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Non-Singular BCI-Algebras

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ARTICLE INFO	ABSTRACT
Published Online: 29 December 2025 **Corresponding Author:** DR. Md Najmul Hoda	The study of BCK/BCI algebras was initiated by Imai Iseki in 1966. These concepts of two classes of abstract algebras studied by eminent authors. Here we study properties of a non-singular BCI algebra which has been introduced by Prasad and Abid under the semi commutative BCI-algebra.

KEYWORDS: BCI-algebra, Semi commutative BCI algebra and Non-singular BCI algebra etc.

1.1 INTRODUCTION

Definition (1.1) : A system $(X, *, 0)$ consisting of non-empty set X, a binary operation * and fixed element 0, is called a BCI-algebra if the following conditions are satisfied.

(BCI 1) $((x * y) * (x * z) * (z * y)) = 0$

(BCI 2) $(x * (x * y)) * y = 0$

(BCI 3) $x * x = 0$

(BCI 4) $x * y = 0 = y * x \Rightarrow x = y$

For all $x, y, z \in X$

Definition (1.2) : A BCI, algebra $(X, *, 0)$ is called

(a) a BCK-algebra if $0 * x\ 0$, for all $x \in X$

(b) a non-singular BCI-algebra if $0 * x = x$ for all $x \in X$

Definition (1.3) : In a BCI-algebra, a partial ordering '≤' is defined as $x \leq y$ iff $x * y = 0$; $x,y\in X$

Theorem (1.1) : Let $(X, *, 0)$ be a BCI-algebra. Then the following result hold

(i) $x * 0 = x$

(ii) $x * (x * (x * y)) = x * y$

(iii) $(x * y) * z = (x * z) * y$

(iv) $x \leq y \Rightarrow x * z \leq y * z$ and $z * y \leq z * x$

Definition (1.5) : A BCI – algebra is said to be

(a) Commutative if $x * (x * y) = y * (y * x)$

(b) Semi-commutative if $x * y = y * y$

For all $x,y \in X$

Proof : Let $x \neq y \neq 0$ and $x * y = 0$. Then the above theorem implies $y * x = 0$.

So condition (BCI 4) implies $x = y$. This is contradiction. Hence $x * y \neq 0$

Also $x * y = x \Rightarrow (x * y) * x = x * x$.

$\Rightarrow (x * x) * y = 0 \Rightarrow y = 0$

which is also a contradiction.

If $x * y = y$ then $y * x = y \quad \Rightarrow (y * x) * y = y * y$

$\Rightarrow (y * y) * x = 0$

$\Rightarrow x = 0$

which is also a contradiction.

Hence the result

The above result very useful in providing example of non-singular BCI-algebra.

Theorem (1.2) : Let $(X, *, 0)$ be a non-singular BCI-algebras. Let a, b, c be three non-identical and non-zero element of X such that $a * b = C$ then $a * c = b$ and $b * c = a$.

Proof : We have $a * c \quad = a * (a * b)$

$= (a * b) * a = (a * a) * b$

$= 0 * b = b$

Also $b * c \quad = b * (a * b)$

$= (a * b) * b$

$= (b * a) * b = (b * b) * a = 0 * a = a$

Corollary (1.1) : Every non-zero element of a non-singular BCI-algebra is an atom.

Proof : If $x * a = b$ Then the above theorem implies $x * b = a$. So $x * (x * a) = a$ for any $x \in X$ this proves that a is an atom of X.

Theorem (1.3) : In the binary table of a non-singular BCI-algebra no two elements of particular row (or a particular column) are identical.

Proof : Let $x_0, x_1, x_2,\ldots x_n$ be elements of a non singular BCI-algebra X. If possible suppose that $x_i * x_j = x_i * x_k$

Where $J \neq K$ then

$(x_i * x_J) * x_i = (x_J * x_k) * x_i$

$\Rightarrow \quad (x_i * x_i) * x_J = (x_i * x_i) * x_k$

$\Rightarrow 0 * x_J = 0 * x_k \Rightarrow x_J = x_k$ which is contradiction. So no two elements of ith row are identical

Similarly we see that

$x_i * x_l = x_J * x_l \ (i \neq J)$

⇒ $(x_1 * x_i) * x_1 = (x_1 * x_J) * x_1$

⇒ $(x_1 * x_1) * x_i = (x_1 * x_1) * x_J$

⇒ $0 * x_i = 0 * x_J$

⇒ $x_i = x_J$ which not true.

Hence the result.

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