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Epidemic Modeling of Student Learning Behavior: A Novel Perspective

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ABSTRACT

This study introduces a groundbreaking approach to understanding student learning behavior by applying epidemic modeling. By analogizing the spread of diseases to the dissemination of knowledge, we modified the classic SIR model to create the Student Learning Behaviour (SLB) model. This model simulates the dynamics of student learning by capturing interactions between students, instructors, and learning materials, enabling knowledge acquisition and retention predictions. The study highlights the significance of social interactions outside the classroom and curricular activities in shaping students' learning behavior and motivation. It investigates the impact of students' learning behavior on their academic achievement, emphasizing the crucial role of self-motivation and appearance in driving improvement. This research provides educators and policymakers with a valuable tool for designing targeted interventions and optimizing instructional strategies, offering insights into the complex processes underlying student learning.

KEYWORDS: SIR model, Student Learning Behavior (SLB), academic performance, stability

I. INTRODUCTION

Mathematical models have long been employed to examine pressing societal issues, including pollution, infections, networks, and rumors. Similarly, learning is a vital aspect of society that impacts every individual. By leveraging the SEIR model, typically used to study infectious diseases, we can analyze student learning behavior. This approach enables a deeper understanding of students' learning behavior, a crucial aspect of their lives. Behavior plays a significant role in a student's ability to thrive in society, and unawareness can hinder their progress. Good behavior is essential for maintaining discipline and improving academic performance. However, behavior can also be a barrier to effective learning, making it challenging for students to achieve their learning objectives. Mathematical modeling can help address this issue by providing a framework for understanding and modifying student learning behavior. The ultimate goal is to develop mathematical skills that facilitate problem-solving, solution validation, and the simplification of complex processes. By applying mathematical models to real-world scenarios, we can better comprehend the dynamics of student learning behavior.

Student's learning basically depends on the numerous factors that are based on the conditions of the learners, like family environment, friends, institutions, social, etc. Many students have found that their family environment does not support their needs; they are in need while they want to produce something productive, and they get affected by such disruptions as mentioned above. This is a big reason to stop the pace of their learning and also affect their mindset to motivate and facilitate. There are problems identified by the teachers in the classroom that the teachers have been assigned other tasks that are more important than teaching. A single teacher cannot possibly be able to attend to every student in a large class equally. Compared to frontline students, backbencher students receive less attention from teachers. Without the attention of the teacher, the learning process of students becomes slower. Therefore, some students are slow learners. Similarly, teaching methodology also plays a crucial role in decreasing the interest of students in studying. Consequently, many students are dealing with the issue of a slow learning process. Compared to other children of the same age, slow learners have demonstrated slower thinking and academic achievement. “Slow learners" are typically defined as students with an IQ that is below average. Slow learners struggle to keep up with peers and fail to meet the expectations of teachers and parents.

Early in 2023, Khalid A. Bin Abdulrahman et al. [1] explored the relationship between academic performance and motivation among medical students in Riyadhin. In 2022, Mutiawati and J. Rahmah et al. [2] described the impact of social interaction and motivation in the student learning behavior. In 2022, Al-Osaimi and Fawaz [3] gave an idea that learning is best principle for academic achievement of student. In 2020, Rasha M. Abdelrahman and R. Hidayat [4, 5] conducted a study on the impact of academic motivation and metacognitive awareness on the academic achievement of Ajman University students. Using TIMSS 2015 data, M. Chen and D. Hastedt [6] elucidated the connection between students' non-cognitive characteristics and their achievement in maths and science. This model offered a more standard classical model for student learning behaviour, based on the biological epidemic model [7, 8] as well as the SIR model. In literature reviews, several analysts have used the epidemic model to show that some curricular activities affect students' academic achievement as well as overall development in a direct or indirect manner [9, 10, 11, and 12]. With regard to education, the study of slow learners is defined under inclusive education in four categories: slow learner individuals, slow learner individuals but not disturbed others, slow learner individuals but disturbed others, and improved class of slow learners. In 2016, Dasaradhi, K. et al. [13] discussed the ways to enhance slow learners' capacity for learning. Malik (2009), Shaw (2010) and Chauhan (2011) suggest that slow learners have an IQ between 76 and 89, are slightly different from normal children, and have limited problem-solving abilities [14, 15, and 16]. The slow learner is generally identified as having a low ability to reason in specific situations as well as to deal with abstracts and symbols, such as in languages, numbers, and concepts. In 2009, Fithriyana, E. [17], studied game therapy based on local wisdom in the cognitive development of slow- learners. In 2015, Verma, S. [18] studied dealing with a slow learner. Khaira, U., et al. [19] looked the assessment procedure for mathematics learners whose learning process is very slow. In 2021, F. Setyawan et al. [20] explored logical and analytical thinking in mathematics modeling for those who learn slowly. In 2023, Nurlistiawati et al. [21] looked those students who learn mathematics slowly might enhance their mathematical skill by using ideas of realistic mathematics education. In 2015, Varachanon, P. [22] defined the nature study of the learning behaviour of nursing students at the Royal Thai Navy College of Nursing. In 2001, Brauer, F. et al. [23] studied the mathematical models in population biology and epidemiology. In 2005, Magdalena, S. M. [24] analysed the correlation between student learning behaviour, objectives of learning, and output of learning among Romanian students.

This study's objective is to offer a framework for slow learners to develop strong logical and critical thinking skills through rigorous computational and mathematical models. Using the SIR mathematical model, we characterized the aberrant learning behaviour and a qualitative methodology for slow learners. Social interaction and logical thinking (censorious discerning) are generally regarded as the main goals of learning. Therefore, social interaction and logical thinking play a crucial role in learning. The discussion learning method effectively addresses slow learners in higher education by focusing on rigorous questions. Therefore, parents and teachers can effectively support slow learners by encouraging them to study in groups, and interacting with peers of their age increases a child's confidence.

II. MODEL FORMATION

In this model A (t), B (t), C (t), and D (t) represent slow learner individuals, slow learner individuals but not disturbed others, slow learner individuals but disturbed others, and an improved class of slow learners at time t, respectively. Then the schematic representation of the mathematical model for slow learners is shown in figure 1.

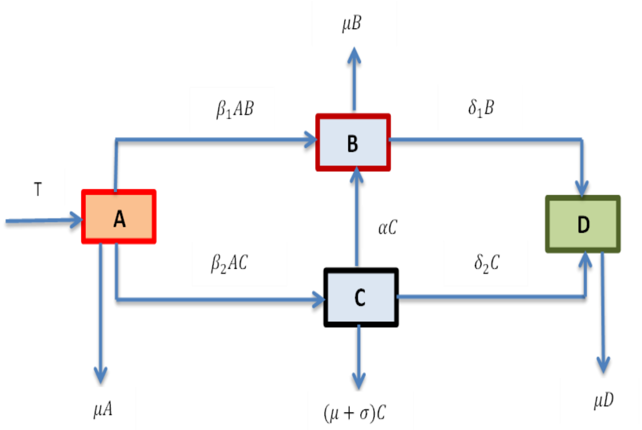

Figure [1]: Flow Diagram for the System

Table-1 Nomenclature

Symbol	Description
A	Slow-learner individuals
B	Slow learner who does not disturb others
C	Slow learner individuals but disturbed others
D	Improved class of slow learners
T	Constant admitted learners
β_1	Transmission rate between individuals A and B
β_2	Transmission rate between individuals A and C
α	The rate at which the individuals C join the other individuals B
δ_1	Improvement rate of individuals B
δ_2	Improvement rate of individuals C
σ	Quit rate slow learners through slow learning process
μ	Natural quit rate of the learning process
N	Total Strength

A. *Mathematical Formulation of the Model*

As per our assumption for slow learners, the mathematical formulation of the above basic compartmental SEIR model is given as follows:

$$\left.\begin{aligned}\frac{dA}{dt} &= T - \beta_1 AB - \beta_2 AC - \mu A \\ \frac{dB}{dt} &= \beta_1 AB + \alpha C - \delta_1 B - \mu B \\ \frac{dC}{dt} &= \beta_2 AC - \alpha C - \delta_2 C - \sigma C - \mu C \\ \frac{dD}{dt} &= \delta_1 B + \delta_2 C - \mu D\end{aligned}\right] \quad (1)$$

B. Model Analysis

We aim to demonstrate that the feasible region remains positively invariant. Specifically, we seek to prove that, $\lim_{t\to\infty} N(t) \le \frac{T}{\mu}$ from the model, where the total population N is given by

$N = A + B + C + D$

Differentiation with respect to t,

$$\frac{dN}{dt} = \frac{dA}{dt} + \frac{dB}{dt} + \frac{dC}{dt} + \frac{dD}{dt}$$

By using (1) we find that $\frac{dN}{dt} + \mu N = T$

By solving this first order partial differential equation we get, $N(t) = \frac{T}{\mu} + c_1 e^{-\mu t}$

Initially $N(0) = \frac{T}{\mu} + c_1 e^{-\mu .0}$ therefore $c_1 = N(0) - \frac{T}{\mu}$

Now, $N(t) = \frac{T}{\mu} + \left(N(0) - \frac{T}{\mu}\right) e^{-\mu t}$ $\quad N(0) = N_0$

$$N(t) = \frac{T}{\mu}(1 - e^{-\mu t}) + N_0 e^{-\mu t}$$

thus $\lim_{t\to\infty} N(t) \le \frac{T}{\mu}$

The system of equations given in (1) feasible area may now be described as

$$\varphi = \left\{(A,B,C,D) \in R^4 : A > 0, B \ge 0, C \ge 0, D \ge 0 \text{ and } N = A + B + C + D \le \frac{T}{\mu}\right\}$$

. From the perspective of system (1), this feasible area is positively invariant. Thus, the basic model is effectively posed mathematically and epidemiologically.

C. Basic reproductive number R_0

The Next Generation Method (NGM) is the most common method used for calculating R_0 therefore, it is calculated by NGM. The success or failure of any model depends on the basic reproduction number R_0. If so $R_0 \le 1$, then the model is stable in a feasible area, and if $R_0 > 1$, then the model is unstable in a feasible region φ. since $\frac{dD}{dt} > 0$, This is a prerequisite for the occurrence of an epidemic. The system (1)'s basic reproduction number is as follows:

To determine R_0 from the mathematical model of individual classes B and C

$$\frac{dB}{dt} = \beta_1 AB + \alpha C - \delta_1 B - \mu B$$

$$\frac{dC}{dt} = \beta_2 AC - \alpha C - \delta_2 C - \sigma C - \mu C$$

With the help of infectious individual classes B and C, we get transmission matrix F and transition matrix V.

$$F = \begin{bmatrix}\beta_1 A & 0 \\ 0 & \beta_2 A\end{bmatrix}$$ and

$$V = \begin{bmatrix}\delta_1 + \mu & -\alpha \\ 0 & \alpha + \sigma + \delta_2 + \mu\end{bmatrix}$$

Determine V^{-1}

Now

$V^{-1} =$

$$\frac{1}{(\delta_1+\mu)(\alpha+\sigma+\delta_2+\mu)}\begin{bmatrix}\alpha + \sigma + \delta_2 + \mu & \alpha \\ 0 & \delta_1 + \mu\end{bmatrix}$$

Determine FV^{-1}

$FV^{-1} =$

$$\frac{1}{(\delta_1+\mu)(\alpha+\sigma+\delta_2+\mu)}\begin{bmatrix}\beta_1 A & 0 \\ 0 & \beta_2 A\end{bmatrix} . \begin{bmatrix}\alpha + \sigma + \delta_2 + \mu & \alpha \\ 0 & \delta_1 + \mu\end{bmatrix}$$

$$FV^{-1} = \begin{bmatrix}\frac{\beta_1 A}{(\delta_1+\mu)} & \frac{\beta_1 A\alpha}{(\delta_1+\mu)(\alpha+\sigma+\delta_2+\mu)} \\ 0 & \frac{\beta_2 A}{(\alpha+\sigma+\delta_2+\mu)}\end{bmatrix}$$

The greatest eigenvalue of the matrix FV^{-1} defines the value of basic reproduction number R_0. Now the characteristic matrix of matrix FV^{-1}

$$|FV^{-1} - \lambda| = 0$$

$$\begin{vmatrix}\frac{\beta_1 A}{(\delta_1+\mu)} - \lambda & \frac{\beta_1 A\alpha}{(\delta_1+\mu)(\alpha+\sigma+\delta_2+\mu)} \\ 0 & \frac{\beta_2 A}{(\alpha+\sigma+\delta_2+\mu)} - \lambda\end{vmatrix} = 0$$

Now the eigenvalue is expressed as $\lambda_1 = \frac{\beta_1 A}{(\delta_1+\mu)}$ or $\lambda_2 = \frac{\beta_2 A}{(\alpha+\sigma+\delta_2+\mu)}$

Now to obtain the eigenvalue of free equilibrium Pollution put $A = \frac{T}{\mu}$, the basic reproduction number R_0 is expressed as $R_0 = (R_1, R_2)$

$$R_1 = \frac{\beta_1 T}{\mu(\delta_1+\mu)} \quad \text{and} \quad R_2 = \frac{\beta_2 T}{\mu(\alpha+\sigma+\delta_2+\mu)}$$

III. EXAMINATION OF EQUILIBRIUM POINTS FOR STABILITY

This section determines the system's equilibrium points (1) and look into their stability. We examined equilibrium points based on stability. Two equilibrium points characterize the model in this instance.

- Point of free equilibrium of aberrant learning behaviour
- Point of endemic equilibrium (aberrant learning behaviour consistently exists yet has a specific range)

Equilibrium points of the system (1) may be found by resolving the following equations:

$$\frac{dA}{dt} = T - \beta_1 AB - \beta_2 AC - \mu A$$

$$\frac{dB}{dt} = \beta_1 AB + \alpha C - \delta_1 B - \mu B$$
$$\frac{dC}{dt} = \beta_2 AC - \alpha C - \delta_2 C - \sigma C - \mu C$$
$$\frac{dD}{dt} = \delta_1 B + \delta_2 C - Md$$

(1) Equilibrium point free from aberrant learning behaviour $E_0(A_0, B_0, C_0, D_0) = (\frac{T}{\mu}, 0, 0, 0)$

When there is no aberrant learning behaviour among the students, then the equilibrium point is free from the impact of learning behaviour.

(2) Point of endemic equilibrium (aberrant learning behaviour consistently exists yet has a specific range) $E^*(A_0, B_0, C_0, D_0) = (A^*, B^*, C^*, D^*)$

When a learner starts to exhibit aberrant learning behaviour or when a group of students that is "influenced" by learning behaviour but not zero. In the study of slow learners, the point of endemic equilibrium explores the consequence social interactions and motivational effect. Now the endemic equilibrium point $E^*(A_0, B_0, C_0, D_0) = (A^*, B^*, C^*, D^*)$

Where $A^* = \frac{k_1}{\beta_2}$

$$B^* = \frac{(k_1\alpha\mu - T\beta_2\alpha)}{-\beta_1 k_1 \alpha - k_1\beta_2(\delta+\mu) + \beta_1 {k_1}^2} = \frac{k_2}{k_3}$$

$$C^* = \frac{k_2}{k_3}\left[\frac{\beta_2(\delta+\mu) - \beta_1 k_1}{\alpha\beta_2}\right] = \frac{k_2}{k_3}.k_4$$

$$D^* = \frac{\delta_1}{\mu}\frac{k_2}{k_3} + \frac{\delta_2}{\mu}\frac{k_2}{k_3}\left[\frac{\beta_2(\delta+\mu) - \beta_1 k_1}{\alpha\beta_2}\right] = \frac{k_2}{\mu\,\alpha\beta_2 k_3}[\delta_1\alpha\beta_2 + \delta_2\beta_2(\delta+\mu) - \delta_2\beta_1 k_1]$$

$$D^* = \frac{\delta_1}{\mu}\frac{k_2}{k_3} + \frac{\delta_2}{\mu}\frac{k_2}{k_3}.k_4$$

Where $k_1 = \alpha + \sigma + \delta_2 + \mu$

$$k_2 = k_1\alpha\mu - T\beta_2\alpha)$$
$$k_3 = -\beta_1 k_1 \alpha - k_1\beta_2(\delta+\mu) + \beta_1 {k_1}^2$$
$$k_4 = \left[\frac{\beta_2(\delta+\mu) - \beta_1 k_1}{\alpha\beta_2}\right]$$

A. Equilibrium point free from aberrant learning behaviour of students and its stability

Lemma 1: If $R_0 \le 1$, the aberrant behaviour of the student does not affect the population, then the equilibrium point free from aberrant learning behavior of students is locally asymptotically stable in a feasible area and unstable if $R_0 > 1$, learning behaviour does not spread.

Proof: The jacobian matrix of system (1) is defined as

$$J_{E_0} = \begin{bmatrix} -\beta_1 B - \beta_2 C - \mu & -\beta_1 A & -\beta_2 A & 0 \\ \beta_1 B & \beta_1 A - \delta_1 - \mu & \alpha & 0 \\ \beta_2 C & 0 & \beta_2 A - (\alpha+\sigma+\delta_2+\mu) & 0 \\ 0 & \delta_1 & \delta_2 & -\mu \end{bmatrix}$$

At free equilibrium $E_0 = (\frac{T}{\mu}, 0,0,0)$, the characteristic equation of system (1)

$$J_{E_0} = \begin{bmatrix} -\mu & -\beta_1 T/\mu & -\beta_2 T/\mu & 0 \\ 0 & \beta_1 T/\mu - \delta_1 - \mu & \alpha & 0 \\ 0 & 0 & \beta_2 T/\mu - (\alpha+\sigma+\delta_2+\mu) & 0 \\ 0 & \delta_1 & \delta_2 & -\mu \end{bmatrix}$$

Now the cofactor expansion of the above jacobian matrix, two eigen values of the above matrix

$$\lambda_1 = -\mu \quad \text{or} \quad \lambda_2 = -\mu$$

Cofactor expansion of the above jacobian matrix

$$J_{E_0} = \begin{bmatrix} \beta_1 T/\mu - \delta_1 - \mu & \alpha \\ 0 & \beta_2 T/\mu - (\alpha+\sigma+\delta_2+\mu) \end{bmatrix}$$

Now the other two eigen values λ_3 & λ_4 can be determined by finding the determinant of the remaining above matrix and expressed as

$$\lambda_3 = \beta_1 T/\mu - \delta_1 - \mu$$
$$\lambda_4 = \beta_2 T/\mu - (\alpha+\sigma+\delta_2+\mu)$$

All of the Eigen values in the above Jacobian matrix must be negative when $R_0 < 1$ for asymptotic stability.

Here $\lambda_1 = -\mu < 0$

$$\lambda_2 = -\mu < 0$$
$$\lambda_3 = \beta_1 T/\mu - \delta_1 - \mu < 0 \quad \text{and}$$
$$\lambda_4 = \beta_2 T/\mu - (\alpha+\sigma+\delta_2+\mu) < 0$$
$$\beta_1 T/\mu - \delta_1 - \mu < 0$$
$$\beta_2 T/\mu - (\alpha+\sigma+\delta_2+\mu) < 0$$
$$\frac{\beta_1 T}{\mu} < \delta_1 + \mu$$
$$\beta_2 T/\mu < (\alpha+\sigma+\delta_2+\mu)$$
$$\frac{\beta_1 T}{\mu(\delta_1+\mu)} < 1$$
$$\frac{\beta_2 T}{\mu\,(\alpha+\sigma+\delta_2+\mu)} < 1$$
$$R_1 < 1$$
$$R_2 < 1$$

Hence the equilibrium point $E_0 = (\frac{T}{\mu}, 0,0,0)$ free from aberrant learning behavior of students is locally asymptotically stable in a feasible area if $R_0 \le 1$.

B. Stability of endemic equilibrium points

Lemma 2: If $R_0 > 1$ and $R_0 < 1$, then the endemic equilibrium point $E^*(A_0, B_0, C_0, D_0) = (A^*, B^*, C^*, D^*)$ is locally asymptotically stable and unstable in a feasible area respectively.

Proof: Jacobian matrix of system (1) at endemic equilibrium point $E^*(A^*, B^*, C^*, D^*)$ is defined as

$$J_{E^*} = \begin{bmatrix} -\beta_1 B^* - \beta_2 C^* - \mu & -\beta_1 A^* & -\beta_2 A^* & 0 \\ \beta_1 B^* & \beta_1 A^* - \delta_1 - \mu & \alpha & 0 \\ \beta_2 C^* & 0 & \beta_2 A^* - (\alpha+\sigma+\delta_2+\mu) & 0 \\ 0 & \delta_1 & \delta_2 & -\mu \end{bmatrix}$$

$$E^*(A^*, B^*, C^*, D^*) = E^*\left(\frac{k_1}{\beta_2}, \frac{k_2}{k_3}, \frac{k_2 k_4}{k_3}, \frac{\delta_1}{\mu}\frac{k_2}{k_3} + \frac{\delta_2}{\mu}\frac{k_2 k_4}{k_3}\right)$$

$$J_{E^*} = \begin{bmatrix} -\beta_1 \frac{k_2}{k_3} - \beta_2 \frac{k_2 k_4}{k_3} - \mu & -\beta_1 \frac{k_1}{\beta_2} & -\beta_2 \frac{k_1}{\beta_2} & 0 \\ \beta_1 \frac{k_2}{k_3} & \beta_1 \frac{k_1}{\beta_2} - \delta_1 - \mu & \alpha & 0 \\ \beta_2 \frac{k_2 k_4}{k_3} & 0 & 0 & 0 \\ 0 & \delta_1 & \delta_2 & -\mu \end{bmatrix}$$

Now the cofactor expansion of above jacobian matrix, one eigen value of above matrix $\lambda_1 = -\mu$

Cofactor expansion of above jacobian matrix

$$J_{E^*} = \begin{bmatrix} -(\beta_1 + k_4\beta_2)\frac{k_2}{k_3} - \mu & -\beta_1 \frac{k_1}{\beta_2} & -k_1 \\ \beta_1 \frac{k_2}{k_3} & \beta_1 \frac{k_1}{\beta_2} - \delta_1 - \mu & \alpha \\ \beta_2 \frac{k_2 k_4}{k_3} & 0 & 0 \end{bmatrix}$$

Now the characteristic equation of jacobian matrix J_{E^*}

$$|J_{E^*} - \lambda I| = 0$$

$$c_0\lambda^3 - c_1\lambda^2 + c_2\lambda - c_3 = 0$$

Here $c_0 = 1$

$$c_1 = -(\beta_1 + k_4\beta_2)\frac{k_2}{k_3} + \beta_1 \frac{k_1}{\beta_2} - \delta_1 - 2\mu$$

$$c_2 = \frac{k_1 k_2 k_4}{k_3}(\beta_2 - \beta_1) + \left\{\delta_1 \beta_1 \frac{k_2}{k_3} + \left(\beta_2 \frac{k_2 k_4}{k_3} + \mu\right)(\delta_1 + \mu)\right\}$$

$$c_3 = -\frac{k_2 k_4}{k_3}\left[\alpha\beta_1 k_1 + k_1\beta_2\{\delta_1 + \mu\} - k_1{}^2 \beta_2\right]$$

Explore the stability of the above characteristic equation from the Routh Hurwitz stability criterion. Now

$$s_0\lambda^3 + s_1\lambda^2 + s_2\lambda + s_3 = 0$$

Where $s_0 = c_0$, $s_1 = -c_1$, $s_2 = c_2$ and $s_3 = -c_3$

In accordance with the stability criteria of Routh-Hurwitz, if $S_1 > 0$, $S_2 > 0$ and $S_1 S_2 - S_3 > 0$. In that case, the characteristic equation's roots are all negative. Therefore, in feasible area the endemic equilibrium point $E^*(A^*, B^*, C^*, D^*)$ is locally asymptotically stable if $R_0 > 1$ and unstable if $R_0 < 1$.

IV. VALIDATION OF THE MODEL

Figure 1 illustrates the proposed model, which adapts the SIR framework to investigate student learning behavior and its impact on academic progress. To solve the differential equations, MATLAB was employed using the Runge-Kutta method. Notably, the results obtained from this model outperform those reported in [10] for both scenarios where the basic reproduction number $R_0 < 1$ and $R_0 > 1$.

V. RESULT AND DISCUSSION

In the present study, the epidemic mathematical model is simulated for $R_0 < 1$ and $R_0 > 1$, as applicable that explore the student learning behavior.

Example-1: This scenario examines the local stability analysis of a student's aberrant learning behavior. The results indicate that the equilibrium point of aberrant learning behavior is locally asymptotically stable within a feasible region. A detailed analysis of the equilibrium points, supported by numerical simulations and graphical representations (Figures 2-4), reveals insights into ineffective students' aberrant learning behavior, with corresponding simulated data presented in Table 2.

The following are the data utilized in the simulation: the initial values are A = 0.5, B = 0.2, C = 0.2 and D = 0.1 with the following other parametric values T = 0.8, μ = 0.4, α = 0.2, β1 = 0.15, β2 = 0.18, δ1 = 0.3, δ2 = 0.3, σ = 0.1. The reproduction number R_0 or epidemic threshold value, is computed as 0.36. It is plainly seen from the Figures. 2, 3 and 4 that, the equilibrium point E_0 is stable, when $R_0 < 1$. Under the condition where R0 < 1, the system exhibits local stability of the infection-free equilibrium. The proportion of 'slow learner individuals' increases significantly over time and reaches a high equilibrium. The proportion of 'improved learners' also increases, though to a much lesser extent, eventually reaching a stable equilibrium. Conversely, the proportion of 'slow learners but not disturbed others' and 'slow learners but disturbed others' decreases over time, approaching zero. This indicates that when the basic reproduction number is less than one, the 'slow learner individuals' dominate the student population, and the 'slow learners' groups diminish over time, while some students eventually become 'improved learners'."

Example-2: Local stability of endemic equilibrium point has been numerically pretended to depict situation by the graphical representation as shown in figures 5, 6, and 7. Here, the starting point is considered the same as above: A = 0.5, B = 0.2, C = 0.2 and D = 0.1 with the given values of parameters T = 0.8, μ = 0.4, β1 = 0.72, β2 = 0.7, δ1 = 0.2, δ2 = 0.2, σ = 0.1. The value of reproduction number R_0 is calculated as 1.5. It is clearly observed in Figures 5, 6, and 7 that the Equilibrium point $E^*(A_0, B_0, C_0, D_0) = (A^*, B^*, C^*, D^*)$ to be stable for non-zero values, demonstrating the stability of endemic equilibrium, when $R_0 > 1$. The graph illustrates the local stability of the infection-free equilibrium when R0 > 1. Specifically, it shows the changes in the proportions of different student categories over time: "slow learner individuals," "slow learners but not disturbed others," "slow learner but disturbed others," and "improved learners." The "improved learners" category shows the greatest rate of increase over time, while the "slow learners but disturbed others" category decreases, eventually approaching zero. The "slow learner individuals" initially increase, and then decrease to stabilize at a specific value over time. Finally, the "slow learners but not disturbed others" increase to a stable value over time and the corresponding data for this effective aberrant learning behavior, which is listed in Table [3].

Table-2 Slow learners distribution of individual classes against time for an aberrant learning behavior $R_0 < 1$

Parameter Values	Time (t)	Slow learner individuals (A)	Slow learner individuals but not disturb others (B)	Slow learner individuals but disturbed others (C)	Improved learners (D)
A = 0.5	0	0.5	0.2	0.2	0.1
B = 0.2	1.0632	0.9894	0.127	0.0799	0.1498
C = 0.2	4.2312	1.6822	0.0342	0.0075	0.214
D = 0.1	7.6167	1.9084	0.0089	0.0008	0.2326
T = 0.8	9.7139	1.9579	0.0039	0.0002	0.2363
$\mu = 0.4$	11.1416	1.9752	0.0022	0.0001	0.2376
$\alpha = 0.2$	14.8016	1.9937	0.0005	0	0.2388
$\beta_1 = 0.15$	17.43	1.9977	0.0002	0	0.2391
$\beta_2 = 0.18$	19.7824	1.999	0.0001	0	0.2392
$\delta_1 = 0.3$,	22.2824	1.9996	0	0	0.2392
$\delta_2 = 0.3$	24.368	1.9998	0	0	0.2392
$\sigma = 0.1$	25	1.9999	0	0	0.2392

Table-3 Slow learners' distribution of individual classes against the time for an aberrant learning behavior $R_0 > 1$

Parameter Values	Time (t)	Slow learner individuals (A)	Slow learner individuals but not disturb others (B)	Slow learner individuals but disturbed others (C)	Improved learners (D)
A = 0.5	0	0.5	0.2	0.2	0.1
B = 0.2	1.0632	0.9894	0.127	0.0799	0.1498
C = 0.2	4.2312	1.6822	0.0342	0.0075	0.214
D = 0.1	7.6167	1.9084	0.0089	0.0008	0.2326
T = 0.8	9.7139	1.9579	0.0039	0.0002	0.2363
$\mu = 0.4$	11.1416	1.9752	0.0022	0.0001	0.2376
$\alpha = 0.1$	14.8016	1.9937	0.0005	0	0.2388
$\beta_1 = 0.72$	17.43	1.9977	0.0002	0	0.2391
$\beta_2 = 0.7$	19.7824	1.999	0.0001	0	0.2392
$\delta_1 = 0.2$,	22.2824	1.9996	0	0	0.2392
$\delta_2 = 0.2$	24.368	1.9998	0	0	0.2392
$\sigma = 0.1$	25	1.9999	0	0	0.2392

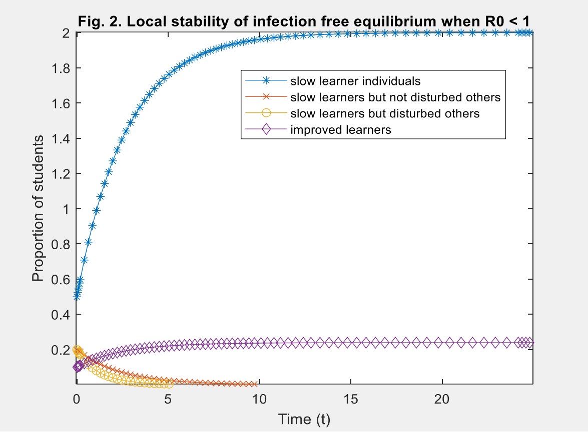

Fig. 2. Local stability of infection free equilibrium when R0 < 1

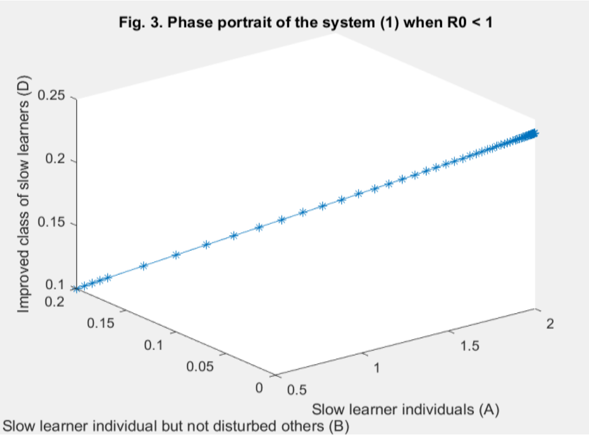

Fig. 3. Phase portrait of the system (1) when R0 < 1

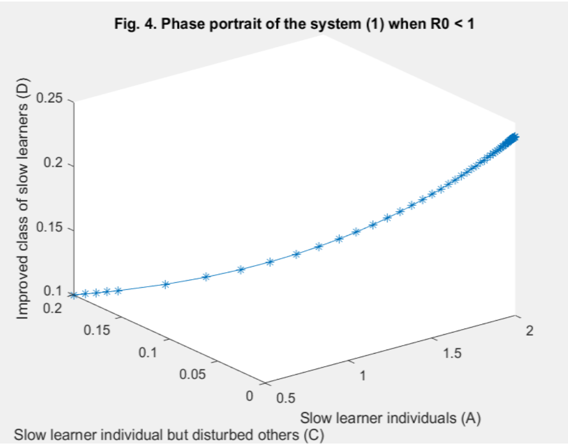

Fig. 4. Phase portrait of the system (1) when R0 < 1

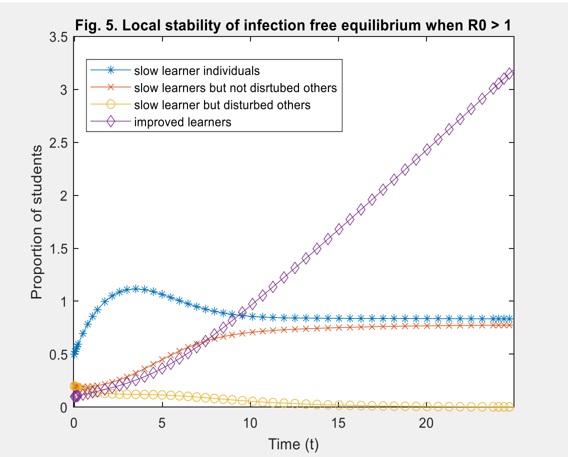

Fig. 5. Local stability of infection free equilibrium when R0 > 1

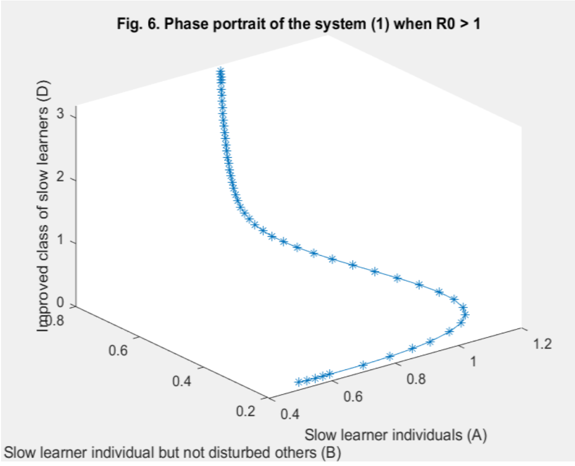

Fig. 6. Phase portrait of the system (1) when R0 > 1

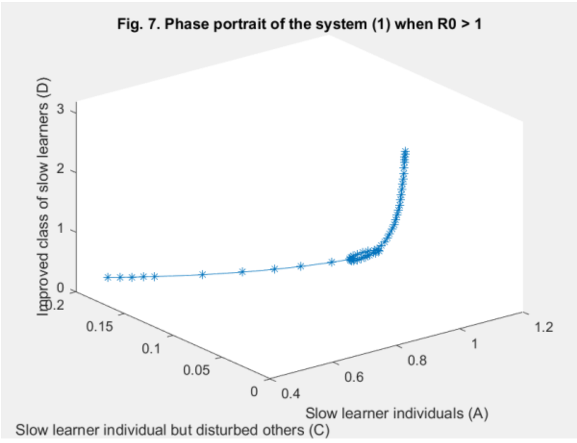

Fig. 7. Phase portrait of the system (1) when R0 > 1

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