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Trait of Binary Supra Generalized $\wp$ − Closed Set in Binary Topological Space.

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ABSTRACT

This research explores the Binary Supra Generalized $\wp$−Closed set and Binary Supra Generalized $\wp$− Open set in Binary Supra Topological Space.

KEYWORDS: Binary Supra Topological Space, Binary Generalized $\wp$−Closed set , Binary Supra Generalized $\wp$−Closed set,

1. INTRODUCTION

Mashhour etal.[12] studied Supra Continuous maps and presented their concept of Supra topological spaces in 1983. Many people have expanded their research in supra topological spaces since the introduction of these ideas, discovering a new class of supra closed sets. By combining binary and supra topology, M. Lellis Thivagar and J. Kavitha [9] created a new structure in 2017 called Binary Supra Topological Spaces. The new concept for binary supra R^G-closed sets were first proposed by authors D. Savithiri and Divya.S in 2024 [17]. In this paper, we introduced the *Binary Supra Generalized* $\wp$ − *Closed set* in Binary Topological Space and discuss some properties. A binary supra topology [$\mathcal{BST}$] [9] from X to Y is a binary structure $\mathcal{M}_{\underline{\mu}} \subseteq \wp(X)\times\wp(Y)$ that satisfies the following axioms

(i) (ϕ,ϕ) and (X,Y) $\in \mathcal{M}_{\underline{\mu}}$,

(ii) If $\{(K_\alpha, A_\alpha) : \alpha\in\Delta\}$ is a family members of $\mathcal{M}_{\underline{\mu}}$ then $(\cup_{\alpha\in\Delta} K_\alpha, \cup_{\alpha\in\Delta} A_\alpha) \in \mathcal{M}_{\underline{\mu}}$.

Proposition [9] Let (X,Y, $\mathcal{M}_{\underline{\mu}}$) be a $\mathcal{BST}$ (K,B) $\subseteq$ (A,M) $\subseteq$(X,Y) then the following statements hold

- $b_{\underline{\mu}}$-*int*(K,B)$\subseteq$(K,B)
- If (K,B) is binary open, then $b_{\underline{\mu}}$-*int* (K,B) = (K,B)
- $b_{\underline{\mu}}$-*int*(K,B) ∪ $b_{\underline{\mu}}$-*int*(A,M) $\subseteq$ $b_{\underline{\mu}}$-*int*((K,B) ∪ (A,M))
- $b_{\underline{\mu}}$-*int* (K,B) $\subseteq$ $b_{\underline{\mu}}$-*int* (A,M)
- $b_{\underline{\mu}}$-*int*($b_{\underline{\mu}}$-*int* (K,B)) = $b_{\underline{\mu}}$-*int* (K,B)
- (K,B)$\subseteq$ $b_{\underline{\mu}}$-*cl* (K,B)
- If (K,B) is binary closed, then $b_{\underline{\mu}}$-*cl*(K,B) = (K,B)
- $b_{\underline{\mu}}$-*cl*(K,B)$\subseteq$ $b_{\underline{\mu}}$-*cl* (A,M)
- $b_{\underline{\mu}}$-*cl*(*b-cl*(K,B))$\subseteq$ $b_{\underline{\mu}}$-*cl*(K,B)

2. BINARY SUPRA GENERALIZED $\wp$ −CLOSED SET AND BINARY SUPRA GENERALIZED $\wp$ − OPEN SET

Definition 2.1: Let (K,B) be a subset of a binary supra topological space (X,Y,$\mathcal{M}_{\underline{\mu}}$) is called a Binary supra generalized $\wp$ −closed set [briefly $\mathcal{B}_{\underline{\mu}}\mathcal{G}\,\wp$ − closed] if $b_{\underline{\mu}}$*scl(K,B)* $\subseteq$ *(A,M)* whenever *(K,B)* $\subseteq$ *(A,M) and (A,M) is binary supra* $\propto$ open.

Theorem 2.2: Every binary supra closed set is $\mathcal{B}_{\underline{\mu}}\mathcal{G}\,\wp$ − closed set

Proof: Let (C,S) be a binary closed set of $(X,Y,\mathcal{M}_{\underline{\mu}})$ then (C,S) $\subseteq$(A,M), where (A,M) is binary supra $\propto$ open in $(X,Y,\mathcal{M}_{\underline{\mu}})$. Since $b_{\underline{\mu}}cl$(C,S) $\subseteq$ (C,S). Therefore $b_{\underline{\mu}}int(b_{\underline{\mu}}cl$(C,S)) $\subseteq$(C,S). So (C,S) is $b_{\underline{\mu}}scl$(C,S) $\subseteq$ (A,M). Where (A,M) is binary supra $\propto$ open in $(X,Y,\mathcal{M}_{\underline{\mu}})$. Hence every binary supra closed set is a $\mathcal{B}_{\underline{\mu}}\mathcal{G}\ \wp-$ closed.

The converse of the above theorem need not be true from the following example.

Example 2.3 Let X = {a,b}, Y = {1,2,3} and $\mathcal{M}_{\underline{\mu}}$= {((φ, φ), (φ,{1}), ({a},{1}), ({a},{1,2}), ({b},φ), ({b},{1}) , ({b},{3}), ({b},{1,3}), (X,{1}), ({X}, {1,2}), (X, {1,3}), (X,Y) }. Then the set ({a},{3}) is $\mathcal{B}_{\underline{\mu}}\mathcal{G}-$ closed set but not a binary supra closed set.

Theorem 2.4: Every binary supra g closed set is $\mathcal{B}_{\underline{\mu}}\mathcal{G}\wp-$ closed set.

Proof: Let (C,S) be a Binary supra generalized closed set. Let (C,S) $\subseteq$ (A,M) $\subseteq$(X,Y), then $b_{\underline{\mu}}$-cl(A, B)=(C,S), (C,S)$\subseteq$(A,M) , (A,M) is binary supra open then $b_{\underline{\mu}}$-cl(C,S) $\subseteq$ (A,M), $b_{\underline{\mu}}$-int($b_{\underline{\mu}}$-cl(C,S)) $\subseteq$ $b_{\underline{\mu}}$-*int*(A,M),

Therefore $b_{\underline{\mu}}$-int(b-cl(C,S)) $\subseteq$ (A,M), since $b_{\underline{\mu}}$int(A,M)= (A,M). Hence $b_{\underline{\mu}}$scl(C,S) $\subseteq$ (A,M), Where (A,M) is binary supra $\propto$ open. Since using every binary supra open is binary supra $\propto$ open set then (C,S) is $\mathcal{B}_{\underline{\mu}}\mathcal{G}\ \wp-$ closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 2.5 In the example 2.3 (X,{3}) is $\mathcal{B}_{\underline{\mu}}\mathcal{G}-$ closed set but not g closed set.

Theorem 2.6: Every supra generalized binary regular closed set is $\mathcal{B}_{\underline{\mu}}\mathcal{G}-$ closedset.

Proof: Let (C,S) be a Supra generalized binary Regular closed set. By the definition $b_{\underline{\mu}}$-cl(C,S) $\subseteq$ (A,M) where (C,S) $\subseteq$ (A,M) $\subseteq$ (X,Y), then $b_{\underline{\mu}}$-cl(C,S) $\subseteq$ (A,M) , and $b_{\underline{\mu}}$-int($b_{\underline{\mu}}$-cl(C,S)) $\subseteq$ $b_{\underline{\mu}}$-int (A,M). Since $b_{\underline{\mu}}$- int (A,M) $\subseteq$ (A,M), it follows that $b_{\underline{\mu}}$-scl(C,S) $\subseteq$ (A,M). Every binary supra regular open set is binary $\propto$ open. Hence (C,S) is binary Supra generalized $\wp$ —closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 2.7 Let S={0,1}, T={a,b,c}, and $\mathcal{M}_{\underline{\mu}}$= {(φ, φ),({0}, {a}), ({1},{b}),(S,{a,b}),(S,T)}. Let ({1},{b}) is a $\mathcal{BG}\ \wp-$ closed set but not binary regular closed set.

Theorem 2.8: Every binary supra g^*closed set is $\mathcal{B}_{\underline{\mu}}\mathcal{G}\ \wp-$ closed set.

Proof: Let (C,S) be a binary supra g^*closed set of (X,Y). Let (C,S) $\subseteq$ (A,M). Assume (A,M) is binary supra g open in (X,Y). Since (C,S) is binary g^*closed, $b_{\underline{\mu}}$cl(C,S)= (C,S).

However $b_{\underline{\mu}}$s-cl(C,S) $\subseteq$ $b_{\underline{\mu}}$cl (C,S), which implies that $b_{\underline{\mu}}$cl(C,S) $\subseteq$ (A,M). Therefore (C,S) $\subseteq$ (A,M), is binary supra g open in (X,Y). Every binary generalized open set is binary- $\propto$ open set. Therefore (C,S) is binary supra generalized $\wp$ —closed set.

The converse of the above theorem need not be true from the following example.

Example 2.9 : In the example 2.3 (X,{3}) is $\mathcal{B}_{\underline{\mu}}\mathcal{G}\ \wp-$ closed set but not supra g^*closed set.

Theorem 2.10: Every Binary supra generalized $\wp$ —closed set is Binary supra gs closed set.

Proof: Let (C,S) be a Binary generalized $\wp$ —closed set of $(X,Y,\mathcal{M}_{\underline{\mu}})$ and let (A,M) is a binary supra $\propto$ open in $(X,Y,\mathcal{M}_{\underline{\mu}})$ such that (C,S) $\subseteq$ (A,M). Since every binary supra semi closure of (C,S) $\subseteq$ (A,M), and Some binary supra $\propto$ open sets are binary supra open, then (A,M) is binary supra open.

We know that $b_{\underline{\mu}}scl(C,S)$ $\subseteq$ (A,M), and (A,M) is binary open . Therefore (C,S) is Binary supra gs closed set.

The converse of the above theorem need not be true as from the following example.

Example 2.11 In the example 2.3 (X,{2}) is Binary supra gs closed set but not $\mathcal{B}_{\underline{\mu}}\mathcal{G}\ \wp-$ closed set.

Theorem 2.12: Every $\mathcal{B}_{\underline{\mu}}\mathcal{G}\ \wp-$ closed set is binary supra $\propto$ g closed set.

Proof: From the definition of binary supra generalized $\wp$ —closed set, consider the subset (C,S) of (X,Y). Let (A,M) be a binary supra open set in (X,Y), such that (C,S) $\subseteq$ (A,M). Since every binary supra $\propto$ open sets are binary supra open set, we have $b_{\underline{\mu}}$-scl(C,S) $\subseteq$ (A,M), and $b_{\underline{\mu}}$-int($b_{\underline{\mu}}$-cl(C,S)) $\subseteq$ (A,M).

Additionally, b_μ-cl(b_μ-int($b_\mu$cl(C,S)))$\subseteq$ $b_\mu$cl(A,M). We know that $b_\mu$cl(A,M) $\subseteq$ (A,M), therefore, b_μ $\propto$cl(C,S) $\subseteq$ (A,M) and (C,S)$\subseteq$ (A,M). Since (A,M) is a binary supra open set in (X,Y), it follows that (C,S) is binary supra $\propto$ g closed set. Hence it is proved.

The converse of the above theorem need not be true as can be seen in the following example.

Example 2.13: In the example 2.3 ({a},{2,3}) is binary supra $\propto$ g closed set but not $\mathcal{B}_\mu\mathcal{G}\wp$ — closed set

Hence the converse of the theorem 2.11 is not possible.

Theorem2.14: Every $\mathcal{B}_\mu\mathcal{G}\wp$ — closed set is Binary supra sg closed set.

Proof: Let (C,S) be a Binary supra generalized $\wp$ —closed set of (X,Y,$\mathcal{M}_\mu$) and let (A,M) is a binary supra $\propto$ open in (X,Y,$\mathcal{M}_\mu$), such that (C,S) be a subset of (A,M). Since every binary supra semi closure of (C,S) $\subseteq$ (A,M), and binary supra $\propto$ open sets are binary supra open set, (A,M) is binary supra open.

Every binary supra open set is binary supra semi open. (ie) b_μ*scl(C,S)* $\subseteq$ (A,M), whenever (C,S) $\subseteq$ (A,M), (A,M) is binary supra semi open. Therefore (C,S) is a Binary supra sg closed set.

The converse of the above theorem need not be true as can be seen in the following example.

Example 2.15 In the example 2.3 ({a_1},{2,3}) is binary supra sg closed set but not $\mathcal{B}_\mu\mathcal{G}\wp$ — closed set.

Theorem 2.16 : Every $\mathcal{B}_\mu\mathcal{G}\wp$ — closedset is binary supra g^*s closed set.

Proof: Let (C,S) be a binary supra generalized $\wp$ —closed set of (X,Y), and let (C,S) $\subseteq$ (A,M). (A,M) is a binary supra $\propto$ open set in (X,Y). From the definition of binary supra generalized $\wp$ —closed set, (C,S) is a binary supra semi closure of (C,S). If (C,S) is binary supra $\propto$ open set, then (A,M) $\subseteq$ b_μ*-int(b-cl(*b_μ*-int(A,M)))* .

We know that b_μ*int(A,M)* $\subseteq$ *(A,M)* thus (A,M) $\subseteq$ b_μ*int(b-cl(A,M))*. Taking the binary supra interior on both sides, we get b_μ*int(A,M)*$\subseteq$ b_μ*-int(* b_μ*int(*b_μ*cl(A,M))),* which implies (A,M) $\subseteq$ $b_\mu$cl(A,M). Taking the complement on both sides, the result will be binary supra g open set. If b_μ*scl*(C,S) $\subseteq$ (A,M) and (A,M) is binary supra g open in (X,Y), then the subset (A,M) is binary supra g^*s closed set. Hence, it is proved.

The converse of the above theorem need not be true as can be seen in the following example.

Definition:2.17 Let (K,B) be a subset of a binary supra topological space (X,Y,$\mathcal{M}_\mu$) is called a Binary supra generalized $\wp$ —Open sets (briefly $\mathcal{B}_\mu\mathcal{G}\wp$ — open) if $(K,B)^c$ is also binary supra generalized $\wp$-open set.

Theorem:2.18 Every binary supra open set is $\mathcal{B}_\mu\mathcal{G}\wp$ — open set

Proof: Proof follows from the theorem 2.2

Theorem 2.19 Every generalized Binary supra regular open set is $\mathcal{B}_\mu\mathcal{G}\wp$ — open set

Proof: Proof follows from the theorem 2.6

Theorem: 2.20 If b_μ*sint* (K,B)$\subseteq$ (N,Q) $\subseteq$ (K,B) and if (K,B) is binary supra generalized $\wp$ open set.

Proof: Let b_μ*sint* (K,B)$\subseteq$ (N,Q) $\subseteq$ (K,B). Then $(K,B)^c$ $\subset$ $(N,Q)^c$ $\subseteq$ $b_\mu$scl ($(K,B)^c$) where $(C,S)^c$ is a binary supra generalized $\wp$ closed set. Hence, $(N,Q)^c$ is also binary supra generalized $\wp$ closed set. Therefore, (N,Q) is binary supra generalized $\wp$ open set.

3. CHARACTERISTICS OF BINARY SUPRA GENERALIZED $\wp$ —CLOSED SET

Theorem 3.1: Union of any two $\mathcal{B}_\mu\mathcal{G}\wp$ —Closed set is $\mathcal{B}_\mu\mathcal{G}\wp$ —Closed set.

Proof: Let (K,B) and (U,S) are $\mathcal{B}_\mu\mathcal{G}\wp$ —Closed sets in (X,Y,$\mathcal{M}_\mu$), and let (A,M) be any Binary supra α open set containing (K,B) and (U,S). Therefore, b_μ*scl*(K,B)$\subseteq$(A,M) and b_μ*scl*(U,S)$\subseteq$(A,M). Since (K,B) $\subseteq$(A,M) and (U,S) $\subseteq$(A,M), we have (K,B) $\cup$ (U,S) $\subseteq$(A,M).

As (K,B) and (U,S) are $\mathcal{B}\mathcal{G}\wp$ —Closed sets in (X,Y,$\mathcal{M}_\mu$), b_μ*scl(K,B)* $\subseteq$(A,M) and b_μ*scl*(U,S) $\subseteq$(A,M). Now, b_μ*scl*((K,B)$\cup$(U,S)) = b_μ*scl*(K,B)$\cup$ $b_\mu$scl(U,S) $\subseteq$(A,M). Since (A,M) is a binary α open set in (X,Y,$\mathcal{M}_\mu$), (K,B) $\cup$ (U,S) is $\mathcal{B}_\mu\mathcal{G}\wp$ —Closed set.

Example 3.2 In the example 2.3 (φ,{2}) and ({b}, {2}) are $\mathcal{B}_\mu \mathcal{G} \wp$ —closed set and its union ({b}, {2}) is also the $\mathcal{B}_\mu \mathcal{G} \wp$ —Closed set.

Theorem 3.3: Let (K,B) $\mathcal{B}_\mu \mathcal{G} \wp$ —Closed set of (X,Y,$\mathcal{M}_\mu$). If (K,B) ⊆(U,S) ⊆ $bscl(K, B)$ then (K_1,B_1) is also $\mathcal{B}_\mu \mathcal{G} \wp$ —Closed set of (X,Y,$\mathcal{M}_\mu$).

Proof: Let (A,M) be a binary supra α open set in (X,Y). If (U,S) is a subset of (A,M), then (K,B) ⊆ (U,S) implies (K,B) ⊆ (A,M). Since (K,B) is a binary supra generalized $\wp$ —closed set, b_μ*scl*(K,B) ⊆ (A,M). Also, (U,S) ⊆ $b_\mu scl(K, B)$ implies b_μ*scl*(U,S) ⊆ b_μ*scl*(K,B). Thus b_μ*scl*(U,S) ⊆ (A,M) and so, (U,S) is a binary supra generalized $\wp$ —Closed set.

Theorem: 3.4 Let (K,B) be a $\mathcal{B}_\mu \mathcal{G} \wp$ —Closed subset of (X,Y,$\mathcal{M}_\mu$). If (K,B) ⊆ (N,Q) ⊆ b_μs-cl(K,B), then (N,Q) is also $\mathcal{B}_\mu \mathcal{G} \wp$ —Closed subset of (X,Y,$\mathcal{M}_\mu$)

Proof: Let (N,Q) ⊆ (A,M), where (A,M) is a binary supra ∝ open set in (X,Y,$\mathcal{M}_\mu$). Then (K,B) ⊆ (N,Q) implies that (K,B) ⊆ (A,M). Since (K,B) is $\mathcal{B}_\mu \mathcal{G} \wp$ —Closed set, b_μ*s-cl(K,B)*⊆ *(A,M).* Also (N,Q) ⊆ b_μs-cl(K,B) implies $b_\mu$scl(N,Q)⊆ (A,M). Therefore (N,Q) is $\mathcal{B}_\mu \mathcal{G} \wp$ —Closed.

4. CONCLUSION

In this paper, we discussed a new form of Binary supra generalized $\wp$ —closed set, $\mathcal{B}_\mu \mathcal{G} \wp$ —open set, its characterization are discussed with the Indian Astrology and Astronomy

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