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Skemp's Theory in Mathematical Learning Ability Using Relational and Instrumental Understanding

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ABSTRACT

This research aims to examine students' ability to understand mathematical concepts based on the Skemp Theory. This type of research is a literature study or literature review obtained from the Skemp book. The steps of this research are designing a review, conducting a review, analyzing, and writing a review. The results of the study show that relational understanding can help students solve problems by thinking more broadly, critically, and structured. Meanwhile, students with instrumental understanding solve problems quickly, correctly, and not using procedures.

KEYWORDS: relational, instrumental, mathematics

INTRODUCTION

Education, as a primary activity in human existence, plays a crucial role in improving and expanding competencies and skills to be applied in daily life. Furthermore, education involves deliberate and structured initiatives to shape learning environments and instructional stages, enabling students to actively develop their potential and acquire self-control, intellectual capacity, morality, as well as personal, social, national, and civic skills.

Mathematics is a fundamental foundation for the advancement of science and technology and is crucial for developing human thinking skills [1]. Mathematics instruction from elementary to higher education levels emphasizes the application of mathematics in everyday life. According to the National Council of Teachers of Mathematics [2], mathematics consists of four components: problem solving, reasoning, communication, and connections. Based on these learning objectives, students are taught mathematics to develop critical, logical, creative, collaborative, and analytical thinking skills.

Mathematical problem solving is one of the key indicators of mathematical competence, particularly in facing global competition such as the Programme for International Student Assessment (PISA). Indonesia ranked 73rd out of 79 participating countries, with an average mathematics competency score of 379. Mathematical problem-solving skills are therefore vital, as each student's ability to find mathematical solutions plays a crucial role in responding to such global challenges.

[3] in his work, explains that what we recognize as mathematics is actually taught and learned in two distinct ways: instrumental understanding and relational understanding. Instrumental understanding refers to a type of understanding in which learners treat concepts as separate entities and rely primarily on memorizing formulas to perform basic calculations. In contrast, relational understanding is an individual's ability to use structured relationships among mathematical concepts, which are considered interconnected, to find solutions and to explain the procedures used.

The results of a study conducted by [4] indicate that students with high mathematical problem-solving skills are able to demonstrate both relational and instrumental understanding effectively, whereas students at moderate and low levels tend to rely solely on instrumental understanding when solving problems. [5] reports that students' potential to demonstrate relational and instrumental understanding at various levels of problem solving remains relatively low.

Based on the foregoing explanation, this study aims to examine the potential of mathematical learning through relational and instrumental understanding based on Skemp's Theory. Therefore, this article is entitled *'Skemp's Theory in Mathematical Learning Ability Using Instrumental and Relational Understanding.*

METHOD

This study employs a qualitative research design. According to [7] qualitative research does not focus on numerical data

but instead utilizes data in the form of images and descriptive narratives. The purpose of this study is to describe mathematical learning ability using instrumental and relational understanding based on Skemp's Theory by adopting a semi-systematic approach to the literature review. This strategy is designed for topics that are conceptually diverse and have been examined in various ways by researchers from different disciplines. Semi-systematic reviews tend to focus on how a particular subject within a specified field evolves over time.

The selection of the research method was conducted to explain the complexity of the subject matter under investigation, namely relational and instrumental understanding in mathematics learning. According to [8] the stages of a literature review include:

Stage 1: Designing the Review. At this stage, the researcher formulates why the literature review needs to be conducted, its objectives, the appropriate analytical methods, and the suitable data search strategies for the ongoing study. After selecting the research theme and objectives, the literature review is used to obtain explanations regarding the implementation of relational and instrumental understanding in mathematics learning. The literature search strategy involves consulting books on Skemp's theory.

Stage 2: Conducting the Review. This stage addresses or follows up on the points outlined in the review design by carrying out the literature analysis process one by one, as detailed in Stage 1. The sample literature selected at this stage includes journals and books on Skemp's theory.

Stage 3: Analysis. This stage involves determining the issues to be reviewed, comparing the issues across journals, reviewing and ensuring the quality of the literature analysis process, and deciding how to write the analysis results so that they align with the research objectives.

Stage 4: Writing the Review. This stage involves presenting the results of the literature analysis in accordance with the research objectives. Activities at this stage include providing clear explanations to answer the research questions and adding information so that the literature review contributes to the body of knowledge or similar studies.

Based on the characteristics of the selected research method, the analysis results are presented in a narrative form, highlighting the definitions, advantages, limitations, and the effects of mathematics learning using instrumental and relational understanding.

RESULTS & DISCUSSION

This study employs a literature review method, drawing data from Skemp's theory books and related journals. The aim of the study is to explain Skemp's theory in relation to mathematical learning ability using relational and instrumental understanding. The results show that in 1976, Skemp observed that what we call mathematics is actually taught and learned in two distinct ways.

1. Relational Understanding

Relational understanding refers to the way learners operate within a mathematical structure based on the connections among mathematical concepts, enabling them to find solutions and understand why particular steps are used (*knowing what to do and why*). In this context, students can perform calculations in a broader context, follow wider procedures and structures that can be applied in problem solving, and relate concepts in a more meaningful way.

Characteristics of Relational Understanding include:

- Connecting new ideas with previous knowledge and integrating them into a conceptual system.
- Identifying patterns and underlying features to express how and why concepts work.
- Engaging in dialogic discourse, examining logic, and providing critical explanations.
- Expressing explanations flexibly.
- Reflecting on the process of developing understanding.

The application of Skemp's theory in relational and instrumental understanding has both short-term and long-term effects on students and teachers, which can be described as follows:

a. Short-Term Effects: Students and teachers develop a deeper understanding of what to do and why. Students attempt to understand relationally what is taught instrumentally and strive to gain deeper comprehension of concepts that are taught superficially.

b. Long-Term Effects: Students and teachers develop relational understanding of mathematics. Students continue learning mathematics and may pursue careers in mathematics-related fields. Conversely, some students may feel insufficiently capable of understanding mathematics, disengage, and abandon mathematics as quickly as possible.

Relational understanding requires students to be able to explain why they perform a particular action. Therefore, students must correctly connect one concept with another and understand the processes involved. Moreover, students with relational understanding tend to have stronger comprehension; even if they do not remember a formula, they can use alternative, linear approaches to solve problems.

Advantages of Relational Understanding include:

a. Easier to adapt to new situations

b. Less likely to be forgotten

c. Effective as a personal learning goal

d. Has the potential for developing mental schemata

According to Skemp, indicators of relational understanding include:

a. The ability to analyze objects based on whether conditions are met to realize a plan

b. The ability to apply plans algorithmically to find solutions

c. The ability to relate different plans or concepts

d. The ability to extend necessary and sufficient conditions of a plan

2. Instrumental Understanding

Instrumental understanding is defined as the ability to operate mathematical structures to find solutions without understanding the underlying reasons. In instrumental understanding, learners merely memorize procedures in sequence and focus on the skill of executing linear steps in solving mathematical problems.

According to [3] teaching that promotes instrumental understanding involves learning a set of fixed plans that allow students to find their way from a specific starting point to a particular endpoint. These plans instruct students on what to do at each decision point. According to Mellin-Olsen and Skemp, instrumental understanding involves a rule-based approach focusing on *how* to perform tasks. Furthermore, [3] explains that instrumental understanding reflects the ability to know *how* to follow mathematical rules without knowing *why* these rules and procedures can be applied (*rules without reason*). Instrumental understanding typically includes:

Typical Forms of Instrumental Understanding:

a. Memorizing formulas

b. Applying formulas to simple calculations

c. Executing steps sequentially

Characteristics of Instrumental Understanding:

a. Treating new ideas as unrelated facts, disconnected from previous knowledge

b. Memorizing facts and following procedures without focusing on how or why

c. Practicing exercises by accepting explanations as given without questioning

d. Expressing explanations rigidly

e. Not reflecting on the process of developing understanding

The application of instrumental understanding has effects on both students and teachers, not only in the short term but also in the long term.

Short-Term Effects include:

a. Students and teachers perform mathematical tasks.

b. Students do not know whether their answers are correct until the teacher confirms them.

c. Everything seems acceptable if there is alignment between the goals of students and teachers.

d. Teachers may feel frustrated because students do not seek to understand the reasoning.

e. Students may feel frustrated and passively follow instructions without questioning how to do it.

Long-Term Effects include:

a. Students learn more rules, develop a shallow understanding of mathematical concepts, and encounter difficulties when a problem does not fit the planned procedure.

b. Students continue studying mathematics to pass exams but tend to abandon mathematics as soon as possible.

c. Students and teachers may develop their relational understanding of mathematics.

3. Mathematics Learning

Mathematics learning refers to the process of providing students with learning experiences through a series of structured activities, enabling them to acquire skills in the mathematics topics being studied. According to Kusaeri, the objectives of mathematics learning include:

a. Students are able to master concepts accurately.

b. Students think logically to solve problems.

c. Students apply problems using tables, symbols, and diagrams.

d. Students understand the practical usefulness of learning mathematics in daily life.

Mathematics learning is always accompanied by mathematical problem solving that must be addressed. Mathematical problem solving is used in mathematics learning as a stage of thinking derived from prior knowledge, employing relevant strategies to address the problem at hand. Mathematical solutions include solving routine and non-routine problems, word problems, and applying mathematics to real-life situations. In solving these problems, several stages are involved, namely:

a. Understanding the problem

b. Developing a solution

c. Reviewing the completed solution

The study conducted by [6] revealed that many students face difficulties in solving word problems. Students with high problem-solving potential completed the problems correctly. Students with moderate abilities solved the problems fairly well, but errors were found in interpreting information, leading to incorrect solutions. Students with low potential struggled to find the answers.

The study by [1] showed that students with a visual learning style were able to meet the criteria for classifying objects based on whether they satisfy the conditions for forming concepts, connecting concepts, and applying plans algorithmically to solutions, but they were not yet able to extend the necessary and sufficient conditions of a plan. Students with an auditory learning style were able to develop the necessary and sufficient conditions of a concept and apply concepts algorithmically to problem solving, but they were not yet able to classify objects according to the conditions for forming concepts or connect different plans. Meanwhile, students with a kinesthetic learning style were able to classify objects based on whether they met the conditions for forming concepts and apply concepts algorithmically to find solutions, but they were not yet able to connect different plans or extend the necessary and sufficient conditions of a concept.

4. Relational and Instrumental Undestanding in Mathematical Problem Solving

Mathematical problem solving supports an individual's thinking process by applying concepts relevant to the problem at hand. In relational understanding, students apply the stages of mathematical solutions to solve problems they encounter. Students with relational understanding are able to classify objects based on whether they meet the conditions for forming concepts, apply concepts algorithmically to solve problems, connect one concept with another, and develop the necessary and sufficient conditions of a concept. As a result, students with relational understanding can solve problems more easily because they know what to do and why. They attempt to understand instrumentally taught concepts relationally and strive to develop a deeper understanding of concepts that were taught superficially.

Instrumental problem solving, on the other hand, is performed to solve a problem without understanding why the procedure can be used. Students merely memorize, apply, and execute procedures without knowing the reasoning behind them. Problem solving using instrumental understanding is easier to follow, and students can arrive at correct answers quickly. According to [3], instrumental problem solving is carried out without following a structured problem-solving procedure; instead, students rely on trial-and-error to find solutions and obtain correct results.

CONCLUSION

1. Problem Solving Using Realtional Understanding

Problem solving using relational understanding encourages students to think more broadly and systematically. Students are able to comprehend problems by analyzing issues according to whether the conditions for realizing a plan are met. They can plan solutions by applying plans algorithmically to problem solving. Moreover, students are able to plan by connecting different plans and extending the necessary and sufficient conditions of a concept. At the final stage of problem solving, which involves reviewing the solution, students can re-examine their results while considering the steps of the procedure.

2. Problem Solving Using Instrumental Understanding

In instrumental problem solving, students solve problems without following a structured problem-solving procedure. As a result, students can complete problems more quickly and obtain correct answers efficiently. This approach is easier to follow, and students primarily rely on memorizing formulas for simple calculations.

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