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Defining New Types of Integers: *k*-PrimeFactors Numbers

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ABSTRACT

Prime numbers are building blocks of integers. Using prime numbers as base, this work defines a class of new types of numbers, namely, *k*-PrimeFactors numbers, for each non-negative integer *k*. Interestingly, *k*-PrimeFactors numbers are defined using prime numbers and they, in turn, generalize their own base, the prime numbers. First 100 *k*-PrimeFactors numbers for initial values of *k* up to 10 are presented for demonstration. The occurrence frequency of these numbers till 1 million is also presented.

KEYWORDS: Prime numbers; composite numbers; *k*-PrimeFactors numbers (new term being defined)

I. INTRODUCTION TO PRIME NUMBERS

One of the most studied types of integers is prime numbers. They have been center of attraction for mathematicians from time immemorial.

Definition (*Prime Number or Prime*) [1] : An integer $p > 1$ which has only two positive integer divisors, namely, 1 and itself (p), is called a prime number or simply a prime.

So, a prime number p has only two integer factorizations, viz., $p = 1 \times p = p \times 1$.

In fact, if commutative property of multiplication of integers is understood, there is only one integer factorization of any prime number p, viz., $p = 1 \times p$.

Positive integers are naturally called natural numbers.

By property of divisibility, every natural number n has always these two divisors or factors, viz. 1 and itself (n). These divisors are called trivial or improper divisors. In case of the very first natural number 1, they coincide and $n = 1$ has only one trivial divisor 1 (which is also itself). This makes 1 unique in one more sense that will follow in a soon appearing context.

A prime number is a natural number, i.e., positive integer, greater than 1, which cannot be written as product of two smaller natural numbers. It doesn't have proper integer divisors.

List of prime numbers goes like :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, …

Positive integers greater than 1, that are not prime numbers, form another type of numbers.

Definition (*Composite Number*) [1] : An integer $n > 1$ which has more than 2 positive integer divisors is called a composite number.

So, composite numbers are integers greater than 1 that are not prime numbers.

Composite numbers do have non-trivial integer divisors, i.e., proper integer divisors, other than 1 and themselves.

List of composite numbers goes like :

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50, …

Each positive integer, except first one 1, is either a prime number or a composite number and, of course, not both. 1 is the only positive integer which neither prime nor composite!

II. HISTORICAL ASPECTS & EARLIER KNOWN PROPERTIES OF PRIME NUMBERS

Prime numbers have been known to mathematicians from ancient times. Their use with explicit type identification dates back to millennia. But the credit of invention of the idea of prime numbers cannot be given to any single person.

The Rhind Mathematical Papyrus [2] from ancient Egyptian civilization dating back to around c. 1500 BCE contains work with integers, where distinctiveness of prime numbers vis-à-vis composite numbers is evident. It seems that Egyptians were aware of what we call prime numbers today enjoy some unique features.

There are clear indications that Pythagoreans [3] realized to certain extent the primality of numbers as early as about c. 500 BCE.

While these are subtle references to early knowledge of prime numbers, work of Euclid c. 300 BCE is firm

understanding. In fact, Euclid proved one of the most important properties of prime numbers : their infinitude, i.e., they being infinite in count [4]. His proof is very elegant.

The procedure for finding prime numbers in a range is given by Eratosthenes in what is popularly known as the Sieve of Eratosthenes [5], one of the best and efficiency wise better algorithms for determining prime numbers with certainty.

III. THE FUNDAMENTAL THEOREM OF ARITHMETIC

Important branches of Mathematics have their Fundamental Theorems. These include Calculus, Differential Geometry, Algebra etc. Arithmetic is a basic branch of mathematics and it is no exception.

Theorem (*Fundamental Theorem of Arithmetic*) [6][7] : Every integer greater than 1 is either itself a prime number or can be expressed as a product of prime numbers in a unique way up to the order of the prime factors.

1 is an exceptional natural number, which, as noted earlier, is neither a prime number nor a composite number; but rest all natural numbers get categorized either as prime or as composite. So, the Fundamental Theorem of Arithmetic classifies each natural number greater than 1 into two types, viz., those that are themselves prime or those that are not themselves prime, but then can be factorized as product of finite number of prime numbers. The first of these types - prime numbers - are being studied extensively from ancient era. They exhibit interesting properties, are source of innumerable unsettled conjectures and their peculiarity lies in fact that as yet there is no simple formula to determine n^{th} prime!

IV. κ-PRIMEFACTORS NUMBERS - NEW NUMBER TYPES DEFINED

The Fundamental Theorem of Arithmetic provides source to define new type of natural numbers greater than 1. Each natural number that is not itself a prime is product of certain fixed number of primes. Using this fact, a new type of number is being defined here.

Definition (*k-PrimeFactors Number*) : For non-negative integer k, a positive integer which is product of k number of prime numbers is called a k-PrimeFactors number.

For each non-negative integral value of k, we get a new type of k-PrimeFactors number. So, k-PrimeFactors number is a class of infinite types of numbers, one for each non-negative integer value of k.

For $k = 0$, we get 0-PrimeFactors numbers. There is only one 0-PrimeFactors number, viz. 1. It is product of zero number of primes, i.e., an empty product as product of primes and by convention; empty product equals multiplicative identity 1.

For $k = 1$, we get 1-PrimeFactors numbers. Owing to grammar rule, they should be called 1-PrimeFactor Numbers, having singular factor form. But, to generalize common terminology, we allow use of 1-PrimeFactors numbers for 1-PrimeFactor Numbers. 1-PrimeFactors numbers are none other than prime numbers themselves. So, the concept of k-PrimeFactors numbers generalizes the concept of its own base, the prime numbers; prime numbers are k-PrimeFactors numbers with $k = 1$.

For $k = 2$, we get 2-PrimeFactors numbers, which are currently well-known by the name semiprimes.

For each positive integral values of $k \geq 3$, we get a different k-PrimeFactors number type.

Each positive integer is of one and only one type of k-PrimeFactors number for some non-negative integer k.

V. FIRST 100 κ-PRIMEFACTORS NUMBERS FOR INITIAL VALUES OF κ

For demonstration purpose, we provide lists of first 100 k-PrimeFactors numbers for initial values of k and numbers of these numbers less than one million.

A. 1-PrimeFactor(s) Numbers, aka, prime numbers :

Table 1: First 100 1-PrimeFactor(s) Numbers

Sr. No.	1-PrimeFactor(s) Numbers (Prime Numbers)									
1-10	2	3	5	7	11	13	17	19	23	29
11-20	31	37	41	43	47	53	59	61	67	71
21-30	73	79	83	89	97	101	103	107	109	113
31-40	127	131	137	139	149	151	157	163	167	173
41-50	179	181	191	193	197	199	211	223	227	229
51-60	233	239	241	251	257	263	269	271	277	281
61-70	283	293	307	311	313	317	331	337	347	349
71-80	353	359	367	373	379	383	389	397	401	409
81-90	419	421	431	433	439	443	449	457	461	463
91-100	467	479	487	491	499	503	509	521	523	541

There are 78498 1-PrimeFactor(s) numbers less than one million.

B. 2-PrimeFactors Numbers, aka, semiprimes :

Table 2 : First 100 2-PrimeFactors Numbers

Sr. No.	2-PrimeFactors Numbers (Semiprimes)									
1-10	4	6	9	10	14	15	21	22	25	26
11-20	33	34	35	38	39	46	49	51	55	57
21-30	58	62	65	69	74	77	82	85	86	87
31-40	91	93	94	95	106	111	115	118	119	121
41-50	122	123	129	133	134	141	142	143	145	146
51-60	155	158	159	161	166	169	177	178	183	185
61-70	187	194	201	202	203	205	206	209	213	214
71-80	215	217	218	219	221	226	235	237	247	249
81-90	253	254	259	262	265	267	274	278	287	289

Sr. No.	2-PrimeFactors Numbers (Semiprimes)									
91-100	291	295	298	299	301	302	303	305	309	314

There are 210035 2-PrimeFactors numbers less than one million.

C. 3-Prime Factors Numbers :

Table 3 : First 100 3-PrimeFactors Numbers

Sr. No.	3-PrimeFactors Numbers									
1-10	8	12	18	20	27	28	30	42	44	45
11-20	50	52	63	66	68	70	75	76	78	92
21-30	98	99	102	105	110	114	116	117	124	125
31-40	130	138	147	148	153	154	164	165	170	171
41-50	172	174	175	182	186	188	190	195	207	212
51-60	222	230	231	236	238	242	244	245	246	255
61-70	258	261	266	268	273	275	279	282	284	285
71-80	286	290	292	310	316	318	322	325	332	333
81-90	338	343	345	354	356	357	363	366	369	370
91-100	374	385	387	388	399	402	404	406	410	412

There are 250853 3-PrimeFactors numbers less than one million.

D. 4-PrimeFactors Numbers :

Table 4 : First 100 4-PrimeFactors Numbers

Sr. No.	4-PrimeFactors Numbers									
1-10	16	24	36	40	54	56	60	81	84	88
11-20	90	100	104	126	132	135	136	140	150	152
21-30	156	184	189	196	198	204	210	220	225	228
31-40	232	234	248	250	260	276	294	296	297	306
41-50	308	315	328	330	340	342	344	348	350	351
51-60	364	372	375	376	380	390	414	424	441	444
61-70	459	460	462	472	476	484	488	490	492	495
71-80	510	513	516	522	525	532	536	546	550	558
81-90	564	568	570	572	580	584	585	620	621	625
91-100	632	636	644	650	664	666	676	686	690	693

There are 198062 4-PrimeFactors numbers less than one million.

E. 5-PrimeFactors Numbers :

Table 5 : First 100 5-PrimeFactors Numbers

Sr. No.	5-PrimeFactors Numbers									
1-10	32	48	72	80	108	112	120	162	168	176
11-20	180	200	208	243	252	264	270	272	280	300
21-30	304	312	368	378	392	396	405	408	420	440
31-40	450	456	464	468	496	500	520	552	567	588
41-50	592	594	612	616	630	656	660	675	680	684
51-60	688	696	700	702	728	744	750	752	760	780
61-70	828	848	882	888	891	918	920	924	944	945

Sr. No.	5-PrimeFactors Numbers									
71-80	952	968	976	980	984	990	1020	1026	1032	1044
81-90	1050	1053	1064	1072	1092	1100	1116	1125	1128	1136
91-100	1140	1144	1160	1168	1170	1240	1242	1250	1264	1272

There are 124465 5-PrimeFactors numbers less than one million.

F. 6-PrimeFactors Numbers :

Table 6 : First 100 6-PrimeFactors Numbers

Sr. No	6-PrimeFactors Numbers									
1-10	64	96	144	160	216	224	240	324	336	352
11-20	360	400	416	486	504	528	540	544	560	600
21-30	608	624	729	736	756	784	792	810	816	840
31-40	880	900	912	928	936	992	1000	1040	1104	1134
41-50	1176	1184	1188	1215	1224	1232	1260	1312	1320	1350
51-60	1360	1368	1376	1392	1400	1404	1456	1488	1500	1504
61-70	1520	1560	1656	1696	1701	1764	1776	1782	1836	1840
71-80	1848	1888	1890	1904	1936	1952	1960	1968	1980	2025
81-90	2040	2052	2064	2088	2100	2106	2128	2144	2184	2200
91-100	2232	2250	2256	2272	2280	2288	2320	2336	2340	2480

There are 68963 6-PrimeFactors numbers less than one million.

G. 7-PrimeFactors Numbers :

Table 7 : First 100 7-PrimeFactors Numbers

Sr. No	7-PrimeFactors Numbers									
1-10	128	192	288	320	432	448	480	648	672	704
11-20	720	800	832	972	1008	1056	1080	1088	1120	1200
21-30	1216	1248	1458	1472	1512	1568	1584	1620	1632	1680
31-40	1760	1800	1824	1856	1872	1984	2000	2080	2187	2208
41-50	2268	2352	2368	2376	2430	2448	2464	2520	2624	2640
51-60	2700	2720	2736	2752	2784	2800	2808	2912	2976	3000
61-70	3008	3040	3120	3312	3392	3402	3528	3552	3564	3645
71-80	3672	3680	3696	3776	3780	3808	3872	3904	3920	3936
81-90	3960	4050	4080	4104	4128	4176	4200	4212	4256	4288
91-100	4368	4400	4464	4500	4512	4544	4560	4576	4640	4672

There are 35585 7-PrimeFactors numbers less than one million.

H. 8-PrimeFactors Numbers :

Table 8 : First 100 8-PrimeFactors Numbers

Sr. No	8-PrimeFactors Numbers									
1-10	256	384	576	640	864	896	960	1296	1344	1408
11-20	1440	1600	1664	1944	2016	2112	2160	2176	2240	2400
21-30	2432	2496	2916	2944	3024	3136	3168	3240	3264	3360
31-40	3520	3600	3648	3712	3744	3968	4000	4160	4374	4416

Sr. No	8-PrimeFactors Numbers									
41-50	4536	4704	4736	4752	4860	4896	4928	5040	5248	5280
51-60	5400	5440	5472	5504	5568	5600	5616	5824	5952	6000
61-70	6016	6080	6240	6561	6624	6784	6804	7056	7104	7128
71-80	7290	7344	7360	7392	7552	7560	7616	7744	7808	7840
81-90	7872	7920	8100	8160	8208	8256	8352	8400	8424	8512
91-100	8576	8736	8800	8928	9000	9024	9088	9120	9152	9280

There are 17572 8-PrimeFactors numbers less than one million.

I. 9-PrimeFactors Numbers :

Table 9 : First 100 9-PrimeFactors Numbers

Sr No.	9-PrimeFactors Numbers									
1-10	512	768	1152	1280	1728	1792	1920	2592	2688	2816
11-20	2880	3200	3328	3888	4032	4224	4320	4352	4480	4800
21-30	4864	4992	5832	5888	6048	6272	6336	6480	6528	6720
31-40	7040	7200	7296	7424	7488	7936	8000	8320	8748	8832
41-50	9072	9408	9472	9504	9720	9792	9856	10080	10496	10560
51-60	10800	10880	10944	11008	11136	11200	11232	11648	11904	12000
61-70	12032	12160	12480	13122	13248	13568	13608	14112	14208	14256
71-80	14580	14688	14720	14784	15104	15120	15232	15488	15616	15680
81-90	15744	15840	16200	16320	16416	16512	16704	16800	16848	17024
91-100	17152	17472	17600	17856	18000	18048	18176	18240	18304	18560

There are 8491 9-PrimeFactors numbers less than one million.

J. 10-PrimeFactors Numbers :

Table 10 : First 100 10-PrimeFactors Numbers

Sr. No.	10-PrimeFactors Numbers									
1-10	1024	1536	2304	2560	3456	3584	3840	5184	5376	5632
11-20	5760	6400	6656	7776	8064	8448	8640	8704	8960	9600
21-30	9728	9984	11664	11776	12096	12544	12672	12960	13056	13440
31-40	14080	14400	14592	14848	14976	15872	16000	16640	17496	17664
41-50	18144	18816	18944	19008	19440	19584	19712	20160	20992	21120
51-60	21600	21760	21888	22016	22272	22400	22464	23296	23808	24000

Sr. No.	10-PrimeFactors Numbers									
61-70	24064	24320	24960	26244	26496	27136	27216	28224	28416	28512
71-80	29160	29376	29440	29568	30208	30240	30464	30976	31232	31360
81-90	31488	31680	32400	32640	32832	33024	33408	33600	33696	34048
91-100	34304	34944	35200	35712	36000	36096	36352	36480	36608	37120

There are 4016 10-PrimeFactors numbers less than one million.

Their relative occurrence frequency till one million is depicted in the following graph.

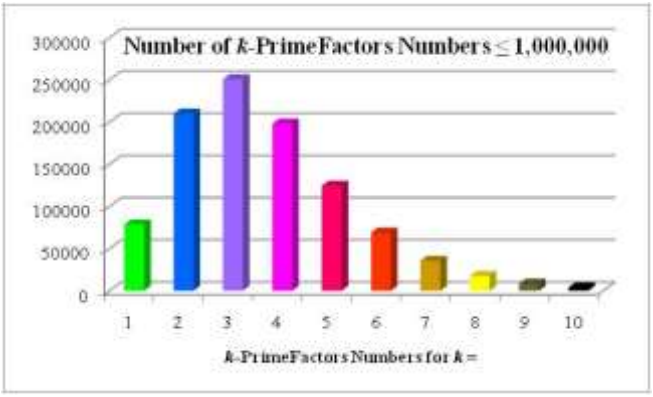

Figure 1 : Number of *k*-PrimeFactors Numbers Less than or Equal to 1 Million

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