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(S, D) Magic Labeling of Subdivision of Some Special Trees

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ARTICLE INFO	ABSTRACT
Published Online: 10 December 2024 **Corresponding Author:** P. Mala	Let G (p,q) be a connected, undirected, simple and non-trivial graph with p vertices and q edges. Let f be an injective function f: V(G) → $\{ s, s+d \ldots . s+(q+1)d\}$ and g be an injective function g: E(G) → $\{d, 2d, \ldots 2(q-1)d\}$.Then the graph G is said to be (s, d) magic labeling if $f(u)+g(uv)+f(v)$ is a constant, for all $u,v \in V(G)$. A graph G is called (s,d) magic graph if it admits (s,d) magic labeling. In this paper the existence of (s, d) magic labeling of subdivision on some special trees are found.

KEYWORDS: subdivision on coconut tree, symmetrical tree, Regular bamboo tree, olive tree and spider graph SP $(1^n 2^m)$

I. INTRODUCTION

Labeling is the process of assigning values to the vertices, edges, or both of a graph under specific conditions. The concept was first introduced by Rosa (1967) and Graham and Sloane (1967) and gained prominence through its application in graph theory by 1980. Researchers have shown significant interest and enthusiasm in exploring graph labeling techniques. Joseph A. Gallian provides an extensive overview of the topic in his comprehensive discussions on graph labeling. Building on these foundational studies, this paper focuses on a specific type of labeling known as *(S, d)* magic labeling. It investigates and analyzes the applicability of *(S, d)* magic labeling to various subdivision graphs, demonstrating that these graphs inherently possess this labeling property.

II. DEFINITIONS

Definition 2.1 A subdivision of a graph G is a graph formed by subdividing edges of G. Subdividing an edge e with end points u,v results in a graph with one new vertex w and an edge set that replaces e with two new edges uw and wv.

Definition 2.2. A graph $S(G)$ is formed by inserting a new vertex into each edge of graph G.

III. MAIN RESULT

Theorem 3. 1: Subdivision on coconut tree is (S,d) magic graph

Proof:

Let G=$S(CT(m,n))$ be the subdivision on coconut tree. Let u_1 $u_2, u_3 \ldots \ldots u_m$and v_1 $v_2, v_3 \ldots \ldots v_n$be subdivided by w_1 $w_2, w_3 \ldots \ldots w_{m-1}$ and x_1 $x_2, x_3 \ldots \ldots x_n$.

Here $p = 2m + 2n - 1$ and $q = 2(m + n - 1)$

Define $f: V(G) \to \{s, s+d, s+2d, \ldots . s+(q+1)d\}$ to label the vertices as follows

$f(u_{i+1}) = s + 2id{:}0 \le i \le m-1$

$f(w_{i+1}) = s + (2i+1)d{:}0 \le i \le m-2$

$f(x_i) = u_n + id; 1 \le i \le n$

$f(v_i) = x_n + id; 1 \le i \le n$

Define $g: E(G) \to \{d, 2d, 3d, \ldots .2(q-1)d\}$ to label the edges as follows

$g(u_i w_i) = 2s + 2(q-1)d - (f(u_i) + f(w_i): 1 \le i \le m-1$

$g(w_i u_{i+1}) = 2s + 2(q-1)d - (f(w_i) + f(u_{i+1}): 1 \le i \le m-1$

$g(x_i u_m) = 2s + 2(q-1)d - (f(x_i) + f(u_m): 1 \le i \le n$

$g(v_i x_i) = 2s + 2(q-1)d - (f(v_i) + f(x_i): 1 \le i \le n$

Labeling of Vertices of $S(CT(m,n))$				
Value of i	$f(u_{i+1})$	$f(w_{i+1})$	$f(x_i)$	$f(v_i)$
$0 \le i \le m-1$	$s+2id$	−	−	−
$0 \le i \le m-2$	−	$s+(2i+1)d$	−	−
$1 \le i \le n$	−	−	$u_n + id$	$x_n + id$

Labeling of edges of $S(CT(m,n))$				
Value of i	$g(u_i w_i)$	$g(w_i u_{i+1})$	$g(x_i u_m)$	$g(v_i x_i)$
$1 \le i \le m-1$	$2s+2(q-1)d - (f(u_i)+f(w_i))$	$2s+2(q-1)d - (f(w_i)+f(u_{i+1}))$	$-$	$-$
$1 \le i \le n$	$-$	$-$	$2s+2(q-1)d - (f(x_i)+f(u_m))$	$2s+2(q-1)d - (f(v_i)+f(x_i))$

From the above table we find that f *and* g are injective and $(u_i)+f(w_i)+g(u_iw_i), f(w_i)+f(u_{i+1})+g(w_iu_{i+1})$, $f(x_i)+f(u_m)+g(x_iu_m)$ and $f(v_i)+f(x_i)+g(v_ix_i)$are constant equal to 2(s+(q−1)d) .Hence we concluded that the $S(CT(m,n))$ admits (S,d) magic labeling.

Example 3.1: Subdivision on coconut tree $S(CT(5,6))$ are shown below

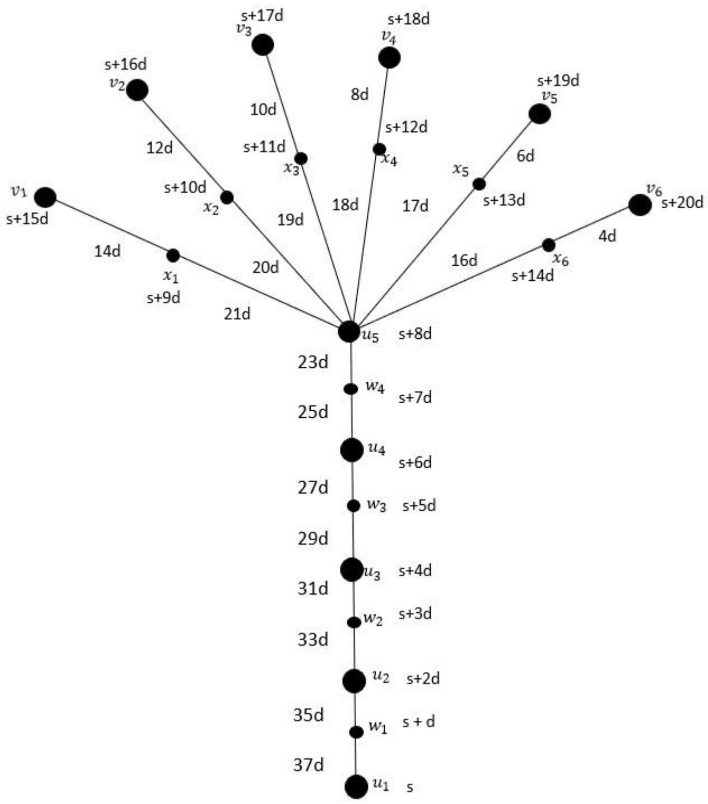

Figure 3.1: Subdivision on coconut tree $S(CT(5,6))$

Theorem 3.2: Subdivision on symmetrical tree admits (S, d) magic labeling

Proof: Let the vertices $u_0, u_1 \dots\dots u_n$ be subdivided w_1 $w_2, w_3 \dots\dots w_n$.let $p = 2^{n+2}-3$ and $q = 2^{n+2}-4$

Define $f: V(G) \to \{s, s+d, s+2d, \dots . s+(q+1)d\}$ to label the vertices as follows

$f(u_0) = s+d$

$$f\left(u_{(2^j-1)+(i-1)}\right) = s + \left(3(j^2-j+1)+(i-1)\right)d; 1 \le j \le n, 1 \le i \le 2^j$$

Let $f(w_0)$=$s-d$

$f\left(w_{(2^j-1)+(i-1)}\right) = f\left(w_{2^{j-1}}-1\right) + 2^j d + (i-1)d; 1 \le j \le n, 1 \le i \le 2^j$

Define $g: E(G) \to \{d, 2d, 3d, \dots .2(q-1)d\}$ to label the edges as follows

$g(w_ju_j) = 2s+2(q-1)d - (f(w_j)+f(u_j)); 1 \le j \le n$

$g(u_iw_{2i+1}) = 2s+2(q-1)d - (f(u_i)+f(w_{2i+1})); \quad 1 \le i \le 2^j$

$g(u_iw_{2(i+1)}) = 2s+2(q-1)d - (f(u_i)+f(w_{2(i+1)}));$ $1 \le i \le 2^j$

Labeling of Vertices of Subdivision on symmetrical tree		
Value of i	$f\left(u_{(2^j-1)+(i-1)}\right)$	$f\left(w_{(2^j-1)+(i-1)}\right)$
$1 \le j \le n, 1 \le i \le 2^j$	$s+\left(3(j^2-j+1)+(i-1)\right)d$	$f\left(w_{2^{j-1}}-1\right)+2^jd+(i-1)d$

Labeling of Edges of Subdivision on symmetrical tree			
Value of i	$g(w_j u_j)$	$g(u_i w_{2i+1})$	$g(u_i w_{2(i+1)})$
$1 \leq j \leq n$	$2s + 2(q-1)d - (f(w_j) + f(u_j))$	$-$	$-$
$1 \leq i \leq 2^j$	$-$	$2s + 2(q-1)d - (f(u_i) + f(w_{2i+1}))$	$2s + 2(q-1)d - (f(u_i) + f(w_{2(i+1)}))$

From the above table we find that f *and* g are injective and $f(w_j) + f(u_j) + g(w_j u_j), f(u_i) + f(w_{2i+1}) + g(u_i w_{2i+1})$ and $f(u_i) + f(w_{2(i+1)}) + g(u_i w_{2(i+1)})$ are constant equal to $2(s+(q-1)d)$ .Hence we concluded that the subdivision on symmetrical tree admits (S,d) magic labeling.

Example 3.2: Subdivision on Symmetrical tree is shown below

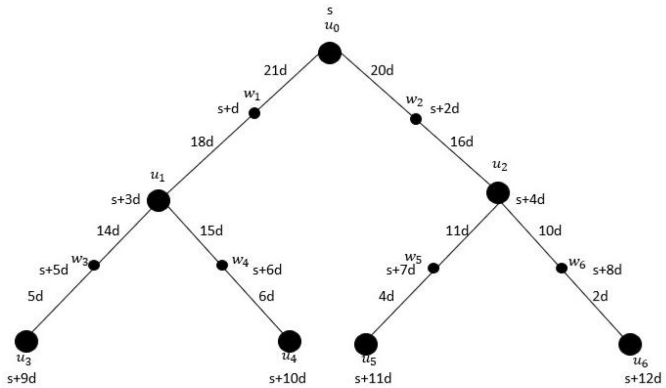

Figure 3.2: Subdivision on Symmetrical tree

Theorem 3.3: Subdivision on Regular bamboo tree admits (s,d) magic labeling

Proof: let $u_0$be a central vertex and let the vertices be $\{u_j^i; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{w_j^i; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{y_k^i; 1 \leq i \leq n, 1 \leq k \leq l\} \cup \{x_k^i; 1 \leq i \leq n, 1 \leq k \leq l\}$

let $p = k(2(n+m)) + 1$ and $q = k(2(n+m))$

Define $f: V(G) \rightarrow \{s, s+d, s+2d, \dots . s+(q+1)d\}$ to label the vertices as follows

$f(u_0) = s$

$f(u_j^i) = s + (n + 2n(i-1) + j)d; 1 \leq i \leq n, 1 \leq j \leq m$

$f(w_j^i) = s + (1 + 2n(i-1) + (j-1))d; 1 \leq i \leq n, 1 \leq j \leq m$

$f(x_k^i) = s + (2nm + i + (i-1)(l-1) + (k-1))d; 1 \leq i \leq n, 1 \leq k \leq l$

$f(y_k^i) = s + (n(2m+l) + i + (i-1)(l-1) + (k-1))d; 1 \leq i \leq n, 1 \leq k \leq l$

Define $g: E(G) \rightarrow \{d, 2d, 3d, \dots . 2(q-1)d\}$ to label the edges as follows

$g(u_j^i w_j^i) = 2s + 2(q-1)d - (f(u_j^i) + f(w_j^i)); 1 \leq i \leq n, 1 \leq j \leq m$

$g(u_j^i w_j^{i+1}) = 2s + 2(q-1)d - (f(u_j^i) + f(w_j^i)); 1 \leq i \leq n-1, 1 \leq j \leq m$

$g(u_0 w_j^1) = 2s + 2(q-1)d - (f(u_0) + f(w_j^1)); i = 1, 1 \leq j \leq m$

$g(u_j^n x_k^i) = 2s + 2(q-1)d - (f(u_j^n) + f(x_k^i)); 1 \leq i \leq n-1, 1 \leq j \leq m, 1 \leq k \leq l$

$g(x_k^j y_k^j) = 2s + 2(q-1)d - (f(x_k^j) + f(y_k^j)); 1 \leq k \leq l, 1 \leq j \leq m$

Labeling of Vertices of Subdivision on Regular Bamboo tree				
$f(u_0) = s$				
Value of $i, j\&k$	$f(u_j^i)$	$f(w_j^i)$	$f(x_k^i)$	$f(y_k^i)$
$1 \leq i \leq n,$ $1 \leq j \leq m$	$s + (n + 2n(i-1) + j)d$	$s + (1 + 2n(i-1) + (j-1))d$	$-$	$-$
$1 \leq i \leq n,$ $1 \leq k \leq l$	$-$	$-$	$s + (2nm + i + (i-1)(l-1) + (k-1))d$	$s + (n(2m+l) + i + (i-1)(l-1) + (k-1))d$

Labeling of Edges of Subdivision on Regular Bamboo tree					
Value of i, j &k	$g(u_j^i w_j^i)$	$g(u_j^i w_j^{i+1})$	$g(u_0 w_j^1)$	$g(u_j^n x_k^i)$	$g(x_k^j y_k^j)$
$1 \le i \le n$, $1 \le j \le m$	$2s + 2(q-1)d - (f(u_j^i) + f(w_j^i)$	$-$	$-$	$-$	$-$
$1 \le i \le n-1$, $1 \le j \le m$	$-$	$2s + 2(q-1)d - (f(u_j^i) + f(w_j^i)$	$-$	$-$	$-$
$i = 1$, $1 \le j \le m$	$-$	$-$	$2s + 2(q-1)d - (f(u_0) + f(w_j^1)$	$-$	$-$
$1 \le i \le n-1$, $1 \le j \le m$, $1 \le k \le l$	$-$	$-$	$-$	$2s + 2(q-1)d - (f(u_j^n) + f(x_k^i)$	$-$
$1 \le k \le l$, $1 \le j \le m$	$-$	$-$	$-$	$-$	$2s + 2(q-1)d - (f(x_k^j) + f(y_k^j)$

From the above table we find that f and g are injective $f(u_j^i) + f(w_j^i)\ g(u_j^i w_j^i)$ $f(u_j^i) + f(w_j^i) + g(u_j^i w_j^{i+1}), f(u_0) + f(w_j^1) + g(u_0 w_j^1),(f(u_j^n) + f(x_k^i) + g(u_j^n x_k^i), f(x_k^j) + f(y_k^j) + g(x_k^j y_k^j)$ are constant equal to $2(s + (q-1)d)$ .Hence we concluded that the subdivision on regular bamboo tree admits (S,d) magic labeling.

Example 3.3: Subdivision on Regular bamboo tree is shown below

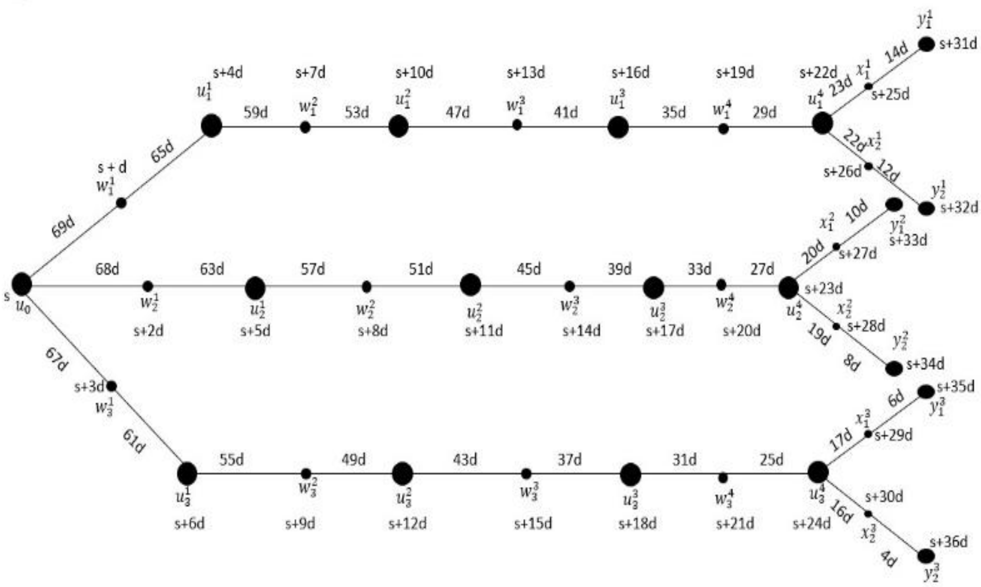

Figure 3.3: Subdivision on Regular bamboo tree

Theorem 3.4: Subdivision on olive tree $S(O_n)$ admits (S,d) magic labeling

Proof Let O_n be the olive tree having n paths of length 1, 2…. n adjoined at on vertex u_0

Let the edges of olive tree is subdivided by w_i^k ;$1 \leq i \leq n, 1 \leq k \leq n$

let $p = n(n+1)+1$ and $q = n(n+1)$

Define $f: V(G) \to \{s, s+d, s+2d, \dots . s+(q+1)d\}$ to label the vertices as follows

$f(u_0) = s$

$f(w_i^k) = s + id; 1 \leq i \leq n, k = 1$

$f(u_i^k) = f(w_i^k) + nd; 1 \leq i \leq n, k = 1$

$f(w_i^k) = s + (\sum_{j=0}^{k-2} 2(n-j) + i)\, d; 1 \leq i \leq n-(k-1), 2 \leq k \leq n$

$f(u_i^k) = f(w_i^k) + nd; 1 \leq i \leq n-(k-1), 2 \leq k \leq n$

Define $g: E(G) \to \{d, 2d, 3d, \dots . 2(q-1)d\}$ to label the edges as follows

$g(u_0 w_i^1) = 2s + 2(q-1)d - (f(u_0) + f(w_i^1); 1 \leq i \leq n$

$g(w_i^k u_i^k) = 2s + 2(q-1)d - (f(w_i^k) + f(u_i^k); 1 \leq k \leq n; 1 \leq i \leq n-(k-1)$

$g(w_i^{k+1} u_i^k) = 2s + 2(q-1)d - (f(w_i^{k+1}) + f(u_i^k); 1 \leq k \leq n-1; 1 \leq i \leq n-k$

Labeling of Vertices of Subdivision on olive tree $S(O_n)$				
Value of i &k	$f(w_i^k)$	$f(u_i^k)$	$f(w_i^k)$	$f(u_i^k)$
$1 \leq i \leq n, k = 1$	$s + id$	$f(w_i^k) + nd$		
$1 \leq i \leq n-(k-1), 2 \leq k \leq n$			$s + (\sum_{j=0}^{k-2} 2(n-j) + i)\, d$	$f(w_i^k) + nd$

Labeling of edges of Subdivision on olive tree $S(O_n)$			
	$g(u_0 w_i^1)$	$g(w_i^k u_i^k)$	$g(w_i^{k+1} u_i^k)$
$1 \leq i \leq n$	$2s + 2(q-1)d - (f(u_0) + f(w_i^1)$	$-$	$-$
$1 \leq k \leq n$; $1 \leq i \leq n-(k-1)$	$-$	$2s + 2(q-1)d - (f(w_i^k) + f(u_i^k)$	$-$
$1 \leq i \leq n-1$; $1 \leq i \leq n-k$	$-$	$-$	$2s + 2(q-1)d - (f(w_i^{k+1}) + f(u_i^k)$

From the above table we find that f and g are injective $(f(u_0) + f(w_i^1) + g(u_0 w_i^1)$ $f(w_i^k) + f(u_i^k) + g(w_i^k u_i^k), f(w_i^{k+1}) + f(u_i^k) + g(w_i^{k+1} u_i^k))$ are constant equal to $2(s + (q-1)d)$ .Hence we concluded that the subdivision on olive tree admits (S,d) magic labeling

Example 3.4: Subdivision on olive tree $S(O_n)$ is shown below

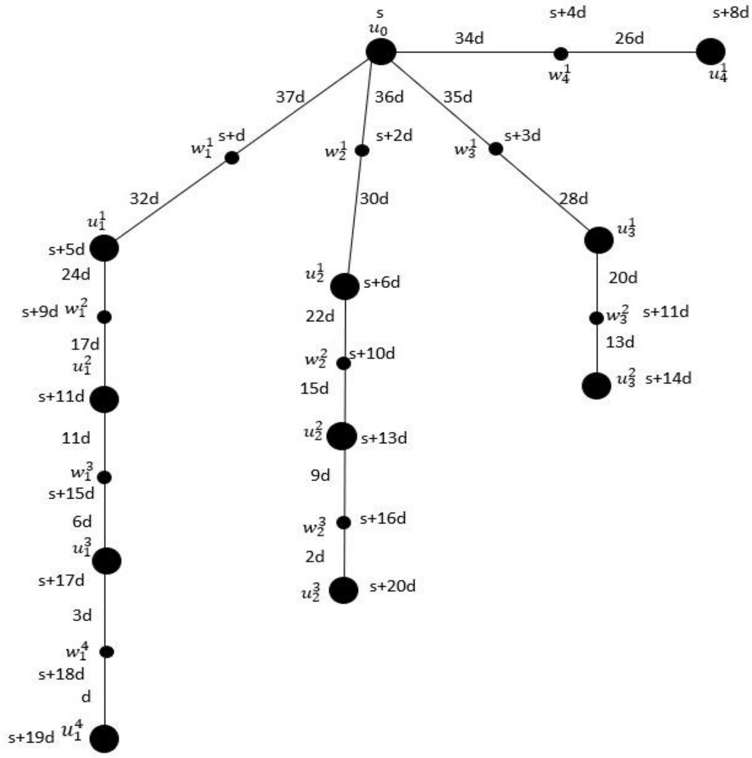

Figure 3.4: Subdivision on olive tree $S(O_4)$

Theorem 3.5: Subdivision on spider graph SP $(1^n 2^m)$ admits (S,d) magic labeling

Proof: Let $G =$ SP $(1^n 2^m)$ .let the edges of spider graph be$\{uv_1, uv_2, \ldots uv_n, uw_1, uw_2, \ldots uw_m, wx_1, wx_2, \ldots wx_m\}$,sub divided by $\{l_i, t_j, y_j\ ; 1 \le i \le n, 1 \le j \le m\}$.

Let $p = 4m + 2n + 1, q = 4m + 2n$

Define $f: V(G) \to \{s, s + d, s + 2d, \ldots . s + (q + 1)d\}$ to label the vertices as follows

$f(u) = s$

$f(t_j) = s + jd; 1 \le j \le m$

$f(y_j) = f(v_n) + jd; 1 \le j \le m$

$f(x_j) = f(y_m) + jd; 1 \le j \le m$

$f(v_i) = f(w_m) + id; 1 \le i \le n$

$f(l_i) = f(t_m) + id; 1 \le i \le n$

$f(w_j) = f(l_n) + jd; 1 \le j \le m$

Define $g: E(G) \to \{d, 2d, 3d, \ldots .2(q - 1)d\}$ to label the edges as follows

$g(ul_i) = 2s + 2(q - 1)d - (f(u) + f(l_i)); 1 \le i \le n$

$g(ut_j) = 2s + 2(q - 1)d - (f(u) + f(t_j)); 1 \le j \le m$

$g((l_i v_i) = 2s + 2(q - 1)d - (f(l_i) + f(v_i)); 1 \le i \le n$

$g((t_j w_j) = 2s + 2(q - 1)d - (f(t_j) + f(w_j)); 1 \le j \le m$

$g((w_j y_j) = 2s + 2(q - 1)d - (f(y_j) + f(w_j)); 1 \le j \le m$

$g((y_j x_j) = 2s + 2(q - 1)d - (f(y_j) + f(x_j)); 1 \le j \le m$

Labeling of vertices of Subdivision on Spider graph						
$f(u) = s$						
Value of i	$f(t_j)$	$f(y_j)$	$f(x_j)$	$f(v_i)$	$f(l_i)$	$f(w_j)$
$1 \le i \le n$	$-$	$-$	$-$	$f(w_m) + id$	$f(t_m) + id$	$-$
$1 \le j \le m$	$s + jd$	$f(v_n) + jd$	$f(y_m) + jd$	$-$	$-$	$f(l_n) + jd$

Labeling of vertices of Subdivision on Spider graph				
Value of i	$g(ul_i)$	$g(l_i v_i)$		
$1 \le i \le n$	$2s + 2(q - 1)d - (f(u) + f(l_i))$	$2s + 2(q - 1)d - (f(l_i) + f(v_i))$	$-$	$-$
Value of j	$g((t_j w_j)$	$g((w_j x_j)$	$g((w_j y_j)$	$g(ut_j)$
$1 \le i \le m$	$2s + 2(q - 1)d - (f(t_j) + f(w_j))$	$2s + 2(q - 1)d - (f(y_j) + f(x_j))$	$2s + 2(q - 1)d - (f(y_j) + f(w_j))$	$2s + 2(q - 1)d - (f(u) + f(t_j))$

From the above table we find that f and g are injective $(f(u) + f(l_i) + g(ul_i)$, $f(u) + f(t_j) + g(ut_j)$, $f(l_i) + f(v_i) + g(l_i v_i)$, $(f(t_j) + f(w_j) + g(t_j w_j), f(x_j) + f(w_j) + g(w_j x_j), f(y_j) + f(w_j) + g(w_j y_j)$ are constant equal to $2(s + (q-1)d)$ .Hence we concluded that the subdivision on spider graph admits (S,d) magic labeling.

Example 3.5: Subdivision on spider graph is shown below

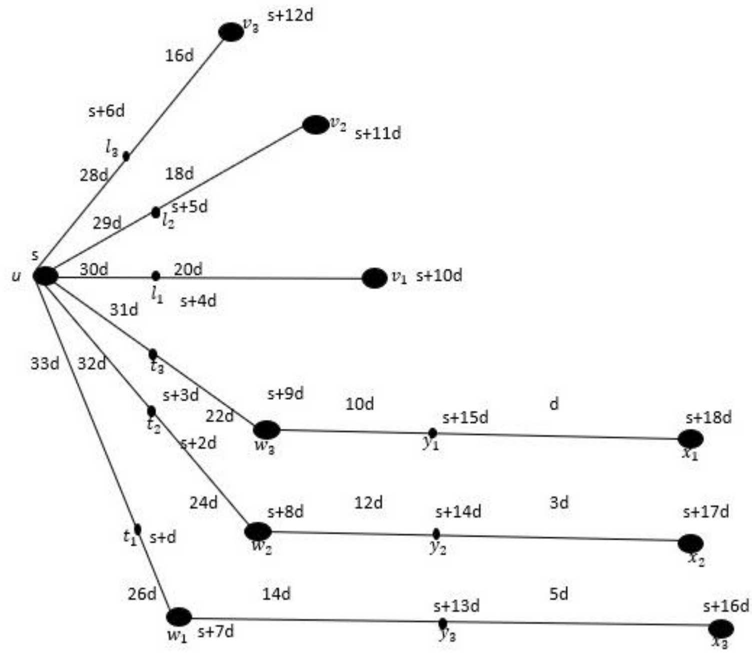

Figure 3.5: Subdivision on spider graph SP ($1^3 2^3$)

IV. CONCLUSION

This study explored and confirmed the existence of *(S, d)* magic labeling in the subdivisions of specific types of trees, including coconut trees, symmetrical trees, regular bamboo trees, olive trees, and spider graphs. Moving forward, the research will focus on extending this concept to other graph families and uncovering potential applications.

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