



Financial Mathematics in the Valuation of Financial Assets: An Analysis of Present Value, Discount Rates and Expected Return

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ARTICLE INFO	ABSTRACT
<p>Published Online: 07 May 2026</p> <p>Corresponding Author: Alberto Merced Castro Valencia</p>	<p>This paper examines the application of financial mathematics to the valuation of financial assets, with particular emphasis on present value, discount rates and expected return as core inputs in investment decision-making. The methodology is quantitative, document-based and analytical: classical and contemporary contributions on the time value of money, the risk–return trade-off and bond valuation are reviewed, and a hypothetical case is developed to illustrate the practical use of financial formulae. The analysis shows that the discount rate is not a secondary parameter but the variable that links risk, opportunity cost and asset price. In the applied example, a bond with a 10% annual coupon, a five-year maturity and a required return of 12% yields a theoretical price of 92,790.45 monetary units, below par value, confirming that when the required return exceeds the coupon rate the instrument trades at a discount. The paper concludes that financial mathematics is indispensable to valuation, asset selection and the internal consistency of corporate investment and financing decisions.</p>
<p>KEYWORDS: present value; discount rate; bond valuation; expected return; financial risk.</p>	

I. INTRODUCTION

The valuation of financial assets occupies a central position in corporate finance, investment management and project appraisal. Every decision to buy, sell, hold or finance an asset rests, explicitly or implicitly, on the discounting of expected cash flows. In this context, financial mathematics provides the formal language needed to translate future amounts into values that are comparable today, estimate required returns and examine the sensitivity of an asset's price to changes in market conditions. The practical relevance of this approach is wide-ranging: it affects the valuation of bonds, equities, leases, derivatives and investment projects, as well as the estimation of the cost of capital and the return required by investors and creditors (Fisher, 1930; Williams, 1938; Damodaran, 2012).

In professional practice, however, it is still common to encounter decisions based on superficial comparables, market multiples or heuristic rules that do not clearly specify the structure of cash flows or the logic of the discount rate. This limitation leads to inconsistent valuations, particularly when instruments with periodic payments, changes in the term structure of interest rates or heterogeneous risk profiles

are assessed. The research problem, therefore, is to determine how financial mathematics integrates present value, the discount rate and expected return so as to produce a technically consistent valuation of financial assets.

The justification for the study is twofold. From an academic perspective, it brings together classical foundations of financial theory with a concrete quantitative application. From a professional perspective, it offers a replicable framework to support investment and financing decisions. The general objective is to analyse the role of financial mathematics in asset valuation, with emphasis on measuring present value and selecting discount rates that are consistent with expected return. The research question is as follows: how do present value, the discount rate and expected return contribute to a consistent valuation of financial assets and to corporate decision-making?

II. LITERATURE REVIEW

The starting point for financial valuation is the time value of money. Fisher (1930) showed that one monetary unit today is not equivalent to one received in the future because economic agents value immediate availability and face

alternative investment opportunities. On this basis, Williams (1938) and Gordon (1962) consolidated the idea that the value of an asset depends on the discounted value of the cash flows it will generate, a principle that remains the backbone of modern valuation. In operational terms, present value brings together two dimensions: the expected size of the cash flow and the rate at which that cash flow must be discounted to reflect time preference and risk.

From a corporate finance perspective, the discount rate represents the opportunity cost of capital. Modigliani and Miller (1958) demonstrated the importance of separating investment policy from financing structure under restrictive assumptions, thereby opening the modern debate on the cost of capital, financial structure and value creation. Later, Tirole (2006) and Damodaran (2012) developed the idea that the relevant rate must be consistent with the risk of the discounted cash flow, the currency of denomination, expected inflation and the structure of the instrument under consideration. Consequently, there is no universal rate: its selection depends on the type of asset and on the information available to the investor about credit risk, duration, liquidity and market conditions.

In fixed income, bond valuation is the clearest expression of this principle. The price of a bond is the sum of the present value of its coupons and its redemption payment at maturity. Tuckman and Serrat (2022) note that each cash flow may be discounted either with a single required rate of return or with a term structure of spot rates, while Bodie et al. (2021) emphasise that the gap between the coupon rate and the required return explains whether the instrument trades at a premium, at par or at a discount. This mechanism is crucial in treasury management, portfolio management and the assessment of corporate debt issuance.

Expected return, in turn, links valuation to risk. Markowitz (1952) laid the foundations of portfolio selection by formalising the relationship between expected return and variance. Subsequently, Sharpe (1964) and Lintner (1965) proposed the capital asset pricing model (CAPM), according to which the expected return of an asset depends on the risk-free rate and its exposure to systematic risk. Although Fama and French (1992) and Fama and French (2004) point to empirical limitations of the CAPM, the risk–return relationship remains an operational reference point for estimating required returns, comparing assets and evaluating performance. In this respect, the contribution of Perold (2004) and Cochrane (2005) is especially relevant: both remind us that the price of any asset may be interpreted as the discounted value of future pay-offs under a coherent rule of compensation for risk.

Taken together, the literature converges on a central idea: financial mathematics does not merely describe capitalisation and discounting operations, but structures the economic reasoning that links prices, cash flows, risk and capital-allocation decisions (Elton et al., 2014; Hull, 2002).

This convergence justifies the use of simple and transparent models for valuation purposes, provided that their assumptions and limitations are made explicit.

III. METHODOLOGY

The study adopts a quantitative, document-based and analytical methodology. It is quantitative because it uses formal capitalisation and discounting expressions to calculate theoretical prices and analyse sensitivity to changes in the required rate. It is document-based because it draws on classical and contemporary academic literature on valuation, portfolio theory and fixed income. It is analytical because it breaks down the asset price into its basic components—coupons, principal, rate and maturity—in order to interpret the individual effect of each variable.

The design is non-experimental and explanatory-applied in scope. Rather than working with a statistical market sample, the study uses a hypothetical case designed to show how a bond valuation model operates. The procedure was carried out in four stages. First, the essential theoretical concepts were identified: present value, future value, the discount rate and expected return. Secondly, the financial formulae linking these concepts were systematised. Thirdly, a fixed-coupon bond was parameterised and its discounted cash flows were calculated. Fourthly, the resulting price was compared under different required-return scenarios in order to discuss its implications for investment decisions.

The choice of a hypothetical example serves both a pedagogical and a methodological purpose: it isolates the mathematical logic of the valuation process without contaminating the analysis with macroeconomic noise or market assumptions that are difficult to control. Even so, the discussion is framed in terms that remain consistent with real-world investment decisions in corporate finance and portfolio management.

IV. ANALYTICAL DEVELOPMENT AND FINANCIAL MODEL

The model begins with two basic relationships from financial mathematics. The first is present value:

$$VP = VF / (1 + i)^n$$

where VP is present value, VF is future value, i is the interest or discount rate, and n is the number of periods. The formula indicates how much a future amount is worth today once the return required by the investor is recognised. If the rate rises, the denominator increases and present value falls.

The second relationship is future value:

$$VF = VP(1 + i)^n$$

which expresses the reverse process: a current amount grows at a given rate for n periods. Both expressions are equivalent and show that capitalising and discounting are symmetrical operations.

For the valuation of a coupon-bearing bond, the following equation is used:

$$P = \sum_{t=1}^n C / (1 + k)^t + VN / (1 + k)^n$$

where P is the bond price, C is the periodic coupon, k is the required market return, VN is the nominal value, and n is the number of periods until maturity. The first term discounts the stream of coupons; the second discounts the principal repaid at maturity.

At a more general level, the expected return of a risky asset is often represented by the CAPM:

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f]$$

where R_f is the risk-free rate, β_i measures the asset's systematic risk, and $E(R_m) - R_f$ is the market risk premium. Although the applied example focuses on a bond and uses a single required rate, this expression is useful because it reminds us that the discount rate should remain consistent with the riskiness of the cash flow being valued (Sharpe, 1964; Lintner, 1965; Fama and French, 2004).

Table I: Operational Definition Of Variables

Symbol	Definition
VP	Present value of the cash flow
VF	Future value of the cash flow
i	Interest or discount rate
n	Number of periods
P	Theoretical bond price
C	Periodic coupon
k	Required return
VN	Nominal value of the bond

V. APPLIED EXAMPLE

The proposed instrument is a bond with a nominal value of 100,000 monetary units, an annual coupon of 10%, a five-year maturity and a required return of 12%. The annual coupon cash flow is:

$$C = 100,000 \times 0.10 = 10,000$$

The discounting of the coupons and the principal is shown in Table II. The calculation uses a single annual rate of 12%, consistent with the return required by the investor for this instrument.

Table II: Bond Valuation With A Required Return Of 12%

Year	Cash flow	Discount factor	Present value
1	10,000.00	0.892857	8,928.57
2	10,000.00	0.797194	7,971.94
3	10,000.00	0.711780	7,117.80
4	10,000.00	0.635518	6,355.18
5	10,000.00	0.567427	5,674.27
5	100,000.00	0.567427	56,742.69
	Total theoretical price		92,790.45

Accordingly, the present value of the coupons amounts to 36,047.76 and the present value of the principal to

56,742.69. The sum of both components yields a theoretical bond price of 92,790.45. Because the price is below nominal value, the bond trades at a discount. The financial interpretation is straightforward: a 10% coupon is insufficient to satisfy a required return of 12%; as a result, the market adjusts the price downwards until the effective return to the purchaser is consistent with that requirement.

VI. RESULTS AND DISCUSSION

The main result confirms a classical principle of fixed-income analysis: bond prices and required returns are inversely related. When the discount rate exceeds the coupon rate, the bond is worth less than par; when both rates coincide, the bond trades approximately at par; and when the required rate is below the coupon rate, the bond trades at a premium (Tuckman and Serrat, 2022; Bodie et al., 2021). Table III summarises this sensitivity.

Table III: Sensitivity Of the Bond Price to Changes in the Required Return

Required return	Theoretical price	Interpretation
8%	107,985.42	Bond at a premium
10%	100,000.00	Bond at par
12%	92,790.45	Bond at a discount

This finding has clear implications for investment decision-making. First, an investor should not assess a bond solely by its coupon rate, but by the required return given its risk, maturity and liquidity profile. Secondly, the discount rate summarises economically relevant information: expected inflation, opportunity cost, perceived risk and market conditions. Thirdly, valuation makes it possible to distinguish between the observed price and intrinsic value under explicit assumptions; that distinction underpins purchase and sale decisions.

The discussion is also relevant to corporate finance. A firm that issues debt with a coupon rate below the return required by the market will need to accept an issue price below nominal value, which implicitly raises its financing cost. From the standpoint of the institutional investor, properly estimating expected return helps prevent overpaying for assets whose profitability does not compensate for their risk. Here the portfolio literature is complementary: the risk-return relationship formalised by Markowitz (1952), Sharpe (1964) and Lintner (1965) reminds us that the discount rate is not arbitrary, but rather a synthesis of exposure to systematic risk and the opportunity set available in the market.

It is important to underline, however, that the usefulness of the model depends on the consistency of its assumptions. If credit risk changes significantly, if the term structure of rates is not flat, or if the bond contains embedded options, the basic formula must be extended. The approach presented here is therefore robust as a baseline model, but it does not

replace more refined market analysis when the instrument includes additional layers of complexity.

VII. CONCLUSIONS

Financial mathematics is an essential tool in the valuation of financial assets because it allows future cash flows to be converted into present prices that are both comparable and economically interpretable. The study has shown that present value is the fundamental mechanism of valuation, while the discount rate acts as the link between time, risk and opportunity cost.

The applied case showed that a bond with a 10% coupon and a required return of 12% should trade at a discount, producing a theoretical price of 92,790.45. This result is not merely arithmetic: it synthesises the economic logic by which the market adjusts an asset's price when the contractual return falls short of the return required by investors.

It is also concluded that a sound estimate of expected return improves corporate decision-making, both in portfolio allocation and in debt issuance and cost-of-capital assessment. Consequently, the teaching and rigorous application of financial mathematics remain indispensable for building transparent, defensible and professionally useful valuations.

VIII. STUDY LIMITATIONS

The analysis has important limitations. First, it uses hypothetical data and therefore does not capture frictions observed in real markets. Secondly, it does not incorporate macroeconomic variables, credit risk, inflation, liquidity or market volatility. Thirdly, valuation is carried out with a single discount rate rather than a full yield curve. These restrictions simplify the model, but they also reduce its ability to represent complex instruments or actual trading conditions.

IX. FUTURE RESEARCH DIRECTIONS

Future research could extend this approach in at least four directions. First, it could examine bond valuation in emerging markets, where sovereign risk premia and lower liquidity alter the required rate. Secondly, it could model interest-rate risk explicitly through duration, convexity and yield-curve shift scenarios. Thirdly, it could incorporate stochastic models of rates and prices for assets whose valuation depends on uncertain paths. Fourthly, it could examine financial instruments in inflationary settings, where the distinction between nominal and real rates is decisive for the interpretation of present value.

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