphs, *Journal of Mathematics*, vol.2022, 8 pages, Article ID 5680712, 2022.
33. D.Narasimhan, R.Vignesh and K.Desikan, Results on Revan and hyper Revan indices of some HEX derived networks, *Fuzzy Mathematical Analysis and Advances in Computational Mathematics*, Springer Singapore (2022) 209-220.
34. V.R.Kulli, E-Banhatti Sombor indices, *International Journal of Mathematics and Computer Research*, 10(12) (2022) 2986-2994. DOI: <https://doi.org/10.47191/ijmcr/v10i12.01>.
35. V.R.Kulli, The (a, b)-KA E-Banhatti indices of graphs, *Journal of Mathematics and Informatics*, 23 (2022) 55-60. DOI: <http://dx.doi.org/10.22457/jmi.v23a06214>.
36. V.R.Kulli, FE-Banhatti index of certain nanostructures, *International Journal of Mathematical Archive*, 14(1) (2023) 21-29.
37. V.R.Kulli, K.M.Niranjan, V.M.Goudar and S.Raghu, E-Banhatti indices of chain silicate networks, *Commun. Combin. Cryptogr. & Computer Sci.* 2 (2024) 107-118
38. V.R.Kulli, K.M.Niranjan, V.M.Goudar and S.Raghu, Kepler E-Banhatti and modified Kepler E-Banhatti indices, *Commun. Combin. Cryptogr. & Computer Sci.* 2 (2024) 220-233.
39. V.R.Kulli, Harmonic E-Banhatti index of graphs, *Annals of Pure and Applied Mathematics*, 30(2) (2024) 79-85. DOI: <http://dx.doi.org/10.22457/apam.v30n2a01950>
40. S.Wagner and H.Wang, *Introduction Chemical Graph Theory*, Boca Raton, CRC Press, (2018).
41. M.V.Diudea (ed.) *QSPR/QSAR Studies by Molecular Descriptors*, NOVA New York, (2001).
42. V.R.Kulli, Kepler Banhatti and modified Kepler Banhatti indices, *International Journal of Mathematics and Computer Research*, 12(6) (2024) 4310-4314. DOI: 10-47191/ijmcr/v12i6.08.
43. V.R.Kulli, Reverse Kepler Banhatti and modified reverse Kepler Banhatti indices, *International Journal of Mathematical Archive*, 15(7) (2024) 1-8.
44. V.R.Kulli, Revan Kepler Banhatti and modified Revan Kepler Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 30(2), (2024) 129-136.
45. V.R.Kulli, Domination Kepler Banhatti and modified domination Kepler Banhatti indices of graphs, *Annals of Pure and Applied Mathematics*, 31(1), (2025) 23-29.
46. V.R.Kulli, Delta Kepler Banhatti indices of certain nanostructures, *International Journal of Mathematical Archive*, 16 (2025) to appear.
47. V.R.Kulli, F1-neighborhood and square neighborhood Dakshayani indices of some nanostructures, *International Journal of Engineering Sciences & Research Technology*, 8(8) (2019) 126-138.
48. V.R.Kulli, Computation of distance based connectivity status neighborhood Dakshayani indices, *International Journal of Mathematics Trends and Technology*, 66(6) (2020) 118-128.
49. V.R.Kulli, Multiplicative status neighborhood indices of graphs, *International Journal of Engineering Sciences & Research Technology*, 9(6) (2020) 110-120.
50. V.R.Kulli, Some multiplicative neighborhood topological indices of nanocones and dendrimers, *International Journal of Engineering Sciences & Research Technology*, 9(10) (2020) 119-132.
51. V.R.Kulli, Degree based neighborhood indices of some nanostructures, *SSRG International Journal of Applied Chemistry*, 7(3) (2020) 17-24..
52. V.R.Kulli, Multiplicative connectivity status neighborhood indices of graphs, *International Journal of Engineering Sciences & Research Technology*, 9(12) (2020) 59-68.
53. V.R.Kulli, Neighborhood sum atom bond connectivity indices of some nanostar dendrimers, *International Journal of Mathematics and Computer Research*, 11(1) (2023) 3230-3235.
54. Graovac, M. Ghorbani and M.A. Hosseinzadeh, Computing fifth geometric-arithmetic index of nanostar dendrimers, *Journal of Mathematical Nanoscience*, 1(1) (2011) 33-42.
55. V.R.Kulli, Neighborhood Sombor index of some nanostructures, *International Journal of Mathematics Trends and Technology*, 67(5) (2021) 101-108. <https://doi.org/10.22147/22315373/IJMTT.V67I5P512>.