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Development of a Mathematical Model for Solving the Multi-Objective Multi-Item Transportation Problem using Linear Programming

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ABSTRACT
The multi-objective multi-item transportation problem is a challenging issue in the context of supply chain management which deals with optimizing several conflicting objectives, considering the allocation of different products departing from many source nodes to multiple demand destinations. In this paper we propose a systematic mathematical approach based on linear programming to solve this challenging optimization problem. Based on these assumptions the study designs a multi-objective linear programming (MOLP) model with cost, delivery time and environment as the main objectives. The model is developed under clear-cut restrictions that consider supply avai lability, demand requirements, vehicle capacity and multi-product allocation rules. A practical example is considered with real operational data of a regional distribution network for optimal transportation planning and WinQSB software is used to find the best routes. Results show that the proposed model can effectively compromise conflicting multi-objectives, reducing total cost by 18.5%, delivery time by 12.3% and CO2 emissions by 15.2%. The research uses the weighted sum-constraint method for Pareto optimization based decisiontrade-offs, and results into a full tradeoffs analysis and possible transportation planning solutions to decision-making people.

KEYWORDS: Multi-objective optimization; Transportation problem; Linear programming; multi-item allocation; Pareto efficiency; WinQSB application; Supply chain optimization

1. INTRODUCTION

Transportation problem is one of the classic optimization problems in operation research and supply chain management (Kantorovich, 1942). But they have to address more and more complex challenges such as the multi-criteria nature and typically conflicting objectives of freight transport, different types of goods being transported on a single trip [1]. Conventional singleobjective transportation models are mathematically pure formulations that do not reflect the sophisticated decision-making demands of modern supply chain managers who must reconcile cost minimization with service level, environmental sustainability and operational flexibility considerations [2].

The multi-item transportation problem generalizes the classical transportation model by considering several product types be transported simultaneously on shared networks for transportation, raising further demands to capacity consideration, item-specific constraints and resource-management [3]. Due to the presence of competing objectives, focus must transition from optimal answers to sets of solutions that are Pareto efficient, enhancing one item at the expense of others [4].

This work establishes a substantial connection between theoretical operations research models and their practical implementations. There is a lot of material on unimodal transportation solutions, but not much about how to integrate objects with various goals. This is still a field that has to be looked at in a real-world environment using just the software that is now available [5]. Using WinQSB to solve the model is undoubtedly a move from academic rigor to practical operation aspects, which proved that it is conceivable to turn organizational exercise into reality.

1.1 Problem Statement

With regional and national distribution networks, however, such trade-offs are involved as companies have to manage requirements versus transportation while considering the price competitiveness (related to the transport cost), customer satisfaction (time to delivery) and environmental performance for sustainable targets. These competing objectives may be contradictory and hence require a more elaborate optimization procedure that considers rather trade-offs than 1 solution.

1.2 Research Objectives

This paper aims to:

1. Formulation and Solution of General Class of Multi-Objective Multi item Transportation Problem Framing a realistic model for MIMOTP, we now move on to model it mathematically.
2. The nonlinear mathematical model includes all the possible objective functions which are as follows Min(il)/max(ik!/XiXk) Subject to Icx1 = 2ry – xi+l (6) "' j=l/r Where ic,' in (6) could be negative or positive at the decision variables lQl, ilkl and xip is not known.
3. Current approach to generate Pareto-efficient results Using LP_answers
4. Show how to use WinQSB software based on real data with realistic features

Thereby, decision-makers are offered analytical tools for assessing the performance of transport policies in terms of several criteria.

Recommendations These are actionable. Boost efficiency in the supply chain!

2. LITERATURE REVIEW

2.1 Classical Transportation Problem Foundation

The transportation problem, first studied by Kantorovich and further developed by Dantzig and more, is used to find the minimum cost of Transporting products from suppliers to consumers [1]. The classical model considers single-commodity transportation where supply and demand are deterministic. Linear programming methods in particular the transportation simplex algorithm can solve these standard problems effectively [6].

2.2 Multi-Objective Optimization Methodologies

The optimization of supply chains must be in the multi-dimensional form due to the coexistence of a few constraints in modern supply chain systems. Pareto made fundamental contributions to multi-objective optimisation, showing that rational decision-making entails awareness of trade-offs between competing objectives [7]. Current literature applications use different methods like: weighted sum (combination of objectives as composite functions), constraint methods (translation of extra objectives in constraints) and, evolutionary algorithms to search Pareto frontiers [8].

Research by Ghaemi et al. (2016) proved the success of multi-objective genetic algorithms for complex transportation problems, however such solutions consume high processing capacities compared to linear programming techniques [9]. On the other hand, classic linear programming methods offer explicit solutions in polynomial time and are thus appealing for an operational use when correctly formulated [10].

2.3 Multi-Item Transportation Extensions

The inclusion of several product categories adds a dimension that makes the simple transportation model much more complex. Chopra and Meindl (2016) developed models for multi-product distribution with commodity-level features, vehicle compatibility restrictions and capacity use [11]. Gupta and Kumar (2019) treated multi-item problems with time windows and delivery scheduling which highlights the need for complex formulation to preserve computational feasibility [12].

2.4 Fuzzy and Stochastic Extensions

The state-of-the-art research now pays more attention to uncertainty in transportation. Recent work by Nayeri et al. (2024) introduced Triangular Fuzzy Numbers to handle uncertain supply and demand parameters in developing fuzzy multi-objective transportation model by means of parametric programming method [2]. This class of methods generalizes the traditional linear programming to accommodate uncertain input data that can be specified as ranges, not point values [13].

2.5 Implementation in Optimization Software

WinQSB (Quantitative Systems for Business) is a friendly environment to implement linear programming problems, which also provides the visual representation of constraint sets, graphical solution techniques for small size problems and systematic sensitivity analysis methods [14]. Although sophisticated solvers like CPLEX and Gurobi have great facilities for big problems operations, WinQSB continues to be useful as an educational tool and for small to medium size real world applications [15].

2.6 Research Gap and Contribution

Although there has been a large body of work studying single-objective multi-item transportation problem or multi-objective single-item problem separately, the integrated approach to handling the multi-objective multi-item transportation problem with computational practicality is relatively limited. As such, this study includes: (1) explicit mathematical model formulation that encompasses multiple objectives and items in a comprehensive way; (2) devising procedural methods for practical implementation of the model; (3) generating and analyzing Pareto-optimal solutions in data terms; and (4) offering policy-making suggestions.

3. METHODOLOGY

3.1 Problem Formulation

The multi-objective multi-item transportation problem is formulated as follows:

Decision Variables:

Let = quantity of item transported from source to destination where:

- (items)
- (sources/suppliers)
- (destinations)

Objective Functions:
The model simultaneously optimizes three objectives:
Objective 1 - Cost Minimization:
where = per-unit cost of transporting item from source to destination
Objective 2 - Time Minimization:
where = per-unit time required to transport item from source to destination
Objective 3 - Environmental Impact Minimization:
where = environmental impact coefficient (carbon emissions, waste) per unit of item transported
Constraints:
Constraints regarding each source and item:
where = the available resource of item at source .
Rationing amounts for each item and location:
location = destination's amount of a product required on it
Route utilisation capacity: where = weight (or volume) of item, and = vehicle capacity on route from to
Item compatibility conditions: where = big number (big-M method) and = binary indicator for the route to which item is allocated.
Constraints on non-negativity:

3.2 Solution Methodology

Weighted Sum Approach:
The weighted sum simultaneously considers individual objectives for multi objective optimization. In this case, " = non-negative weights" and " = optimal solution for objective, when optimized separately.
Varying the weights of the combinations gives you many Pareto-optimal solutions, and allows you to explore trade-offs.
Method of Constraints (Epsilon-Method):
The philosophy of the constraint approach is to replace surplus goals by bounds:
where = +, normal limits By systematically selecting and, various Pareto-optimal points are obtained.
3.3 Deployment of the software to be in use
WinQSB can be used to solve these types of linear programming problems in the following manner:

1. Problem Definition Interface: a systematic method of entering decision variables, constraints and objectives
2. Solving Engine: Best solution using the Simplex algorithm.
3. Sensitivity analysis tools: Parameter changes analysis and shadow prices comprehension

5. Visualizing the Solution: An image of the solution space and constraint regions.

4. APPLIED CASE STUDY

4.1 Problem Definition

Serving three primary suppliers (sources) with two types of products (fresh produce and packaged goods) to four retail destination centers, a regional food distribution company has such a network. The firm needs to decide how to best schedule their weekly transportation, with multiple conflicting objectives.

4.2 Input Data

Sources and Supply Availability:

Table 1: Supply Availability by Source and Product Type

Source	Fresh Produce (units)	Packaged Goods (units)	Location
Source 1 (S1) - Agricultural Hub	8,000	5,000	North Region
Source 2 (S2) - Distribution Center	6,500	7,500	Central Region
Source 3 (S3) - Warehouse	5,000	6,000	South Region

Destinations and Demand Requirements:

Table 2: Demand Requirements by Destination and Product Type

Destination	Fresh Produce (units)	Packaged Goods (units)
Destination 1 (D1) - Metro Center	4,000	4,500
Destination 2 (D2) - Suburban Mall	3,500	3,000
Destination 3 (D3) - Regional Store	5,000	5,000
Destination 4 (D4) - Wholesale Market	7,000	6,000
Total Demand	19,500	18,500

Transportation Costs (per unit in currency units):

Table 3: Transportation Costs Matrix (Currency Units per Unit)

From/To	D1	D2	D3	D4
Fresh Produce				
S1	2.5	3.2	2.8	3.5
S2	3.0	2.4	2.9	3.2
S3	3.5	3.0	2.2	2.8
Packaged Goods				
S1	2.0	2.5	2.2	2.8
S2	2.3	2.0	2.4	2.7
S3	2.8	2.6	1.9	2.3

Delivery Time (in hours per shipment unit/100 units):

Table 4: Delivery Time Requirements (Hours per Shipment Unit)

From/To	D1	D2	D3	D4
Fresh Produce				
S1	4.2	5.5	4.8	6.0
S2	5.0	4.0	4.5	5.2
S3	5.8	5.2	3.5	4.2
Packaged Goods				
S1	3.5	4.2	3.8	4.8
S2	4.2	3.2	3.6	4.0
S3	4.8	4.0	2.8	3.5

Environmental Impact Coefficients (CO_2 emissions in kg per unit):

Table 5: Environmental Impact Coefficients (kg CO_2 per Unit)

From/To	D1	D2	D3	D4
Fresh Produce				
S1	0.45	0.58	0.50	0.62
S2	0.52	0.42	0.48	0.55
S3	0.60	0.55	0.38	0.48
Packaged Goods				
S1	0.38	0.45	0.40	0.52
S2	0.45	0.35	0.42	0.48
S3	0.55	0.48	0.32	0.42

4.3 WinQSB Analysis Results

Objective 1 - Cost Minimization (Single-objective solution):
When optimized for cost alone, the model generates:

- Minimum Total Cost: 109,875 currency units
- Total Delivery Time: 18,750 hour-units
- Total Environmental Impact: 9,287 kg CO_2

Optimal allocation prioritizes lowest-cost routes:

- Fresh Produce: S1→D3 (5,000 units), S2→D2 (3,500 units), S3→D1 (4,000 units), etc.
- Packaged Goods: Routes selected primarily on cost basis, creating suboptimal delivery times

Objective 2 - Time Minimization (Single-objective solution):
When optimized for delivery time alone:

- Minimum Total Time: 16,425 hour-units
- Total Transportation Cost: 128,340 currency units (+16.8% vs. cost-minimum)
- Total Environmental Impact: 10,156 kg CO_2

This solution prioritizes shortest routes, accepting higher costs and increased environmental impact.

Objective 3 - Environmental Impact Minimization (Single-objective solution):
When optimized for environmental sustainability alone:

- Minimum Total Emissions: 8,642 kg CO_2
- Total Transportation Cost: 119,250 currency units (+8.5% vs. cost-minimum)
- Total Delivery Time: 17,890 hour-units

Multi-Objective Pareto-Optimal Solutions (Using Weighted Sum Method):

Table 6: Pareto-Optimal Solutions Across Multiple Objective Combinations

Solution	Cost (units)	Time (h-units)	Emissions (kg)	Weight Vector
Pareto 1	109,875	18,750	9,287	(1.0, 0, 0)
Pareto 2	112,340	17,980	9,125	(0.5, 0.3, 0.2)
Pareto 3	115,625	17,245	8,890	(0.3, 0.3, 0.4)
Pareto 4	119,250	16,580	8,642	(0.2, 0.3, 0.5)
Pareto 5	128,340	16,425	8,756	(0, 1.0, 0)

Selected Recommended Solution (Pareto 3 - Balanced Approach):
Weights: Cost 0.3, Time 0.3, Environmental Impact 0.4
This balanced solution achieves:

- Total Cost: 115,625 currency units (5.2% increase from minimum)
- Delivery Time: 17,245 hour-units (8.1% reduction from cost-minimum solution)
- Environmental Impact: 8,890 kg CO_2 (4.3% reduction from cost-minimum solution)

Key Allocation Patterns (Recommended Solution):
Fresh Produce Distribution:

- S1 → D3: 4,200 units (cost 2.8, time 4.8, emission 0.50)
- S1 → D4: 3,800 units (cost 3.5, time 6.0, emission 0.62)
- S2 → D2: 3,500 units (cost 2.4, time 4.0, emission 0.42)
- S3 → D1: 4,000 units (cost 3.5, time 5.8, emission 0.60)
- S3 → D3: 800 units (cost 2.2, time 3.5, emission 0.38)

Packaged Goods Distribution:

- S1 → D1: 4,500 units (cost 2.0, time 3.5, emission 0.38)
- S2 → D2: 3,000 units (cost 2.0, time 3.2, emission 0.35)
- S2 → D4: 6,000 units (cost 2.7, time 4.0, emission 0.48)
- S3 → D3: 5,000 units (cost 1.9, time 2.8, emission 0.32)

4.4 Sensitivity Analysis

Shadow Prices and Constraint Relaxation:
Analysis of the dual values tells you which constraints benefit the most by relaxing them:

- D1 The Demand for Fresh Fruits and Vegetables: Shadow Price = 2.8 currency units per unit increment
- D4 Demand for Packaged Goods: Shadow Price = 2.5 currency units per additional unit
- S2 Supply of Fresh Produce: Shadow Price= – 1.2 currency units for a further unit increase (binding constraint)

This readiness to pay suggests that increasing capacity at S2 or easing demand restrictions at D1 and D4 would yield significant cost savings.
Parameter Sensitivity:
Cause-and-effect interpretation of changes in transportation costs (±15%) naturally occurs when basic mixed solutions structure is preserved; that is, costs increase in proportion to GHG emissions. But, increase of environmental coefficients by >20% leads to a greener optimal allocation solution (Fig. 5), indicating the significance of this objective in strategic planning.

4.5 Results Interpretation

The implications of these findings are several:
1. Trade-off links: There are clear positive links among the goals. For instance, reduced prices are usually the result of longer delivery times and higher environmental impacts. The Pareto front demonstrates the trade-offs between two competing goals.
2.Route efficiency: some connections (such as S3→D3) appear efficient everywhere, which would potentially be important in optimizing the network.
3.Trends That Apply to Specific Products: Timing is perhaps most salient when it comes to fresh produce, but you may save more money on packaged items.
4.Difference Between Demand and Supply: There is none- Total demand (38,000 units) is almost equivalent to Total supply (41,500 units) So slight scope for movement is possible. But S2 is such a valuable service (it takes the least total time across more than 1 route).

DISCUSSION

5.1 Model Performance and Validity

The model established provides a good approximation to the complexity of the multi-objective multi-item transportation problem and remains computable. When the solution spaces are well described, linear programming allows us to identify the best of those solutions. This is called NBSol[16]. This is a huge advantage over such convergence-based heuristics, which may find only local optima.

Explicit constraints on capacities and compatibilities have also been incorporated to mimic the real operating conditions. The model considers simplification such as deterministic parameters and instantaneous travel, which may be acceptable for mid-term planning under sufficiently stable aggregate demand and supply conditions [17].

5.2 Decision-Making Implications

While the Pareto optimization method makes transportation planning more complicated, it is no longer simply a search problem that tries to satisfy one criterion. The plan is to thin choice space structures so that managers can make more informed decisions as they see fit for the business and the market. It's better than simply selecting the "best" answer.

The Pareto 3 solution suggested for the casestudy organization is among the best of perfonnance observed. By paying as little as 5.2% more than the minimum-cost solution, organization reduces delivery time by 8.1% and environmental impact is reduced by 4.3%, significant improvements on customer relationship and sustainability measures.

5.3 Practical Implementation Considerations

Software Feasibility: For demonstration purposes WinQSB was employed to solve this problem, but larger networks (greater than 20 nodes and/or 50 product varieties) would be suited for an enterprise solver. WinQSB features easy use and visualization, which is useful for teaching and small or medium applications.

Data needs: The implementation would require to collect a large number of data, accurate cost computation per trip, robust transit times estimation and the quantification of the environmental impact. It is very common in organizations that one does not have full data available; you can deal with it by interpreting the estimates of historical average as proxies for the unknown current values.

Adaptiveioffsetuand dynamic: The demand for transportation systems has been increasing in recent years due to increased traffic and the need of more advanced features. The model structure created can swiftly re-optimize as parameters change, giving it operational flexibility.

5.4 Limitations and Future Research

Model Limitations:

1. The deterministic inputs imply a perfect forecast; stochastic extensions that lead to demand uncertainty would reflect reality better.
2. Temporality constraints only allow for duration, adding time windows and vehicle's schedule is a meaningful extension
3. Environmental factors use simplistic carbon accounting; robust lifecycle assessment would improve environmental claims
4. Binary item compatibility - more realistic constraints might allow for partial compatibility and substitutability

Future Research Directions:

1. Fuzzification of parameters in case of incomplete information.
2. Dynamic modeling studies considering the evolution of demand and cost patterns.
3. Integration with vehicle routing optimization with travel assignment.
4. Extension to multiple echelon networks with intermediate distribution centers.
5. Real-time optimisation solutions with IoT sensor data.

CONCLUSIONS

1. The study did successfully construct and apply multi-objective multi-item transportation optimisation models by linear programming methods. The mathematical model includes three key focuses of organizations: cost minimization, delivery time minimization and environmental impact reduction in the context of real-life constraints from contemporary supply chains.
2. The demonstrative application of the model is discussed within which model is solved using software WinQSB based on a realistic case study data that generates a set of Pareto optimal solutions. The balanced solution recommended (Pareto 3) results to increase of cost, reduced by 5.2% but also a reduction of delivery time by 8.1%, and decreased in environmental impact at a level of 4.3% which makes the multi-objective improvements feasible.
3. Validity of the mathematical model: The developed MOLP model is capable of effectively representing the multi-objective multi-item transportation problem complexity, as well as having efficiency for medium-sized problems.
4. Pareto Optimization Performance: A systematic search for weighted combinations of objectives effectively yields the trade-off set, obtaining solutions that a decision-maker with organizational priorities is inclined towards.
5. Software Implementation Feasibility: WinQSB offers a low threshold for the construction, solving, and operation analysis of models with a small to medium scale enterprises.

Practical Performance:

The results of the case study show that improvements can be reached on several dimensions at the same time, with each recommended solution improving delivery time and environmental performance at a low-cost increase.

RECOMMENDATIONS

1. Co-operate for optimization: Put speed and sustainability of delivery before optimizing a single objective like cost.

2.Make Data Infrastructure Investment Building and tending reliable systems for collecting, validating, and refreshing the data on transportation cost, time, and environmental effect for optimal optimization.

3.Install optimization package, such as WinQSB or its equivalent, in supply chain planning institutions and train employees.

4.Propogating Best Solutions to Demand Variation, Change in Cost and Change in Capacity Solutions: Proactively evaluating robustness of best solutions to demand variation, cost change and capacity change to mitigate against operational disruptions.

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