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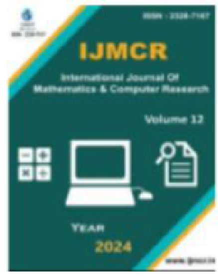

Extended Modified Sub-Equation Method for Solving Nonlinear Travelling Wave Equation

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ABSTRACT
We considered the third order nonlinear Schrodinger equation (TONSE) that models the wave pulse transmission in a time period less than one-trillionth of a second. We extended modified sub-equation method to obtain numerous exact travelling wave solutions containing sets of generalized hyperbolic. Trigonometric and rational solutions that are more general than classical ones. We followed analytical mathematical metod by constructed the transformation groups which help us using vector fields and Lie symmetry groups. We discussed the dynamic behavior and structure of the exact solutions for distinct solution of arbitrary constants .We obtained the symmetry reductions and direct of the equation. In addition to group-invariant solutions which are Jacobi elliptic function and exponential type.

KEYWORDS: Exact Travelling Wave Equation, Schrodinger Equation, Extended Modified Method , Lie Symmetry Groups , Invariant Solutions.

1. INTRODUCTION

It is observed that most of the physical phenomena occurring in nature are mathematically modeled by the evolution equations. However, we know from the empirical result that many important physical processes are the type of nonlinear evolution equations

$$F(x, t, u_t, u_x, u_{xx}, u_{xxx}, \dots) = 0 \quad (1)$$

[1].Well-known Korteweg- de Vries equation

$$u_t + 6uu_x + u_{xxx} = 0 \quad (2)$$

Represents shallow water waves solution which isspecial solitary travelling wave-unchanged wave velocity andshape after interaction is a generalized wave packet. Forthese reasons, a travelling solution is used not only in waterwave theory, but also in optical communication. Solutionsare derived from the sensitive between nonlinear and dispersive terms. It is desirable that travelling wave solution transmission in communication systems should be high speed [7,8]. For example, the (1+1)-dimensional nonlinear Schrodinger equation

$$iq_t + 2|q|^2 q + q_{xx} = 0 \quad (3)$$

We considered the third order equations

$$iq_x + \alpha_2(q_{tt} + 2q|q|^2) - i\alpha_3(q_{ttt}+6q_t|q|^2)=0 \quad (4)$$

From the hierarchy ofthehigherordergivenin[7].Ourmain goal is to obtain exactanalyticalsolutionsofthisequation. There are many methodsintheliteraturetoobtainthe solutions of nonlinearSchrodingerequation.Recently,the sub-equation extendedmethodareintroducedin[8].What makes this methodinterestingisthat,unlikeothermethods that its solutionsincludethegeneralizedtypeofhyperbolic and trigonometricfunctions.Thismethodhasafiniteseries expansion formbalancingprinciple.Inordertoovercomethis deficiency, thismethodwhichisveryeffectiveandpractical for solving nonlineardifferentialequationsinmathematical physics was usedtoobtainthesolutionunderconsideration. Another approachdiscussedinthestudyisLietechnique.In this algorithmicmethodbasedonthefindingof transformation groupsthatleavetheequationinvariant, reduced equationsandgroupinvariantsolutionscanbe obtained. The waveandgroupinvariantsolutionsofequation (4) will be investigatedwiththehelpofthesetwomethods. The reduction of equation(4)isgivenin(4),Liegroups method is employed tostudyequation(4).

2. MATHEMATICAL MODEL

Inspite of the factthat equation (3) is successful in the describing a great number of nonlinear effects, it may be necessary to modify the experimental conditions. Therefore higher-order effects should be considered for the transmission of pulses to sub-picoseconds and femtoseconds which has better performance on the transmitting information. The higher-order integrable hierarchy can be presented as

$$iq_x+\alpha_2(q_{tt}+2q|q|^2)-i\alpha_3(q_{ttt}+6q_t|q|^2)+\alpha_4(q_{tttt}+6q_t^{*}+4q|q_t|^2+8|q|^2q_{tt}+6|q|^4q)-i\alpha_5(q_{ttttt}+10|q|^2q_{ttt}+30|q|^4q_t+10qq_tq_{tt}^{*}+10qq_{tt}^{*}+10qq_tq_{tt}^{*}+10q_tq_t^{2*}+20qq_tq_{tt}^{*})+\cdots=0, \quad (5)$$

were $q(x,t)$ represents the normalized complex amplitude of the optical pulse envelope, asterisk represents the conjugation, $\alpha(I=2,3,4,\ldots)$ are real constant parameters, x denotes the propagation variable and t denotes the transverse variable (time in a moving frame) [11]. We investigated the equation (4) which we have obtained by taking $\alpha_m=0, m=4,5,\ldots$. we aimed to simplify the equation (4). Thus we are seeking solutions of (4) with the following structure

$$q(x,t)=p(\zeta)e^{i\varphi(x,t)}, \varphi(x,t)=-kx+\omega t+\theta, \quad (6)$$

where $\zeta=x-vt$ is the wave variable and $p(\zeta)$ is an amplitude component of the soliton solution. Here v and k are the velocity and frequency of the soliton, respectively. ω is the soliton wave number and θ is phase constant. If we use the transformation in the equation (4) and separate the real and imaginary parts, a pair of relations emerges. The real part equation gives

$$(2\alpha_2+6\alpha_3\omega)p^3+(k-\alpha_2\omega^2-\alpha_3\omega^3)p+(\alpha_2+3\alpha_3w)v^2p''=0, \quad (7)$$

And imaginary part equation reads

$$\alpha_3p'''v^3+p'+(6\alpha_3p^2p'-2\alpha_2\omega p'-3\alpha_3\omega^2p')v=0. \quad (8)$$

Integrating equation (8) once and setting the integration constant to zero we obtain

$$2v\alpha_3p^3+(1+v(-2\alpha_2\omega-3\alpha_3\omega^2))p+\alpha_3v^3p''=0. \quad (9)$$

Equations (7) and (9) will be equivalent provided that

$$\frac{2v\alpha_3}{(2\alpha_2+6\alpha_3\omega)}=\frac{1+v(-2\alpha_2\omega-3\alpha_3\omega^2)}{k-\alpha_2\omega^2-\alpha_3\omega^3}=\frac{\alpha_3v^3}{(\alpha_2+3\omega)v^2}.$$

Hence, one can find the following parametric constraints,

$$\alpha_3=\frac{\alpha^2}{-3w+v}, k=-\frac{-4v\alpha_2\omega^2+2v\alpha_2\omega^2+3\omega^3-v}{-3\omega+v}. \quad (10)$$

Eventually, equations (7) and (9) can be rearranged to be in the form

$$p''+\frac{2}{v^2}p^3+\frac{(1+v(-2\alpha_2\omega-3\alpha_3\omega))^2}{\alpha_3v^3}p=0. \quad (11)$$

The solutions of the equation (11) will be examined using the extended modified sub-equation method.

3. BASIC IDEAS OF THE EXTENDED MODIFIED SUB-EQUATION METHOD

We presentedbriefly themain steps ofthe extendedmodifiedsub-equationmethodfor finding travelling wave equations [7]. Firstly, we considered the general of the type

$$P(u,u_t,u_x,u_{tt},u_{xx},\ldots)=0. \quad (12)$$

Using the wave transformation

$$u(x,t)=U(\zeta), \qquad \zeta=x-tv,$$

we can rewrite equation (12) as the following nonlinear ordinary differential equation

$$Q(U,U',U'',U''',\ldots)=0. \quad (13)$$

Let us assume that the solution of ordinary differential equation (13) can be written as a polynomialof $R(\zeta)$ as follows:

$$U(\zeta)=\sum_{j=-n}^{n}b_jR(\zeta)^i, \qquad b_j\neq0, \quad (14)$$

where $b_j(-n\leq j\leq n)$ are constants which we will be determined later. $R(\zeta)$ in (14) satisfies thenonlinear ordinary differential equation in the form

$$R'(\zeta)=\ln(A)(S_0+S_1R(\zeta)+S_2R(\zeta)^2), \qquad A\neq0,1. \quad (15)$$

The coefficient classifications and corresponding solution forms of (15) are as follows:

Case1: If $\Delta=S_1^2-4S_0S_2<0, S_2\neq0$ then $R(\zeta)=-\frac{S_1}{2S_2}+\frac{\sqrt{-\Delta}\tan_A(\frac{\sqrt{-\Delta}}{2}\zeta)}{2S_2}$

$$R_2(\zeta)=-\frac{S_1}{2S_2}-\frac{\sqrt{-\Delta}\cot_A(\frac{\sqrt{-\Delta}}{2})}{2S_2}, \qquad R_3(\zeta)=-\frac{S_1}{2S_2}+\frac{\sqrt{-\Delta}(\tan_A(\sqrt{-\Delta}\zeta)\pm\sqrt{rp}\sec_A(\sqrt{-\Delta}\zeta))}{2S_2},$$

$$R_4(\zeta)=-\frac{S_1}{2S_2}-\frac{\sqrt{-\Delta}(\cot_A(\sqrt{-\Delta}\zeta)\pm\sqrt{rP}(\sqrt{-\Delta}\zeta))}{2S_2},$$

$$R5(\zeta)=-\frac{S_1}{2S_2}+\frac{-\sqrt{-\Delta}\,(tan_A(\frac{\sqrt{-\Delta}}{4}\zeta)-cot_A(\frac{\sqrt{-\Delta}}{4}))}{4S_2}.$$

Case2: If $\Delta=S^2{}_1-4S_0S_2>0, S_2\neq0$, then

$$R6(\zeta)=-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}tanh_A(\frac{\sqrt{\Delta}}{2}\zeta)}{2S_2}, R_7(\zeta)=-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}coth_A(\frac{\sqrt{\Delta}}{2}\zeta)}{2S_2},$$

$$R8(\zeta)=-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}(coth_A(\sqrt{\Delta}\zeta)\pm\sqrt{rp}csch_A(\sqrt{\Delta}\zeta))}{2S_2},$$

$$R9(\zeta)=-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}(tanh_A(\sqrt{\Delta}\zeta)\pm\sqrt{rp}csch_A(\sqrt{\Delta}\zeta))}{2S_2},$$

$$R10(\zeta)=-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}\,(tanh_a(\frac{\sqrt{\Delta}}{4}\zeta)+coth(\frac{\sqrt{\Delta}}{4}\zeta))}{4S_2}.$$

Case 3: If $S_0=S_2, S_1=0$. Then

$$R11(\zeta)=tan_A(S_0\zeta), R_{12}(\zeta)=-cot_A(S_0\zeta), R_{13}(\zeta)=tan_A(2S_0\zeta)\pm\sqrt{rp}sec_A(2S_0\zeta)$$

$$R14(\zeta)=-cot_A(2S_0\zeta)\pm\sqrt{rp_A}(2S_0\zeta),$$

$$R15(\zeta)=\frac{1}{2}tan_A(\frac{S_0}{2}\zeta)-\frac{1}{2}coth_A(\frac{S_0}{2}\zeta).$$

Case 4: If $S_0=-S_2, S_1=0, then$

$$R16(\zeta)=-tanh_4(S_0\zeta), R_{17}(\zeta)=-coth_4(S_0\zeta), R_{18}(\zeta)-tanh_4(2S_0\zeta)\pm\sqrt{rp}sech_4(2S_0\zeta),$$

$$R19(\zeta)=-coth_4(2S_0\zeta)\pm\sqrt{rp}csch_4(2S_0\zeta),$$

$$R20(\zeta)=-\frac{1}{2}tanh_4(\frac{S_0}{2}\zeta)-\frac{1}{2}coth_4(\frac{S_0}{2}\zeta).$$

Case 5: If $S^2{}_1-4S_0S_2=0\ then$

$$R21(\zeta)=-2\frac{S0(S_1\zeta\ ln(A)+2)}{S1\zeta\ln(A)}.$$

Case 6: If $S_1=\lambda, S_0=m\lambda, m\neq0\ and\ S_2=0\ then$

$$R22(\zeta)=A^{\lambda\zeta}-m.$$

Case 7: If $S1=0, S_2=0\ then$

$$R23(\zeta)=S_0\zeta\ ln(A).$$

Case 8: If $S0=0, S_1=0$ then

$$R24(\zeta)=-\frac{1}{S2\ln(A)\zeta}.$$

Case 9: If $S_0=0, S_1\neq0$ then

$$S25(\zeta)=-\frac{rS_1}{S_2(cosh(S_1\zeta)-sinh_A(S_1\zeta)+r)}.$$

$$R26(\zeta)=-\frac{(cosh_A(S_1\zeta)+sinh_A(S_1\zeta))S_1}{S2(cosh_A(S_1\zeta)+sinh_A(S_1\zeta)+p)}.$$

Case 10:If $S_1=\lambda, S_2=m\lambda, m\neq0$ and $S_0=0$ then

$$R27(\zeta)=\frac{rA^{\lambda\zeta}}{p-mrA^{\lambda\zeta}}.$$

The generalizedtrigonometricandhyperbolicfunctionsusedinthefamiliesgivenaboveareddefinedasfollows:

$$tan_A(\xi)=\frac{-i(rA^{i\xi}-pA^{-i\xi})}{rA^{i\xi}+pA^{-i\xi}},\quad tanh^A(\xi)=\frac{i(rA^{\xi}-pA^{-\xi})}{rA^{\xi}+pA^{-\xi}},$$

$$cot_A(\xi)=\frac{i(rA^{i\xi}+pA^{-i\xi})}{rA^{i\xi}-pA^{-\xi}},\quad coth^A(\xi)=\frac{rA^{\xi}+pA^{-\xi}}{rA^{\xi}-pA^{-\xi}},$$

$$cos_A(\xi)=\frac{rA^{i\xi}+pA^{-i\xi}}{2},\quad coshA(\xi)=\frac{rA^{\xi}+pA^{-\xi}}{2},$$

$$sin_A(\xi)=\frac{-i(rA^{i\xi}-pA^{-i\xi})}{2},\quad sinh_A(\xi)=\frac{rA^{\xi}-pA^{-\xi}}{2},$$

$$csc_A(\xi)=\frac{2i}{rA^{i\xi}-pA^{-\xi}},\qquad cschA(\xi)=\frac{2}{rA^{\xi}-pA^{-\xi}},$$

$$sec_A(\xi)=\frac{2}{rA^{i\xi}+pA^{i\xi}},sech(\xi)=\frac{2}{rA^{\xi}+pA^{-\xi}}\qquad(16)$$

In equation (16), ξ is an independentvariableconstants $r,p>0$ aredeformationparametersin n is a positive integer (14) that can be determined by the balancing procedureconstructedtakingintoaccountthehighestordernonlinearterms and the highest order linear terms in the resulting equation.Byusingequationintoequation (14) and equation (15) consisting of the (13) power of $R(\xi)$ has to be equal tozero.Henceweobtainedanalgebraicsystemofequationsintermsof $b_{-n},\ldots,b_{-1},b_0,b_1,\ldots,b_n$. Bydetermining these parametersandrewritingtheequationusingdetermined (14) parametersananalytic solution $u(x,t)$ is obtained in closed form.

4. EXACT MODIFIED SOLUTIONS OF TRAVELLINGWAVEEQUATION

We obtained the analytical solutions for the amplitude of the travelling wave equations by using the extended modified sub-equation method. Substituting $p(\zeta)=\sum_{i=-nj}^{n} bR^i(\zeta)$ intoequation (11) andbalancing p'' with p^3 yields $n=1$. Therefor equation (11) admits the use of

$$p(\zeta)=b_{-1}R(\zeta)^{-1}+b_0+b_1R(\zeta).\qquad(17)$$

Substituting equation (17) into equation (11) through equation (15) and collecting the coefficients of different powers of $R(\zeta)$ setting each coefficient to zero we get the system of algebraic equations. By solving the resulting system with the help of maple the following results are achieved.

Set 1

After the hung calculations we deduce the following relations between parameters appearing algebraic

$$\omega=\frac{2v^2\alpha_2+3+\sqrt{4v4\alpha2\,9-24v4\alpha2\,2(ln(A))^2 S_2S_0+6v4\alpha2\,2(ln(A))^2S_1^2}}{6v\alpha2},$$

$$b-1=iS_0ln(A)v,\ b_0=\frac{ivln(A)S_1}{2},b_1=0,$$

Where α_2,α_3,v,k are arbitrary constants we now can constructtheexactsolutions ofequation (4) easily for these parameters set through the classification cases which is given in (3).

Case1: If $\Delta=S_1^2-4S_0S_2<0, S_2\neq0$, then we have

$$q1(x,t)=\left(\frac{iS_1ln(A)v}{2}+iv\,ln(A)S_0\times\left(\frac{-S_1}{2S_2}-\frac{\sqrt{-\Delta}cot_A\left(\frac{\sqrt{-\Delta}(x-vt)}{2}\right)}{2S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q2(x,t)=\left(\frac{iS_1ln(A)v}{2}+iv\,ln(A)S_0\times\left(\frac{-S_1}{2S_2}-\frac{\sqrt{-\Delta}\,\overline{cot}_A(x-vt)}{2S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)}$$

$$q3(x,t)=\left(\frac{iS_1ln(A)v}{2}+iv\,ln(A)S_0\times\left(-\frac{S_1}{2S_2}+\frac{\sqrt{-\Delta}(tanA(\sqrt{-\Delta}(x-vt)))\pm\sqrt{rp}\,secA(\sqrt{-\Delta}(x-vt))}{2S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)}$$

$$q4(x,t)=\left(\frac{iS_1ln(A)v}{2}+iv\,ln(A)S_0\times\left(\frac{-S_1}{2S_2}-\frac{\sqrt{-\Delta}(cotA(\sqrt{-\Delta}(x-vt)))\pm\sqrt{tp}A(\sqrt{-\Delta}(x-vt))}{2S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)}$$

$$q5(x,t)=\left(\frac{iS_1ln(A)v}{2}+iv\,ln(A)S_0\times\left(-\frac{S_1}{2S_2}+\frac{\sqrt{-\Delta}\left(tanA\left(\frac{\sqrt{-\Delta}(x-vt)}{4}\right)-cotA(\sqrt{-\Delta}(x-vt))\right)}{4S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)}.$$

Case 2: If $\Delta=S^2_1-4S_0S_2>0, S_2\neq 0$, then we obtain

$$q6(x,t)=\left(\frac{iS_1 ln(A)v}{2}+ivln(A)S_0\times\left(-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}tanh_A(\sqrt{\Delta}(x-vt))}{2S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q7(x,t)=\left(\frac{iS_1 ln(A)v}{2}+iv\,ln(A)S_0\times\left(-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}coth_A(\frac{\sqrt{\Delta}(x-vt)}{2})}{2S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q8(x,t)=\left(\frac{iS_1 ln(A)v}{2}+ivln(A)S_0\times\left(-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}(coth_A(\sqrt{\Delta}(x-vt)))\pm\sqrt{rp}csch_A(\sqrt{\Delta}(x-vt))}{2S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q9(x,t)=\left(\frac{iS_1 ln(A)v}{2}+ivln(A)S_0\times\left(-\frac{S_1}{2S_2}-\frac{\sqrt{\Delta}(tanh_A(\frac{\sqrt{\Delta}(x-vt)}{4})+coth_A(\frac{\sqrt{\Delta}(x-tv)}{4}))}{4S_2}\right)^{-1}\right)\times e^{i(-kx+\omega t+\theta)}.$$

Case 3: If $S0=S_2, S_1=0$, then we yield

$$q10(x,t)=\frac{iv\,lnS_0e^{i(-kx+\omega t+\theta)}}{tan_A(S_0(x-vt))},$$

$$q11(x,t)=-\frac{iv\,ln(A)S_0e^{i(-kx+\omega t+\theta)}}{cot_A(S_0(x-vt))},$$

$$q12(x,t)=\frac{iv\,ln(A)S_0e^{i(-kx+\omega t+\theta)}}{tan_A(2S_0(x-vt))\pm\sqrt{rt}sec_A(2S_0(x-vt))},$$

$$q13(x,t)=\frac{iv\,ln(A)S_0e^{i(-kx+\omega t+\theta)}}{-cot_A(2S_0(x-vt))\pm\sqrt{rp_A}(2S_0(x-vt))},$$

$$q14(x,t)=\frac{2iv\,ln(A)S_0e^{i(-kx+\omega t+\theta)}}{tan_A(\frac{S_0(x-vt)}{2})-cot_A(\frac{S_0(x-vt)}{2})}.$$

Case 4:P If $S0=-S_2, S_1=0$, then one obtains

$$q15(x,t)=\frac{-iv\,ln(A)S_0e^{i(-kx+\omega t+\theta)}}{tan_A(S_0(x-vt))},$$

$$q16(x,t)=\frac{-iv\,ln(A)S_0e^{i(-kx+\omega t+\theta)}}{coth_A(S_0(x-vt))},$$

$$q17(x,t)=\frac{iv\,ln(A)S_0e^{i(-kx+\omega t+\theta)}}{-coth_A(2S_0(x-vt))}\pm\sqrt{rp}csch_A(2S_0(x-vt)).$$

Case 5: If $S21-4S_0S_2=0$ then we attain

$$q18(x,t)=\left(\frac{iS_1 ln(A)v}{2}-\frac{iv(ln(A))^2S_1^2(x-vt)}{2(S_1(x-vt)ln(A)+2)}\right)\times e^{i(-kx+\omega t+\theta)}. \quad (19)$$

Case 6: If $S1=\lambda, S_0=m\lambda, m\neq 0$ *and* $S_2=0$ then we derive

$$q19(x,t)=\left(\frac{iS_1 ln(A)v}{2}+\frac{iv\,ln(A)S_0}{A^{\lambda(x-vt)}-m}\right)e^{i(-kx+\omega t+\theta)}.$$

Case 9: If $S_0=0, S_1\neq 0$ then we construct

$$q20(x,t)=(iS_1 ln(A)v)2^{-1}e^{i(-kx+\omega t+\theta)}.$$

Case 10: If $S_1=\lambda, S_2=m\lambda, m\neq 0$ and $S_0=0$ then we get

$$q21(x,t)=(i\lambda\,ln(A)v)2^{-1}e^{i(-kx+\omega t+\theta)}.$$

Set 2

After some calculations, the following relations are obtained between the parameters in the system of algebraic equations:

$$\omega = \frac{2v^2+3+\sqrt{4v^4\alpha^2+9-24v\alpha^2 \ (ln(A))SS+6v^4\alpha^2 \ (ln(A))S^2}}{6v\alpha_2},$$

$$b-1=0, b_0=\frac{iv\, ln(A)S_1}{2}, b_1=iS_2 ln(A)v,$$

where α_2,α_3,v,k arearbitraryconstants.Accordingtoclassificationcasesfortheseparametersin(3) we can construct the exact solutionsofequation (4) as follows:

case 1:If $\Delta=S^2{}_1-4S_0S_2<0, S_2\neq 0, then$

$$q_{22}(x,t)=\left(\frac{1}{2}i ln(A)v\sqrt{-\Delta}tan_A\left(\frac{\sqrt{-\Delta}(x-vt)}{2}\right)\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q_{23}(x,t)=\left(\frac{1}{2}i ln(A)v\sqrt{-\Delta}cot_A\left(\frac{\sqrt{-\Delta}(x-vt)}{2}\right)\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q_{24}(x,t)=\left(\frac{i\, ln(A)v\sqrt{-\Delta}(tanA(\sqrt{-\Delta}(x-vt)) \pm\sqrt{-\Delta}secA(\sqrt{-\Delta}(X-VT))}{2}\right)\times e^{i(-KX+\omega T+\theta)},$$

$$q_{25}(x,t)=-\left(\frac{i\, ln(A)v\sqrt{-\Delta}\left(cotA\left(\frac{\sqrt{-\Delta}(x-vt)}{2}\right)\right) \pm\sqrt{rp}A\left(\frac{\sqrt{-\Delta}(x-vt)}{2}\right)}{2}\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q_{26}(x,t)=\left(\frac{i\, ln(A)v\sqrt{-\Delta}(tanA(\sqrt{-\Delta}(x-vt)) -cotA(\sqrt{-\Delta}(x-vt))}{4}\right)\times e^{i(-kx+\omega t+\theta)}.$$

Case 2: If $\Delta=S^2{}_1-4S_0S_2>0, S_2\neq 0$ $then$

$$q_{27}(x,t)=\left(-i\, ln(A)v\sqrt{\Delta}tanh_A\left(\frac{\sqrt{\Delta}(x-vt)}{2}\right)\right)2^{-1}\times e^{i(-kx+\omega t+\theta)},$$

$$q_{28}(x,t)=\left(-i\, ln(A)v\sqrt{\Delta}coth_A\left(\frac{\sqrt{\Delta}(x-vt)}{2}\right)\right)2^{-1}\times e^{i(-kx+\omega t+\theta)},$$

$$q_{29}(x,t)=-\left(\frac{i\, ln(A)v\sqrt{\Delta}(cothA(\sqrt{-\Delta}(x-vt)) \pm\sqrt{rp}cschA(\sqrt{-\Delta}(x-vt))}{S2}\right)\times e^{i(-kx+\omega t+\theta)},$$

$$q_{30}(x,t)=\left(\frac{-i\, ln(A)v\sqrt{\Delta}\left(tanhA\left(\frac{\sqrt{-\Delta}}{4}(x-vt)\right)\right) +cothA\left(\frac{\sqrt{-\Delta}}{4}(x-vt)\right)}{4}\right)\times e^{i(-kx+\omega t+\theta)} \quad .(20)$$

Case 3: If $S_0=S_2$, $S_1=0$, then

$$q_{31}(x,t)=iS_0 ln(A)v tan_A(S_0(x-vt))\times e^{i(-kx+\omega t+\theta)},$$

$$q_{32}(x,t)=-iS_0 ln(A)v cot_A(S_0(x-vt))\times e^{i(-kx+\omega t+\theta)},$$

$$q33(x,t)=iS_0 ln(A)v\,(tan_A(2S_0(x-vt)))\pm\sqrt{rp}sec_A(2S_0(x-vt))e^{i(-kx+\omega t+\theta)},$$

$$q34(x,t)=iS_0 ln(A)v\,(-cot_A(2S_0(x-vt)))\pm\sqrt{rp}_A(2S_0(x-vt))e^{i(-kx+\omega t+\theta)},$$

$$q35(x,t)=\frac{iS_0 ln(A)v}{2}\left(tan_A\left(\frac{S_0(x-vt)}{2}\right)\right)-cot_A\left(\frac{S_0(x-vt)}{2}\right)e^{i(-kx+\omega t+\theta)}.$$

Case 4: If $S_0=-S_2, S_1=0$ then

$$q36(x,t)=(-iS_2 ln(A)v tanh_A(S_0(x-vt)))\times e^{i(-kx+\omega t+\theta)},$$

$$q37(x,t)=(iS_2 ln(A)v(-coth_A(2S_0(x-vt))))\pm\sqrt{rp}csch_A(2S_0(x-vt))e^{i(-kx+\omega t+\theta)}.$$

Case 5: If $S^2{}_1-4S_0S_2-0$ then

$$q38(x,t)=\left(\frac{-2iS_2vS_0(S_1(x-vt)ln(A)+2)}{S^2{}_1(x-vt)}+\frac{iv\,ln(A)S_1}{2}\right)e^{i(-kx+\omega t+\theta)}.$$

Case 6: If $S_1=\lambda, S_0=m\lambda, m\neq 0$ and $S_2=0$ then

$$q39(x,t)=\frac{iv\,ln(A)\lambda}{2}e^{i(-kx+\omega t+\theta)}.$$

Case 7: If $S_0=0, S_1=0$ then

$$q_{40}(x,t)=\frac{-iv}{(x-vt)}e^{i(-kx+\omega t+\theta)}.$$

Case 8: If $S_0=0, S_1\neq 0$ then

$$q41(x,t)=\left(\frac{-i\,ln(A)vrS_1}{cosh_A(S_1(x-vt))-sinh_A(S_1(x-vt)+r)}+\frac{iv\,ln(A)S_1}{2}\right)e^{i(-kx+\omega t+\theta)} \qquad (21)$$

5.DISCUSSIONANDRESULTS :

We appliedLiesymmetryanalysisfor equation(1)

$$iq_1+\alpha_2(q_\alpha+2q|q|^2)-i\alpha_3(q_{ttt}+6q_t+|q|^2)=0 \quad (1)$$

Firstly, weassumedthat: $q(x,t)=u(x,t)e^{iv(x,t)}$, (2) where u and v are real valued functions and we substituted equation(2) into and split up real imaginaryparts so we obtained

$$-v_xu+\alpha_2u_{tt}-\alpha_2uv_t^2+2\alpha_2u^3+3\alpha_3u_{tt}v_t+3\alpha_3u_tv_{tt}+\alpha_3uv_{ttt}-\alpha_3uv_t^3+6\alpha_3uv_t^3=0,$$
$$2\alpha_2u_tv_t+\alpha_2uv_{tt}+3\alpha_3u_tv_t^2-\alpha_3u_{ttt}-6\alpha_3uu_t+u_x+3\alpha_3uv_{tt}v_t=0 \qquad (3)$$

Witha small parameter (3), the correspondingvector field for these transformationsis

$$X=\zeta(x,t,u,v)\frac{\partial}{\partial x}+t(x,t,u,vv)\frac{\partial}{\partial t}+\eta(x,t,u,v)\frac{\partial}{\partial u}+\phi(x,t,u,v)\frac{\partial}{\partial y} \qquad (4)$$

When (4) isa vector field (or generator) the transformationgroup ofthe equationor systemconsidered is

$$\frac{d\hat{x}}{d\epsilon}=\zeta(\hat{x},\hat{t},\hat{u},\hat{v}),\ \hat{x}|_x=0=x,$$
$$\frac{d\hat{t}}{d\epsilon}=\tau(\hat{x},\hat{t},\hat{u},\hat{v}),\qquad \hat{t}|_\epsilon=0=t,$$
$$\frac{d\hat{u}}{d\epsilon}=\eta(\hat{x},\hat{t},\hat{u},\hat{v}),\ \hat{u}|_\epsilon=0=u.$$
$$\frac{d\hat{u}}{d\epsilon}=\eta(\hat{x},\hat{t},\hat{u},\hat{v}),\ \hat{v}|=0=v,$$

The third prolongation formula $Pr^{(3)}$ Xis

$$Pr^{(3)}X=x+\phi^x\frac{\partial}{\partial v_x}+\phi^t\frac{\partial}{\partial v_t}+\eta^{tt}\frac{\partial}{\partial u_t}+\frac{\partial}{\partial u_{tt}}+\phi^{tt}\frac{\partial}{\partial v_{tt}}+\phi^{ttt}\frac{\partial}{\partial v_{ttt}}+\eta^x\frac{\partial}{\partial u_x}+\phi^t\frac{\partial}{\partial v_t}+\eta^t\frac{\partial}{\partial u_t}+\phi^{tt}\frac{\partial}{\partial v_{tt}}+\eta^{ttt}\frac{\partial}{\partial u_{ttt}} \qquad (5)$$

Where $\eta^x,\phi^x,\phi^t,\eta^t,\phi^{tt},\phi^{ttt},\eta^{tt},\phi^{ttt}$ are extendedinfinitesimal.Hence thesystemof(3)hasthefollowinginvariantconditions

$$\eta(-v_x-\alpha v^2+6\alpha u^2+\alpha_3v_{ttt}-\alpha_3v_t^3+18\alpha_3v_tu)-\phi u+\eta^x(\alpha_3+3\alpha_3v_2^2)+\phi^t(-2\alpha_2uv_t-3\alpha_3u_{tt}-3\alpha_3uv_1^2+6\alpha_3u)+\alpha_3(3\eta^t v_{tt}+3\phi u_t+\phi^{tt}u)=0,$$
$$\eta(\alpha_2vtt-12\alpha_3u_tu+3\alpha_3v_tv_{tt})+\eta^t(2\alpha_2v_2+3\alpha_3v_t^2-6\alpha_3u)+\phi^t(2\alpha_2v_t+6\alpha_3u_tv_t+3\alpha_3v_{tt})+\phi^{tt}(\alpha_2u+3\alpha_3uv_t)-\eta^{ttt}\alpha_3+\eta^x=0.$$

With thehelpoftheobtainedequationpair andthevaluesof extendedinfinitesimals we gotan overdeterminedsystemofpartial differential equation.Solving overdeterminedsystem of PDEs we can be obtained the following :

$$\zeta=-3c_1x+c_3,$$
$$\tau=-ct+_1\frac{2\alpha^2{}_2c_{11}}{3\alpha_3}x+c_4,$$
$$\eta=c_1u,$$

$$\phi = \frac{\alpha_2 c_1}{3\alpha_3} t + c_2 \quad , \qquad (6)$$

From what we have discussed we can arrive at thefollowing facts : There are many methods that we can usetosolve Travelling wave equation but we preferred to usetheexact modified sub-equation method because it’s a methodthat works to solve mathematical problems in a physicalmanner and describes the follow of water in differentgeometric shapes. It also works to move from one solutiontoanother solution in a stable , smoot , logical and symmetricalwayfor nonlinear partial differential equations which basedonthe vector field method and its characterized byaccuracy

6. CONCLUSION

We considered the third ordinary nonlinearSchrodinger equation which enables studies and advancesinthespeedof information transmission that plays a majorroleinfieldssuch as ultra short pulses, optical fiber applied physics, communication system, etc,…. To contributetothestudiesof the higher order Schrodinger equation andthespecialcases of this equation in the literature .We consideredequation(4) which was considered $\propto_2$= and $\frac{1}{2}$ we studied the nonautonmouscharacteristics of the W-Shapedsolutionsand have modifiedthedarboux transformationmethodtothefirst and secondordersrespectively. As farweknewtheexact invariantsolutionsofthis equation whichincludegeneralized hyperbolic and trigonometric functions which we investigatedforthefirst time. We believethatthesolutions we haveobtainedarenew. One of theadvantagesofthe appliedmethodsinthat it contains moregeneralsolutions than mostofthemethods in the literature.Theresults obtainedbytheapplication of these methodshaveshownthat this methodiseffective, strong and applicabletoother problemsinmathematical physics.

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