



The Three Machines Sequencing Problems

Dr. A. Sridhar¹, Swathi²

¹Associate Professor, Department of Mathematics, Government Thirumagal Mill's College, Gudiyatham, Tamilnadu, India.

²Post Graduate Student, Department of Mathematics, Government Thirumagal Mill's College, Gudiyatham, Tamilnadu, India.

ARTICLE INFO	ABSTRACT
Published Online: 12 May 2025	Sequencing problems have found numerous applications across various domains due to their ability to understand patterns and relationships within sequential data. A sequencing model refers to tasks where you need to determine the optimal order or arrangement of items or events according to certain criteria. They find applications in various fields. Sequencing models are powerful tools that require a solid understanding of mathematical concepts, problem formulation, and appropriate solution techniques to effectively address complex real-world challenges
Corresponding Author: Dr. A. Sridhar	
KEYWORDS: Sequencing models, positive difference, max- min, john's method.	

1. INTRODUCTION

Sequencing problems are quite common in real life. They arise whenever there is a choice of determining the order in which a number of jobs can be performed. These problems play a very important role in manufacturing setups for the optimal use of resources and/or the customer's satisfaction. Many times, manufacturing firms produce goods made up of several components, which are sourced from other manufacturing firms. The final product undergoes many processes before it is taken to the market for sale. Since several machines are used in the production process, it is important to sequence the production process optimally so that the performance measures of the firm are satisfied. Let us suppose that we have n machines and m jobs. Then we have $(n!)^m$ possible sequences.

2. BASIC DEFINITIONS

Number of machines - It refers to the number of service facilities through which a job must pass before it is assumed to be completed.

Processing time - This is the time required by each job on each machine.

Precessing order - This refers to the order (sequence) in which machines are required for completing the job.

Idle time on a machine - This is the time during which a machine does not have a job to process.

Total elapsed time - This is the time interval between starting the first job and completing the last job, including the idle

time (if any), in a particular order by the given set of machines.

No passing rule - This means that the passing is not allowed, i.e., the same order of jobs is maintained over each machine. If n jobs are to be processed through two machines A and B in the order AB , then this means that each job will go to machine A first and then to B .

3. BASIC ASSUMPTIONS OF THE MODEL

- The processing time on each machine is known.
- The time required to complete a job is independent of the order of jobs in which they are to be processed.
- No machine can process more than one job simultaneously.
- The time taken by each job in changing over from one machine to another is negligible.
- Each job, once started on a machine is to be performed up to completion on that machine.
- The order of completion of job has no significance, i.e., no job is to be given priority.
- A job starts on the machine as soon as the job and the machine both are idle.

4. TYPES OF JOB SEQUENCING PROBLEM

Various types of job sequencing problem arise in the world. All sequencing problems cannot be solved.

Type I. n jobs are to be processed on 2 machines, say, machine A and machine B in the order AB . This means that

“The Three Machines Sequencing Problems”

each job is to be processed first on machine A and then on machine B.

Type II. n jobs are to be processed on 3 machines A, B and C in the order ABC,

Type III. n jobs are to be processed on m machines in the given order

Type IV. 2 jobs are to be processed on m machines in the given order.

Here, we consider only type II job sequencing problems with two different types of algorithms.

5. CONDITIONS

An optimal solution to this problem can be obtained if either or both of the following conditions hold good:

(a) The minimum processing time on machine A is atleast as great as the maximum processing time on machine B.

$$\text{That is, } \min t_{1j} \geq \max t_{2j}, \text{ for } j=1,2,3,\dots,n$$

(b) The minimum processing time on machine C is atleast as the maximum processing time on machine B.

$$\text{That is, } \min t_{3j} \geq \max t_{2j}, \text{ for } j=1,2,3,\dots,n.$$

6. ALGORITHM FOR PROPOSED MODEL I

Step 1: Examine the processing times of the given jobs on all three machines and if either one or both the above conditions hold, then go to step 2; otherwise, the algorithm fails.

Step 2: Introduce two fictitious machines, say G and H, with corresponding processing times given by,

i) $t_{Gj} = t_{1j} - t_{2j}, j=1,2,3,\dots,n$ i.e., the processing time of machine G is a positive difference of $t_{1j} - t_{2j}, j=1,2,3,\dots,n.$

ii) $t_{Hj} = t_{2j} - t_{3j}, j=1,2,3,\dots,n$ i.e., the processing time of machine H is a positive difference of $t_{2j} - t_{3j}, j=1,2,3,\dots,n.$

Step 3: Eliminate the job which has already been assigned from further consideration, and repeat steps 1 and 2 until an optimal sequence is found.

Example: 6.1

Find the optimal job sequence, total elapsed time and idle time for the following

Jobs	1	2	3	4	5	6	7
Machine A	4	9	8	5	10	9	8
Machine B	5	4	3	6	2	5	4
Machine C	7	8	6	12	6	7	13

Solution:

Convert 3 machine problem into 2 machine is

Jobs	1	2	3	4	5	6	7
Machine G	1	5	5	1	8	4	4
Machine H	2	4	3	6	4	2	9

The optimal sequence is

1	4	7	5	2	3	6
---	---	---	---	---	---	---

The total minimum elapsed time is obtained from the following table:

Jobs	Machine A		Machine B		Machine C	
	In time	Out time	In time	Out time	In time	Out time
1	0	4	4	9	9	16
4	4	9	9	16	16	28
7	9	17	17	28	28	41
5	17	27	27	41	41	47
2	27	36	36	47	47	55
3	36	44	44	55	55	61
6	44	53	53	61	61	68

“The Three Machines Sequencing Problems”

Total minimum elapsed time = 68 mins

Idle time for Machine A = 15 mins

Idle time for Machine B = 39 mins

Idle time for Machine C = 9 mins

7. ALGORITHM FOR PROPOSED MODEL I

Step 1:

Examine the processing times of the given jobs on all three machines and if either one or both the above conditions in section 5 hold, then go to next step; otherwise, the algorithm fails.

Step 2:

Introduce two fictitious machines, say G and H, with corresponding processing times given by,

- i) Compare machine A and machine B in each job choose the maximum value. Assume this maximum value as a processing time of G
- ii) Compare machine B and machine C in each job choose the maximum value. Assume this maximum value as a processing time of H

Example 7.1 Find the optimal job sequence, total elapsed time and idle time for the following

Jobs	1	2	3	4	5	6	7
Machine A	4	9	8	5	10	9	8
Machine B	5	4	3	6	2	5	4
Machine C	7	8	6	12	6	7	13

Solution:

Convert 3 machine problem into 2 machine as follows

Jobs	1	2	3	4	5	6	7
Machine G	5	9	8	6	10	9	8
Machine H	7	8	6	12	6	7	13

The optimal sequence is

1	4	7	2	6	3	5
----------	----------	----------	----------	----------	----------	----------

The total minimum elapsed time is obtained from the following table:

Jobs	Machine A		Machine B		Machine C	
	In time	Out time	In time	Out time	In time	Out time
1	0	4	4	9	9	16
4	4	9	9	15	16	28
7	9	17	17	21	28	41
2	17	26	26	30	41	49
6	26	35	35	40	49	56
3	35	43	43	46	56	62
5	43	53	53	55	62	68

Total minimum elapsed time = 68 mins

Idle time for Machine A = 15 mins

Idle time for Machine B = 39 mins

Idle time for Machine C = 9 mins

8. CONCLUSION

The real-world application based on sequencing problem is considered and one of its models is solved. The sequencing model is considered the main part and the existing algorithm to find optimal sequence and total elapsed time are studied in detail. In all proposed models we get same answers in john's method.

REFERENCES

1. H. A. Taha, Operations Research, Eighth Edition, Pearson Education India, 2008.
2. A. Sridhar, R. Allah Pitchai, “Two Server Queueing System with Single and Batch Service”, *International Journal of Applied Operational Research*, Vol. 4, No. 2, pp. 15-26, Spring 2014.
3. A. Sridhar, R. Allah Pitchai, “Analyses of a Markovian queue with two heterogeneous servers and working vacation”, *International Journal of Applied Operational Research*, Vol. 5, No. 4, pp. 1-15, Autumn 2015.
4. A. Sridhar, R. Allah Pitchai, “A Single and Batch Service Queueing System With Additional Service Station”, *International Journal of Advanced Computer and Mathematical Sciences*, Vol4, Issue3, 2013, pp199-209.
5. A. Sridhar, R. Allah Pitchai, “New Approach to Solve Unbalanced Transportation Problems Using Least Cost Method”, *Journal of Emerging Technologies and Innovative research*, September 2018, Volume 5, Issue 9.
6. A. Sridhar, R.S. Suganthi, “Solution of Transportation Problems Using Summation Method”, *International Journal of Mathematical Archive*, 10(5), 2019, 40-44.
7. A. Fischer and C. Helmberg, “The symmetric quadratic travelling salesman problem,” *Mathematical Programming A*, vol. 142, no. 1-2, pp. 205–254, 2013.
8. A. Fischer, F. Fischer, G. Jager, J. Keilwagen, P. Molitor, and I. Grosse, “Exact algorithms and heuristics for the quadratic travelling salesman problem with an application in bioinformatics,” *Discrete Applied Mathematics*, vol. 166, pp. 97–114, 2014.