International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 13 Issue 12 December 2025, Page no. –6013-6017

Index Copernicus ICV: 57.55, Impact Factor: 8.615

DOI: 10.47191/ijmcr/v13i12.10

Mathematical Modeling of Key Factors Affecting Dust Particles and Soil Erosion in the Atmosphere

Tursun Shafiev[1], Bobojonova M. A.[2]

[1,2]Bukhara State University, Uzbekistan

ARTICLE INFO

Published Online:
23 December 2025

Corresponding Author:
Tursun Shafiev

ABSTRACT

This study presents a comprehensive investigation of the key factors influencing the transport and diffusion of dust particles in the atmosphere, with particular attention to wind-driven soil erosion. The mathematical modeling approach incorporates the physical and mechanical properties of particles, including size, density, shape, and aerodynamic characteristics, allowing for an accurate representation of particle dispersion processes. The model accounts for environmental variables such as wind speed and direction, atmospheric turbulence, soil moisture, and surface conditions. Computational experiments were conducted to simulate the dynamics of dust particle movement along the x, y, and z axes. The results are illustrated through graphs demonstrating the temporal evolution of particle velocities and trajectories under varying conditions. The findings highlight the significant impact of particle properties and environmental factors on atmospheric transport and soil erosion, providing insights for effective land management and environmental protection strategies. This study investigates the main factors affecting the transport and diffusion of dust particles in the atmosphere. The analysis is based on equations that account for the physical and mechanical properties of particles, reflecting the dispersion processes of these dust particles in the atmosphere. Computational experiments were conducted to obtain the results, and graphs are presented illustrating the dynamics of particle movement along the x, y, and z directions.

KEYWORDS: dust particle transport, soil erosion, numerical simulation, atmospheric dispersion, environmental factors

I. INTRODUCTION

Significant achievements in various sectors of the economy are clearly demonstrated in the modern world. These successes are based on an increase in production, the development of agriculture, an increase in the share of exports and the expansion of the service sector. However, scientific and technological progress, a sharp increase in the population and the intensification of land use, as well as the widespread introduction of mechanization in agriculture have a serious impact on the natural stability of soils.

This is especially noticeable as a result of improper use of land, non-compliance with agrotechnical requirements, reduction of forests and disruption of land reclamation processes, which leads to an annual increase in soil erosion. Soil erosion is the process of destruction and washing away of a fertile layer under the influence of water or wind, as a result of which a strategically important, irreplaceable natural resource is lost. This, in turn, causes a decrease in crop yields, an imbalance in the ecological balance and degradation of landscapes.

Currently, effective implementation of projects in agriculture, land reclamation, animal husbandry, construction and infrastructure development is impossible without constant monitoring of soil erosion processes and the introduction of measures to prevent it.

For Uzbekistan, the problem of soil erosion is particularly relevant and is closely related to changes in the natural environment in the Aral Sea region. The decrease in the water surface of the Aral Sea has led to the formation of open, dry areas from which mineral particles and salt marsh dust rise in the wind. These aerosols spread over considerable distances, settling on agricultural land and natural landscapes. This phenomenon enhances the processes of soil degradation and salinization, reduces their natural stability and accelerates erosive changes. As a result, the necessary measures to monitor soil conditions and rational land use are becoming particularly important to ensure the ecological stability of the region.

An analysis of the scientific literature on mathematical modeling of wind erosion and soil dusting has shown that significant theoretical and practical results have been achieved in this area. Research makes it possible to better understand the mechanisms of soil particle transport by wind, assess the scale of erosion processes and predict the impact on agricultural land and natural landscapes. For Uzbekistan, especially in the Aral Sea region, such approaches are of practical importance, as they help to develop recommendations on rational land use and reduce the negative effects of soil erosion.

In particular, the article [1] presents in a simplified form the processes of wind erosion, which are the movement of soil mass under the influence of air currents. The authors have shown that in order to identify the causes of the movement of these masses, it is necessary to be able to measure this movement – to measure the flow of solid phase (i.e. soil) during erosion.

An analysis of the scientific literature on mathematical modeling of wind erosion and soil dusting has shown that significant theoretical and practical results have been achieved in this area. In the work of the authors [2], modern models of wind erosion for arid territories were proposed, which make it possible to predict the zones of greatest risk and develop measures to protect agricultural lands. Also, the study [3] examined in detail the influence of soil surface parameters, humidity and vegetation cover on the intensity of wind erosion, which is especially important for semi-arid regions of Central Asia. The authors of [4] demonstrated numerical methods for modeling dust emissions from treated agricultural lands, emphasizing the impact of irrigation and agrotechnical techniques on reducing erosion risk. Special attention can be paid to the work [5], which summarized the methods of numerical and empirical erosion modeling, emphasizing the need to integrate monitoring processes into rational land use planning.

In the research [6-8], the author developed a three-dimensional linear mathematical model taking into account the physico-mechanical properties of particles, which reflects the process of movement and diffusion of harmful substances in the atmosphere, as well as a three-dimensional nonlinear mathematical model for migration and diffusion of harmful substances in the atmosphere, taking into account the changing speed of movement of particles of pollutants, weather conditions and climate of the area under consideration and its topography.

The problem of the Aral Sea is very problematic for the population of Uzbekistan and for the territories bordering it. The authors' article [9, 10] conducted research on this aspect to analyze the environmental situation in the Aral Sea region of Uzbekistan. The main share of emissions of harmful substances in the Aral Sea region is accounted for by dust, salts and toxic chemicals carried away from the dried-up bottom of the Aral Sea. Thus, when mathematically modeling the atmospheric scattering process, it is necessary to take into account the physical and mechanical properties of particles and the main forces acting on them.

Despite the fact that the processes of blowing and transferring soil particles occur continuously, the intensity of erosion and the distribution of dust on agricultural and natural lands are influenced by a variety of natural and anthropogenic factors. These include wind speed and direction, soil moisture, vegetation cover, terrain, and land cultivation methods. These factors determine how quickly and widely soil particles spread, affecting erosion processes and the quality of the soil layer.

There are also natural and climatic factors that affect the process of dispersion of harmful substances in the atmospheric air. Natural factors include: features of the air mass circulation regime; stability of ambient temperature; changes in atmospheric pressure and humidity; temperature, temperature variability, frequency and duration of its recurrence; wind speed, stagnation of airflow and low wind speed (0-1 m/s); duration of areas in the area under consideration; geological, hydrological structure and the topography of the territory; the condition of the soil and plants (soil type, water permeability, porosity, soil erosion, soil composition); can be called such background indicators of atmospheric pollution.

II. STATEMENT OF THE PROBLEM

Due to the fact that soil temperature, humidity, wind speed and direction are constantly changing in natural conditions, the processes of wind erosion and dust particle transport occur in constantly changing conditions. Adverse weather conditions, terrain features, and the presence of hills can increase the impact of these factors, slowing down the natural subsidence and accumulation of soil particles. At the same time, rain, high humidity and air temperature contribute to accelerating the deposition of dust, reducing the rate of erosion and improving the preservation of a fertile soil layer.

During the day, the heating of the soil surface by the sun causes a rise in warm air and the formation of additional turbulence, which contributes to increased wind erosion and the transport of dust particles. By evening, the air temperature and turbulence on the surface decrease, slowing down the processes of dust blowing and spreading.

In addition, the physical and mechanical properties of soil particles have a significant effect on the intensity of their transport. The size, shape, and density of the particles determine their aerodynamic characteristics, air flow resistance, and settling rate. In mathematical modeling of wind erosion processes, these parameters must be taken into account in order to accurately assess the extent of soil transfer and predict areas of greatest risk of erosion.

The physical and mechanical properties of particles affect their horizontal and vertical movement in atmospheric air. The horizontal velocity of the particles, along with the wind speed, is influenced by their drag coefficient and shape.

The vertical movement of soil particles lifted by the wind from the surface and distributed in the atmosphere also depends on a number of important parameters. Despite the fact that particles of different shapes and densities can move at similar speeds, they do not settle to the ground at the same time due to differences in aerodynamic drag. This effect plays a key role in modeling wind erosion, determining the rate and zone of deposition of dust and sand particles on agricultural and natural lands.

In general, if we assume that the motion of a particle is affected only by its resistance, then the force acting on the particle can be written in vector form as follows:

$$\bar{F}_r = -F_r \frac{\vec{V}}{V},$$

where - F_r is the absolute value of the acting force.

Using F_r, it is possible to derive the differential equation of motion of a body thrown in a horizontal direction:

$$m\frac{d^2x}{dt^2} = -\frac{F_r}{V}\frac{dx}{dt};$$

$$m\frac{d^2y}{dt^2} = -mG - \frac{F_r}{V}\frac{dy}{dt}.$$

In the second part of the above equation, the Archimedes force acting on the body is also taken into account, which is equal to

$$G = \left(1 - \frac{\rho_h}{\rho_z}\right)$$

where g – acceleration of free fall, ρ_h – air density, ρ_z – the density of the particle in question.

The vertical velocity of a particle does not increase indefinitely due to air resistance. Consequently, the particle velocity eventually reaches a certain value (if only air resistance is taken into account). If the particle's sinking speed reaches, which can be achieved $v_{\max}$, then the drag force remains equal to its weight (taking into account the Archimedes force):

$$\bar{F}_r = mG.$$

When analyzing the factors affecting the horizontal and vertical velocity of particles, it is impossible to proceed to the next parameter without determining their resistance[11-13]. The magnitude of this force depends on the velocity field of the air flow around the particles. At low flow rates, the flow is laminar. In this case, the motion of the particle can be considered as a relative motion that does not interfere with the flow of air.

When soil particles move in the air stream, an internal friction force occurs between the air layers. This force is a manifestation of the viscosity of the air and is characterized by a dynamic viscosity coefficient. The value $\eta = 1{,}8\cdot 10^{-5}$ $Па\cdot с$ ($Па{\cdot}с = кг{\cdot}м^{-1}{\cdot}с^{-1}$) is usually used for air, but this parameter varies depending on the ambient temperature. Taking into account the viscosity of air is important in the mathematical modeling of the transport of soil particles by wind, as it affects the speed of movement of particles, their deposition and distribution over the territory.

It is convenient to use the expression of the resistance force together with the coefficient of resistance in the considered speed range. At low speeds, the nature of the drag changes, and the transition of a particle from the laminar to the turbulent regime depends on its velocity. There is no direct relationship between the drag coefficient and the particle velocity. The relationship between them is most often expressed in terms of the Reynolds number, which takes into account the size, velocity, and viscosity of the medium.

III. SOLUTION METHOD

To account for the factors influencing the processes of wind erosion and the transport of dust particles, consider the following equations that describe the coordinate velocities of soil particles as they move under the influence of wind, turbulence, and other natural factors. These equations make it possible to quantify the transfer rate, movement directions, and deposition zones of particles on agricultural and natural lands:

$$m\frac{du_p}{dt} = 3\pi\eta d_p(u - u_p); \tag{1}$$

$$m\frac{dv_p}{dt} = 3\pi\eta d_p(v - v_p); \tag{2}$$

$$m\frac{dw_p}{dt} = 3\pi\eta d_p(w - w_p) - mg; \tag{3}$$

and the corresponding initial conditions:

$$u_p\big|_{t=0} = u_p^0;\ v_p\big|_{t=0} = v_p^0;\ w_p\big|_{t=0} = w_p^0;$$

where u, v, w — wind speed by direction; g - acceleration of free fall; t – time; d_p - particle diameter; m - particle mass; η – dynamic viscosity coefficient.

To ensure stability in the process of solving this problem by numerical methods, we approximate equations (1)-(3) by an implicit-finite difference scheme as follows:

$$\frac{u_p^{\,n+\frac{1}{3}} - u_p^{\,n}}{\Delta t/3} = \frac{3\pi\eta d(u - u_p^{\,n+\frac{1}{3}})}{m},$$

$$\frac{v_p^{\,n+\frac{1}{3}} - v_p^{\,n}}{\Delta t/3} = \frac{3\pi\eta d(v - v_p^{\,n+\frac{1}{3}})}{m},$$

$$\frac{w_p^{\,n+\frac{1}{3}} - w_p^{\,n}}{\Delta t/3} = \frac{3\pi\eta d(w - w_p^{\,n+\frac{1}{3}}) - mg}{m}.$$

Opening the brackets, we will reduce the analogous terms and find the velocities of the particle in the considered directions:

$$u_p^{n+\frac{1}{3}} = \frac{mu_p^n + \Delta t\pi\eta du}{m + \Delta t\pi\eta d},$$

$$v_p^{n+\frac{1}{3}} = \frac{mv_p^n + \Delta t\pi\eta dv}{m + \Delta t\pi\eta d},$$

$$w_p^{n+\frac{1}{3}} = \frac{mw_p^n + \Delta t\pi\eta dw - mg\Delta t/3}{m + \Delta t\pi\eta d}.$$

In the same way, applying the procedure described above to the OY coordinate, we arrive at the following expression:

$$u_p^{n+\frac{2}{3}} = \frac{mu_p^{n+\frac{1}{3}} + \Delta t\pi\eta du}{m + \Delta t\pi\eta d},$$

$$v_p^{n+\frac{2}{3}} = \frac{mv_p^{n+\frac{1}{3}} + \Delta t\pi\eta dv}{m + \Delta t\pi\eta d},$$

$$w_p^{n+\frac{2}{3}} = \frac{mw_p^{n+\frac{1}{3}} + \Delta t\pi\eta dw - mg\Delta t/3}{m + \Delta t\pi\eta d}.$$

Exactly the same, applying the procedure described above to the coordinate, we arrive at the following expression:

$$u_p^{n+1} = \frac{mu_p^{n+\frac{2}{3}} + \Delta t\pi\eta du}{m + \Delta t\pi\eta d},$$

$$v_ч^{n+1} = \frac{mv_p^{n+\frac{2}{3}} + \Delta t\pi\eta dv}{m + \Delta t\pi\eta d},$$

$$w_p^{n+1} = \frac{mw_p^{n+\frac{2}{3}} + \Delta t\pi\eta dw - mg\Delta t/3}{m + \Delta t\pi\eta d}.$$

IV. EXPERIMENTAL RESULTS

Time dynamics u_p, v_p, w_p, which are the components of the corresponding velocities of the aforementioned particles in the x,y, and z directions can be seen in Figures 1-3. A number of computer experiments were conducted to study the effect of the particle velocity of harmful substances on the migration and diffusion of harmful substances in the atmosphere.

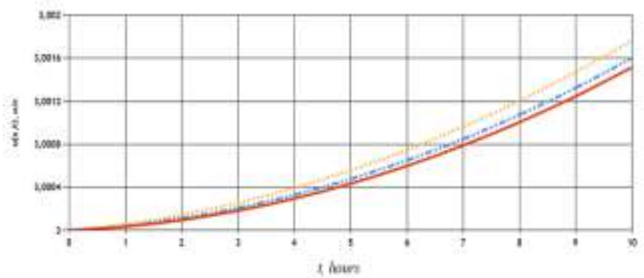

Fig.1. The change in the velocity of various dust particles lifted into the atmospheric air over time along the Ox axis with an initial velocity of $u_0 = 3$ m/s.

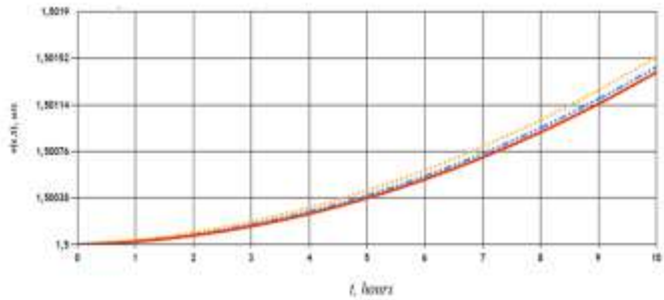

Fig.2. The change in the velocity of various dust particles lifted into the atmospheric air over time along the Oy axis with an initial velocity of $v_0 = 1,5$ m/s.

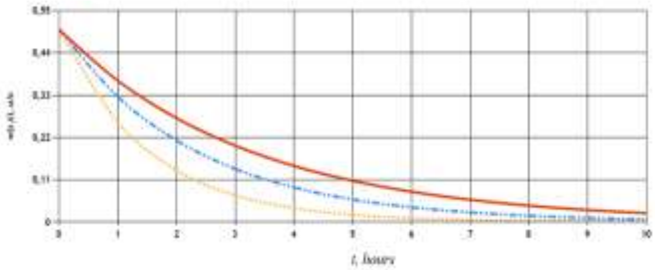

Fig.3. The change in the velocity of various dust particles lifted into the atmospheric air over time along the Oz axis with an initial velocity of $w_0 = 0,5$ m/s.

V. DISCUSSION

Figures 1-3 show the dynamics of changes in particle velocities u_p, v_p, w_p along the directions x, y, z. In the course of computational experiments, dust particles with various physical and mechanical properties were selected. Comparing the obtained results, we can say that their physical and mechanical properties are very important in determining the particle velocity.

Figure 1 shows the results of computational experiments to determine the velocity of dust particles of various fractions (fine dust with a diameter of 1-5 microns and a density of 2.5 g/cm^3, medium dust with a diameter of 5-10 microns and a density of 2.6 g/cm^3, coarse dust with a diameter of 10-20 microns and a density of 2.7 g/cm^3) along the Ox coordinate in time at an initial velocity of $u_0 = 3$ m/s. The physical and mechanical properties of these particles take into account their aerodynamic drag, shape coefficient (sphericity 0.8–0.95) and the force of friction in atmospheric air, which determines the trajectory and deceleration under the influence of gravity and turbulence. Based on the results of the conducted computational experiments, we can observe that the velocity of dust particles increases significantly over time due to the initial impulse of ascent and the subsequent influence of ascending air currents.

Figure 2 shows the results of computational experiments to determine the velocity of dust particles of various fractions (fine dust with a diameter of 1-5 microns and a density of 2.5 g/cm3, medium dust with a diameter of 5-10 microns and a density of 2.6 g/cm3, coarse dust with a diameter of 10-20 microns and a density of 2.7 g/cm3) according to the Oy coordinate in time at ($v_0 = 1{,}5$) m/s . The physical and mechanical properties of the particles, including the coefficient of aerodynamic drag (0.4–0.6), sphericity (0.8–0.95) and moment of inertia, determine their behavior in the transverse direction under the influence of turbulence and lateral air flows. Similarly, based on the results of computational experiments conducted here, we can observe that the velocity of dust particles increases significantly over time.

Figure 3 shows the results of computational experiments to determine the velocity of dust particles of various fractions along the Oz (vertical) coordinate in time at $w_0 = 0{,}5$ m/s. Taking into account gravitational deposition, thermal mobility and air viscosity, the physical properties of the particles (specific surface area, coefficient of friction 0.02–0.05) lead to the predominance of resistance forces over lifting. Similarly, based on the results of computational experiments conducted here, we can observe the dynamics of the velocity of dust particles, depending on their fractional composition.

VI. CONCLUSION

Based on the results obtained as a result of computational experiments, it is confirmed that the main role in the movement of dust particles in the directions Ox, Oy and Oz is played by their physical and mechanical properties (mass, size, density, shape). Based on the results of computational experiments, it can be concluded that the lower the mass density and particle size, the higher their horizontal velocity (according to Ox and Oy) and the lower their vertical velocity (according to Oz) due to the increased influence of gravity and diffusion.

REFERENCES

1. Bahodirov Z. Prospects for modeling wind erosion // Prospects for the development of science in Uzbekistan. 2022. P. 60–62. (in Russian)
2. Lobb D.A., Cornish P.S. Wind erosion and dust emission modeling in arid regions // Aeolian Res. 2019. Vol. 38. P. 100–112.
3. Zobeck T.M., Van Pelt R.S. Wind-induced dust generation and transport mechanics on a bare agricultural field // J. Hazard. Mater. 2006. Vol. 132, № 1. P. 26–38.
4. Li S., Shao M., Wang T. Simulation of dust emission from arable lands in Central Asia // Environ. Model. \& Softw. 2015. Vol. 72. P. 1–12.
5. Shi P., Shao Y. Soil erosion and its modeling in arid and semi-arid regions of China // Prog. Phys. Geogr. 2011. Vol. 35, № 1. P. 80–103.
6. Shafiev T., Nazarov S. Studies of the influence of vegetation cover on the process of transfer and diffusion of harmful substances in the atmosphere // E3S Web of Conferences. EDP Sciences, 2023. Vol. 431.
7. Ravshanov N., Shafiev T.R., Daliev S. Study of the main factors affecting the spread of aerosol particles in the atmosphere // Journal of Physics: Conference Series. IOP Publishing Ltd, 2021. Vol. 1791, № 1.
8. Ravshanov N., Shafiev T.R. Nonlinear mathematical model for monitoring and predicting the process of transfer and diffusion of fine-dispersed aerosol particles in the atmosphere // J. Phys. Conf. Ser. 2019. Vol. 1260, № 10.
9. Ravshanov N., Daliev S.K., Tagaev O. Numerical simulation of two aquarius horizons // Int. J. Adv. Trends Comput. Sci. Eng. 2020. Vol. 9, № 4. P. 6549–6554.
10. Ravshanov N., Ravshanov Z., Bolnokin V.E. Modeling the salt-dust aerosols distribution in the atmosphere, taking into account the soil erosion // IOP Conference Series: Materials Science and Engineering. Institute of Physics Publishing, 2020. Vol. 862, № 6. P. 1–7.
11. Ravshanov N., Abdullaev Z., Shafiyev T. Mathematical model and numerical algortm to study the process of aerosol particles distribution in the atmosphere // International Conference on Information Science and Communications Technologies: Applications, Trends and Opportunities, ICISCT 2019. 2019. P. 1–7.
12. Tikhonov V.I., Samarskiy A.A. Levelling mathematical physics. Moscow: FIZMATLIT, 1979. 742 p. (in Russian)
13. Shafiev T. Mathematical model for monitoring and forecasting the process of aerosol particle propagation in the atmosphere // Problems of computational and applied mathematics. 2020. № 1(25). P. 69–84. (in Russian)