



On Fixed Points of Expansion Mappings in Menger Spaces

A.S. Saluja

Department of Mathematics, Institute for Excellence in Higher Education (IEHE), Bhopal – 462016, Madhya Pradesh, India.

ARTICLE INFO	ABSTRACT
<p>Published Online: 20 March 2025</p> <p>Corresponding Author: A.S.Saluja</p>	<p>The aim of this paper is to establish a common fixed point theorem for expansion mappings involving six mappings in a Menger space, utilizing the concepts of semi-compatibility and weak compatibility while considering the continuity of the mappings.</p> <p>2020 Matheematics Subject Classification: Primary: 54H25; Secondary: 54E50, 47H10.</p>
<p>KEYWORDS: Compatible maps, Semi compatible maps, Weak compatible maps, Expansion mappings, t-norm.</p>	

1. INTRODUCTION

Metric spaces have undergone numerous generalizations over time. One such generalization is the Menger space, introduced by Menger in 1942 [12], where distribution functions replace nonnegative real numbers as metric values. Schweizer and Sklar [22] further explored this concept, and significant developments in Menger space theory were later contributed by Sehgal and Bharucha-Reid [18].

Sessa [19] introduced the concept of weakly commuting maps in metric spaces, which was later extended by Jungck [8] to include compatible maps. Mishra [13] further expanded this idea in Menger spaces by incorporating the notions of weak compatibility and compatibility of self-maps.

In this paper, we establish a common fixed point theorem for expansion mappings involving six mappings in Menger spaces, considering the conditions of weak compatibility and semi-compatibility.

2. PRELIMINARY

Definition 2.1- A triangular norm $*$ (shortly t -norm) is a binary operation on the unit interval $[0,1]$ such that for all $a, b, c, d \in [0,1]$ the following conditions are satisfied.

- (a) $a * 1 = a$
- (b) $a * b = b * a$
- (c) $a * b \leq c * d$ whenever $a \leq c$ & $b \leq d$
- (d) $a * (b * c) = (a * b) * c$

Examples of t -norm are $a * b = \max \{a + b - 1, 0\}$ and $a * b = \min \{a, b\}$.

Definition 2.2- A distribution function is a function $F : [-\infty, \infty] \rightarrow [0, 1]$ which is left continuous on \mathbb{R} , non-decreasing and $F(-\infty) = 0, F(\infty) = 1$.

We will denote by \mathcal{D} the family of all distribution functions on $[-\infty, \infty]$. H is special element of \mathcal{D} defined by

$$H(t) = \begin{cases} 0 & \text{if } t \notin \mathbb{R} \\ 1 & \text{if } t > 0 \end{cases}$$

If X is nonempty set, $F : X \times X \rightarrow \mathcal{D}$ is called a probabilistic distance on X and $F(x, y)$ is usually denoted by F_{xy} .

Definition 2.3 (Schweizer and Sklar [16])-The ordered pair (X, F) is called a probabilistic metric space (shortly PM space) if X is nonempty set and F is a probabilistic distance satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,

- (FM0) $F_{xy}(t) = 1$ iff $x = y$
- (FM1) $F_{xy}(0) = 0$
- (FM2) $F_{xy}(t) = F_{yx}(t)$
- (FM3) $F_{xz}(t) = 1, F_{zy}(s) = 1 \Rightarrow F_{xy}(t + s) = 1$

The ordered triplet $(X, F, *)$ is called Menger space if (X, F) is a PM-space, $*$ is a t -norm and the following condition is also satisfies: for all $x, y, z \in X$ and $t, s > 0$,

$$(FM 4) \quad F_{xy}(t + s) \geq F_{xz}(t) * F_{zy}(s).$$

Proposition 2.4 (Sehgal and Bharucha-Reid [13]) - Let (X, d) be a metric space. Then the metric d induced a distribution function F defined by $F_{xy}(t) = H(t - d(x, y))$ for all $x, y \in X$ and $t > 0$. If t -norm $*$ is $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$ then $(X, F, *)$ is a Menger space. Further, $(X, F, *)$ is complete Menger space if (X, d) is complete.

Definition 2.5 (Mishra [14]) - Let $(X, F, *)$ be a Menger space and $*$ be a continuous t -norm.

(a) A sequence $\{x_n\}$ in X is said to be convergent to some point x in X iff for every $\epsilon > 0$ and $l \in (0, 1)$ there exist an integer $n_0 = n_0(\epsilon, l)$ such that $F_{x_n x}(\epsilon) > 1 - l$ for all $n \geq n_0$.

(b) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if for $\epsilon > 0$ and $l \in (0, 1)$ There exist an integer $n_0 = n_0(\epsilon, l)$ such that $F_{x_n x_{n+p}}(\epsilon) > 1 - l$ for all $n \geq n_0$ and $p > 0$.

(c) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Remark 2.6- If $*$ is continuous t -norm, it follows from (FM 4) that the limit of sequence in Menger space is uniquely determined.

Definition 2.7 (Singh and Jain [21]) - Self maps A and B of Menger space $(X, F, *)$ are said to be weakly compatible if they commute at their coincidence points, that is $ABx = BAx$ for some $x \in X$.

Definition 2.8 (Mishra [14]) - Self maps A and B of Menger space $(X, F, *)$ are said to be compatible if $F_{ABx_n BAx_n}(t) \geq 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \rightarrow x$ for some x in X as $n \rightarrow \infty$.

Definition 2.9- Self maps A and B of Menger space $(X, F, *)$ are said to be a semi compatible if $\lim_{n \rightarrow \infty} F_{ABx_n Bx}(t) \geq 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = x, \lim_{n \rightarrow \infty} Bx_n = x$ as $n \rightarrow \infty$ for some x in X .

Lemma 2.10- Let $\{x_n\}$ be a sequence in a Menger space $(X, F, *)$ with continuous t -norm $*$ and $t * t \leq t$. If there exist a constant $k > 1$ such that $F_{x_{n-1}x_n}(kt) \geq F_{x_n x_{n+1}}(t)$ For all $t > 0$ and $n = 1, 2, \dots$, then $\{x_n\}$ is a Cauchy sequence in X .

Lemma 2.11- Let $(X, F, *)$ be a Menger space. If there exist $k > 1$ such that $F_{xy}(kt) \geq F_{xy}(t)$ For all $x, y \in X$ and $t > 0$, then $x = y$.

3. MAIN RESULTS

Theorem 3.1- Let A, B, S, T, L and M be self maps on a complete Menger space $(X, F, *)$ with $t * t \leq t$ for all $t \in [0, 1]$ satisfying,

- (a) $ST(X) \subseteq L(X)$, $AB(X) \subseteq M(X)$
- (b) For all $x, y \in X$ and $t > 0$ there exist a constant $k > 1$ such that

$$F^2_{LxMy}(kt) \geq \max \left\{ \begin{array}{l} F_{ABxLx}(t/2) * F_{LxSTy}(t/2) * F_{MySTy}(t) \\ F_{LxSTy}(kt/2) * F_{MySTy}(kt/2) \end{array} \right\}$$

- (c) Either AB or L is continuous.
- (d) $AB = BA, ST = TS, LB = BL$ and $MT = TM$.
- (e) Pair (L, AB) is semi compatible and (M, ST) is weak compatible.

Then A, B, S, T, L and M have unique common fixed point in X .

Proof: - Let $x_0 \in X$ is any arbitrary point. As $ST(X) \subseteq L(X)$ and $AB(X) \subseteq M(X)$, there exist $x_1 \in X, x_2 \in X$ such that $STx_0 = Lx_1 = y_1$ and $ABx_1 = Mx_2 = y_2$, inductively we have $STx_{2n-1} = Lx_{2n} = y_{2n}$ and $ABx_{2n} = Mx_{2n+1} = y_{2n+1}$ for $n = 1, 2, 3, \dots$

Step (1) - Using (b) with $x = x_{2n}$ and $y = x_{2n+1}$

$$F^2_{Lx_{2n}Mx_{2n+1}}(kt) \geq \max \left\{ \begin{array}{l} F_{ABx_{2n}Lx_{2n}}(t/2) * F_{Lx_{2n}STx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{Lx_{2n}STx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{array} \right\}$$

$$F^2_{y_{2n}y_{2n+1}}(kt) \geq \max \left\{ \begin{array}{l} F_{y_{2n+1}y_{2n}}(t/2) * F_{y_{2n}y_{2n+2}}(t/2) * F_{y_{2n+1}y_{2n+2}}(t) \\ F_{y_{2n}y_{2n+2}}(kt/2) * F_{y_{2n+1}y_{2n+2}}(kt/2) \end{array} \right\}$$

$$F^2_{y_{2n}, y_{2n+1}}(kt) \text{ \& } F_{y_{2n+1}, y_{2n+2}}(t) F_{y_{2n}, y_{2n+1}}(kt)$$

$$F_{y_{2n}, y_{2n+1}}(kt) \text{ \& } F_{y_{2n+1}, y_{2n+2}}(t)$$

Similarly it can be found that

$$F_{y_{2n+1}, y_{2n+2}}(kt) \text{ \& } F_{y_{2n+2}, y_{2n+3}}(t)$$

Therefore for all n even or odd we have $F_{y_n, y_{n+1}}(kt) \text{ \& } F_{y_{n+1}, y_{n+2}}(t)$.

Thus $\{y_n\}$ is a Cauchy sequence. Since X is complete then $\{y_n\}$ converges to some point z in X . Or all subsequence $\{STx_{2n-1}\}, \{Lx_{2n}\}, \{Mx_{2n+1}\}$ and $\{ABx_{2n}\}$ also converges to z .

Case (a) – When map AB is continuous.

Step (2)– Since $\lim_{n \rightarrow \infty} Lx_{2n} = z$ and $\lim_{n \rightarrow \infty} ABx_{2n} = z$. Since

AB is continuous map then $\lim_{n \rightarrow \infty} ABLx_{2n} = ABz$ and

$$\lim_{n \rightarrow \infty} ABABx_{2n} = ABz$$

Also (AB, L) is semi compatible map then if

$$\lim_{n \rightarrow \infty} ABx_{2n} = \lim_{n \rightarrow \infty} Lx_{2n} = z$$

Therefore $\lim_{n \rightarrow \infty} LABx_{2n} = ABz$.

Using (b) with $x = ABx_{2n}, y = x_{2n+1}$

$$F^2_{LABx_{2n}, Mx_{2n+1}}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{ABABx_{2n}, LABx_{2n}}(t/2) * F_{LABx_{2n}, STx_{2n+1}}(t/2) * F_{Mx_{2n+1}, STx_{2n+1}}(t) \\ &F_{LABx_{2n}, STx_{2n+1}}(kt/2) * F_{Mx_{2n+1}, STx_{2n+1}}(kt/2) \end{aligned} \right\}$$

Limiting $n \rightarrow \infty$ We have

$$F^2_{zABz}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{ABzABz}(t/2) * F_{zABz}(t/2) * F_{zz}(t) \\ &F_{zABz}(kt/2) * F_{zz}(kt/2) \end{aligned} \right\}$$

$$F^2_{zABz}(kt) \text{ \& } F_{zABz}(t/2) F_{zABz}(kt/2)$$

$$F^2_{zABz}(kt) \text{ \& } F_{zABz}(t) F_{zABz}(kt)$$

$$F_{zABz}(kt) \text{ \& } F_{zABz}(t)$$

$$ABz = z$$

Step (3)- Using (b) with $x = z, y = x_{2n+1}$

$$F^2_{Lz, Mx_{2n+1}}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{ABzLz}(t/2) * F_{Lz, STx_{2n+1}}(t/2) * F_{Mx_{2n+1}, STx_{2n+1}}(t) \\ &F_{Lz, STx_{2n+1}}(kt/2) * F_{Mx_{2n+1}, STx_{2n+1}}(kt/2) \end{aligned} \right\}$$

Now limiting $n \rightarrow \infty$ we have

$$F^2_{zLz}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{zLz}(t/2) * F_{zLz}(t/2) * F_{zz}(t) \\ &F_{zLz}(kt/2) * F_{zz}(kt/2) \end{aligned} \right\}$$

$$F^2_{zLz}(kt) \text{ \& } F_{zLz}(t/2) F_{zLz}(kt/2)$$

$$F^2_{zLz}(kt) \text{ \& } F_{zLz}(t) F_{zLz}(kt)$$

$$F_{zLz}(kt) \text{ \& } F_{zLz}(t) \Rightarrow Lz = z$$

Step (4) - Since $LB = BL$ then $LBz = BLz = Bz$

Also $AB = BA \Rightarrow ABBz = BABz = Bz$

By using (b) with $x = Bz, y = x_{2n+1}$

$$F^2_{LBz, Mx_{2n+1}}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{ABzLBz}(t/2) * F_{LBz, STx_{2n+1}}(t/2) * F_{Mx_{2n+1}, STx_{2n+1}}(t) \\ &F_{LBz, STx_{2n+1}}(kt/2) * F_{Mx_{2n+1}, STx_{2n+1}}(kt/2) \end{aligned} \right\}$$

$$F^2_{LBz, Mx_{2n+1}}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{BzBz}(t/2) * F_{zBz}(t/2) * F_{zz}(t) \\ &F_{zBz}(kt/2) * F_{zz}(kt/2) \end{aligned} \right\}$$

$$F^2_{zBz}(kt) \text{ \& } F_{zBz}(t/2) F_{zBz}(kt/2)$$

$$F^2_{zBz}(kt) \text{ \& } F_{zBz}(t) F_{zBz}(kt)$$

$$F_{zBz}(kt) \text{ \& } F_{zBz}(t) \Rightarrow Bz = z$$

Step (5) - Since $Lz = z$ and $ST(X) \subseteq L(X)$

Then there exists a point $w \in X$ such that $Lz = STw$

By using (b) with $x = x_{2n}, y = w$

$$F^2_{Lx_{2n}, Mw}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{ABx_{2n}, Lx_{2n}}(t/2) * F_{Lx_{2n}, STw}(t/2) * F_{Mw, STw}(t) \\ &F_{Lx_{2n}, STw}(kt/2) * F_{Mw, STw}(kt/2) \end{aligned} \right\}$$

Now limiting $n \rightarrow \infty$ we have

$$F^2_{zMw}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{zz}(t/2) * F_{zLz}(t/2) * F_{MwLz}(t) \\ &F_{zLz}(kt/2) * F_{MwLz}(kt/2) \end{aligned} \right\}$$

$$F^2_{zMw}(kt) \text{ \& } F_{zMw}(t) F_{zMw}(kt/2)$$

$$F^2_{zMw}(kt) \text{ \& } F_{zMw}(t) F_{zMw}(kt)$$

$$F_{zMz}(kt) \text{ \& } F_{zMw}(t) \Rightarrow Mw = z$$

Therefore $STw = Mw$

Since (ST, M) is weak compatible, then $STMw = MSTw$

$$\Rightarrow STz = Mz$$

Step (6) – Using (b) with $x = x_{2n}, y = z$

$$F^2_{Lx_{2n}, Mz}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{ABx_{2n}, Lx_{2n}}(t/2) * F_{Lx_{2n}, STz}(t/2) * F_{Mz, STz}(t) \\ &F_{Lx_{2n}, STz}(kt/2) * F_{Mz, STz}(kt/2) \end{aligned} \right\}$$

$$F^2_{zMz}(kt) \text{ \& } \max \left\{ \begin{aligned} &F_{zz}(t/2) * F_{zMz}(t/2) * F_{Mz, Mz}(t) \\ &F_{zMz}(kt/2) * F_{Mz, Mz}(kt/2) \end{aligned} \right\}$$

$$F^2_{zMz}(kt) \text{ \& } F_{zMz}(t/2) F_{zMz}(kt/2)$$

$$F^2_{zMz}(kt) \text{ \& } F_{zMz}(t) F_{zMz}(kt)$$

$$F_{zMz}(kt) \text{ \& } F_{zMz}(t) \Rightarrow Mz = z.$$

Step (7) – Since $MT = TM$ therefore $MTz = TMz = Tz$

Again Since $ST = TS$ therefore $STTz = TSTz = Tz$

Now using (b) with $x = x_{2n}, y = Tz$

$$F^2_{Lx_{2n}MTz}(kt) \text{ £max} \left\{ \begin{array}{l} F_{ABx_{2n}Lx_{2n}}(t/2) * F_{Lx_{2n}STz}(t/2) * F_{MTzSTz}(t) \\ F_{Lx_{2n}STz}(kt/2) * F_{MTzSTz}(kt/2) \end{array} \right\}$$

Now limiting $n \in \mathbb{N}$ we have

$$F^2_{zTz}(kt) \text{ £max} \left\{ \begin{array}{l} F_{zz}(t/2) * F_{zTz}(t/2) * F_{TzTz}(t) \\ F_{zTz}(kt/2) * F_{TzTz}(kt/2) \end{array} \right\}$$

$$F^2(kt) \text{ £ } F_{zTz}(t/2).F_{zTz}(kt/2)$$

$$F^2_{zTz}(kt) \text{ £ } F_{zTz}(t).F_{zTz}(kt)$$

$$F_{zTz}(kt) \text{ £ } F_{zTz}(t) \Rightarrow Tz = z$$

Since $STz = z \Rightarrow Sz = z$

Therefore $Az = Bz = Sz = Tz = Lz = Mz = z$, that is z is the common fixed point of the six maps.

Case (b) – When map L is continuous.

Step (8) – Since $\lim_{n \in \mathbb{N}} ABx_{2n} = \lim_{n \in \mathbb{N}} Lx_{2n} = z$ and

$$\lim_{n \in \mathbb{N}} LABLx_{2n} = \lim_{n \in \mathbb{N}} LLx_{2n} = Lz$$

Also pair (AB, L) is semi compatible pair since

$$\lim_{n \in \mathbb{N}} ABx_{2n} = \lim_{n \in \mathbb{N}} Lx_{2n} = z$$

$$\lim_{n \in \mathbb{N}} ABLx_{2n} = Lz$$

By using (b) with $x = Lx_{2n}, y = x_{2n+1}$

$$F^2_{LLx_{2n}Mx_{2n+1}}(kt) \text{ £max} \left\{ \begin{array}{l} F_{ABLx_{2n}LLx_{2n}}(t/2) * F_{LLx_{2n}STx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{LLx_{2n}STx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{array} \right\}$$

Now limiting $n \in \mathbb{N}$ we have

$$F^2_{zLz}(kt) \text{ £max} \left\{ \begin{array}{l} F_{zLz}(t/2) * F_{zLz}(t/2) * F_{zz}(t) \\ F_{zLz}(kt/2) * F_{zz}(kt/2) \end{array} \right\}$$

$$F^2_{zLz}(kt) \text{ £ } F_{zLz}(t/2).F_{zLz}(kt/2)$$

$$F^2_{zLz}(kt) \text{ £ } F_{zLz}(t).F_{zLz}(kt)$$

$$F_{zLz}(kt) \text{ £ } F_{zLz}(t) \Rightarrow Lz = z.$$

Step (9) - Again using (b) with $x = z, y = x_{2n+1}$

$$F^2_{LzAx_{2n+1}}(kt) \text{ £max} \left\{ \begin{array}{l} F_{ABzLz}(t/2) * F_{LzSTx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{LzSTx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{array} \right\}$$

Now limiting $n \in \mathbb{N}$ we have

$$F^2_{zz}(kt) \text{ £max} \left\{ \begin{array}{l} F_{zABz}(t/2) * F_{zz}(t/2) * F_{zz}(t) \\ F_{zLz}(kt/2) * F_{zz}(kt/2) \end{array} \right\}$$

$$F^2_{zz}(kt) \text{ £ } F_{zABz}(t/2).F_{zLz}(kt/2)$$

$$F^2_{zz}(kt) \text{ £ } F_{zABz}(t).F_{zLz}(kt)$$

$$F_{zz}(kt) \text{ £ } F_{zABz}(t)$$

$$\Rightarrow 1 \text{ £ } F_{zABz}(t) \Rightarrow ABz = z$$

$$\Rightarrow ABz = Az = z.$$

Rest proof is same as **step-4** onward.

Uniqueness: - Let u be another common fixed point of A, B, S, T, L and M then

$$Au = Bu = Su = Tu = Lu = Mu = u.$$

By using (b) with $x = u, y = x_{2n+1}$

$$F^2_{LuMx_{2n+1}}(kt) \text{ £max} \left\{ \begin{array}{l} F_{ABuLu}(t/2) * F_{LuSTx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{LuSTx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{array} \right\}$$

Now limiting $n \in \mathbb{N}$ we have

$$F^2_{uz}(kt) \text{ £max} \left\{ \begin{array}{l} F_{uu}(t/2) * F_{uz}(t/2) * F_{zz}(t) \\ F_{uz}(kt/2) * F_{zz}(kt/2) \end{array} \right\}$$

$$F^2_{uz}(kt) \text{ £ } F_{uz}(t/2).F_{uz}(kt/2)$$

$$F^2_{uz}(kt) \text{ £ } F_{uz}(t).F_{uz}(kt)$$

$F_{uz}(kt) \text{ £ } F_{uz}(t) \Rightarrow u = z$. This completes the proof of the theorem.

Example 3.2: - Let $x \in [0,1]$ with the metric d defined by

$$d(x, y) = |x - y| \text{ and defines } F_{xy}(t) = H(t - d(x, y))$$

for all $x, y \in X, t > 0$. Clearly $(X, F, *)$ is a complete Menger space where t-norm $*$ is defined by $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$. Let A, B, S, T, L and M be maps from X into itself defined as $A(x) = x/3, B(x) = x, S(x) = x/2, T(x) = x, L(x) = x/3, M(x) = x/4$ for all $x \in X$. Then

$$L(X) = \{0, 1/3\} \in \{0, 1/2\} = ST(X) \text{ and}$$

$$M(X) = \{0, 1/4\} \in \{0, 1/3\} = AB(X). \text{ Clearly}$$

$$AB = BA, MT = TM, ST = TS, LB = BL \text{ and } AB, L$$

are continuous. Moreover $\{x_n\}$ is a sequence such that $\lim_{n \in \mathbb{N}} x_n = 0$.

Since $\lim_{n \in \mathbb{N}} ABx_n = \lim_{n \in \mathbb{N}} Lx_n = 0$ for $0 \in X$,

then $\lim_{n \in \mathbb{N}} ABLx_n = \lim_{n \in \mathbb{N}} LABx_n$. Therefore (AB, L) is compatible.

Also M and ST are weakly compatible at 0.

Condition (b) of main theorem satisfy equal condition for $k = 2, t = 1/3$ and $x = 1, y = 1/2$ where $\{1, 1/2\} \in X$

and less than condition for $k = 2, t = 1/20$ and

$x = 0, y = 1/2$ where $\{0, 1/2\} \in X$. Thus all the

condition of main theorem satisfied and 0 is the unique common fixed point of A, B, S, T, L and M .

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Corollary: - Let A, B, S, T be self maps on a complete Menger space $(X, F, *)$ with $t * t \leq t$ for all $t \in [0, 1]$ satisfying,

(a) $A(X) \subset T(X), B(X) \subset S(X)$

(b) For all $x, y \in X$ and $t > 0$ there exist a constant $k > 1$ such that

$$F_{AxBy}^2(kt) \leq \max \left\{ \begin{array}{l} F_{SxAx}(t/2) * F_{AxTy}(t/2) * F_{ByTy}(t) \\ F_{AxTy}(kt/2) * F_{ByTy}(kt/2) \end{array} \right\}$$

(c) Either A or S is continuous .

(d) Pair (A, S) is semi compatible and (B, T) is weak compatible .

Then A, B, S & T have unique common fixed point in X .

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