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A Bayesian Approach to Modeling and Evaluating Aircraft Service Queues at Adi Soemarmo International Airport

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ABSTRACT

Over time, the transportation sector has also experienced significant progress. One of the easily observable phenomena in daily life is queuing at public transportation facilities, particularly at airports. Airport service facilities cannot avoid queuing events due to the large number of arriving aircraft; however, queues can be minimized with an efficient system. The purpose of this research is to understand the aircraft queuing system at the Apron of Adi Soemarmo International Airport in Surakarta. The Bayesian method is employed to combine previous research and this study to obtain new information. The sample distributions (Weibull and inverse Gaussian) and prior distributions (Inverse Gaussian and Weibull) obtained from previous research are combined with the likelihood function of the sample to derive the posterior distribution. After calculating the posterior distribution, it is found that the queuing model for aircraft at Adi Soemarmo International Airport is (GAMMA/GAMMA/3):(GD/∞/∞), where it meets the steady-state conditions, and the queuing system at Adi Soemarmo International Airport, based on performance measures, is considered satisfactory.

KEYWORDS: Adi Soemarmo International Airport, Queue, Bayesian, Posterior, System Performance.

I. INTRODUCTION

Over time, the transport sector has developed quite rapidly. One phenomenon that is easily observed is queuing in the public transport sector. The state of waiting is part of the situation that can be found in the process of random events in service facilities in general. In everyday life it can also be easy to find queues; in this situation queues occur when there are clients/customers who have to wait in a line to get services from service facilities.

Queues at public facilities can be found at Adi Soemarmo-Surakarta Airport in Boyolali, Central Java, which is a class A airport serving domestic and foreign flights. The queue of aircraft that occurs at the airport is due to the large number of aircraft landing with various types of aircraft. The type of aircraft observed in this study is regular passenger aircraft. The aircraft queue that occurs for all incoming and outgoing airline aircraft as customers are served, while the airport acts as a server. A queue process is calculated from the time the aircraft arrives until it is served and leaves the airport. The number of flights between today and the next day is different, so the resulting queue is also different (Bronson, 1991).

The selection of the distribution of customer arrival and service data greatly affects the queuing model used. Based on empirical data, information is obtained that the data statistically follows several theoretical probability distributions. Therefore, the Bayesian method approach can be used to combine several appropriate distributions by considering the results of previous research. Since the current research data distribution is a change in the previous research data with additional information, the Bayesian method can be used to combine the two pieces of information to obtain the posterior distribution. To get the posterior distribution, the prior distribution from the previous research data and the likelihood function based on the latest research data are needed (Armero dan Bayarri, 1995).

The prior distribution in this study was obtained from research conducted by Widiawati et al. (2010). This data is used as initial information on the grounds that the object is the same, namely observing the aircraft service queue at Adi Soemarmo-Surakarta International Airport, and researchers are interested in finding models and performance measures of the aircraft queuing system at the airport after the Covid-19 pandemic. The information needed from previous research is arrival data and aircraft service time as prior information. This prior distribution is then combined with empirical data used as the basis for the formation of the posterior distribution. Therefore, this study aims to measure the performance of the aircraft queuing system at Adi Soemarmo International Airport Surakarta after Covid-19 with a Bayesian approach.

The following sections of this article are organised as follows. Section 2 describes the queueing theory of several data distributions and system performance measures, complemented by the formation of posterior distributions using the Bayesian approach. Section 3 presents the research methodology for data analysis. Section 4 presents the processing results and analysis of system performance. Section 5 presents conclusions and recommendations for future research.

II. METHOD DETAILS

Profile of Adi Soemarmo-Surakarta International Airport

PT Angkasa Pura has the authority and responsibility to manage and regulate activities at the airport, which manages 15 airports in Indonesia, one of which is Adi Soemamarmo Airport. Adi Soemamarmo Airport was inaugurated as an international class airport that serves various flights, local (domestic) flights and international flights. Adi Soemarmo Airport is located as far as 14 kilometers northwest of Surakarta City (Solo), precisely in Boyolali Regency, Central Java, Indonesia

Description of Queue Theory

Queue refers to a set of data or entities waiting to obtain a service or process. *Queue process* occurs when the customer/customer needs service comes. In the queue method, it is a process that has a relationship with customers who arrive at *Service Point*, queue if not served, be served, and finally leave *Service Point*. In this case, the customer who arrives there cannot directly use the service and has to wait for the service to be used because all the *Server* is busy.

Important Components of the Queue System

In a queue system, an important component that supports a queue is the distribution of time between customer arrivals (*Arrival Model*), distribution of service time required by customers, service facilities (*server),* service order, queue input source, call source.

Steady *State Levels*

'Steady state' refers to the state in which the queue system has reached a long-term equilibrium, which is the result of comparing the average time between entity (customer) arrivals (λ) with the average number of entities (customers) that have been served per unit time (μ), and the symbol 's' is the number of servers. The steady-state condition can be written with the following equation:

$$\rho = \frac{\lambda}{s\mu} \tag{1}$$

To meet *the steady state* conditions $\rho < 1$.

Exponential Distribution and Poisson Process

The Poisson process is a stochastic process to describe events that occur randomly and independently in a certain time interval or space. There are 3 assumptions for the poisson process itself, namely non-overlapping (*independent*), homogeneity of time, regularity.

Theorem I

$$P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}; n \geq 0 \tag{2}$$

The random variable that follows a poisson distribution with the parameter λt is the number of arrivals that occur at time interval t (Gross and Harris, 1998).

An exponential distribution is a continuous random distribution where the variables are independent and have no history. This distribution is often used in the queue system/

Theorem 2

When the process of arrival with parameter λ follows the poisson process, then it can be stated that the random variable for the time between consecutive arrivals follows an exponential distribution with $\frac{1}{\lambda}$ as the parameter (Gross and Harris, 1998).

$$E(T) = \frac{1}{\lambda} \tag{3}$$

Distribution Compatibility Test

Statistical methods to test the suitability or similarity of data distributions with theoretical distributions or predefined reference distributions. The *Kolmogorov Smirnov* test is the steps required to test the alignment of continuous data.

Here are the stages:

a) Establishing a Hypothesis
   H_0 : The distribution of the sample is equal to the specified distribution
   H_1 : The distribution of the sample is not the same as the specified distribution
b) Significance Levels
   In Kolmogorov Smirnov, the level of significance used is $\alpha = 5\%$
c) Test statistics
   $D = \underset{1 \leq i \leq r}{Max}\{Max[|F_0(x_i) - S(x_i)|. |F_0(x_i) - S(x_{i-1})|]\}$
   where:
   r : the number of different x-values
   $S(x_i)$: Cumulative Chance Function of the Third Sample of the Population
   $S(x_{i-1})$: Cumulative Probability Function of the 1st Sample of the Population
   $F_0(x_i)$: the cumulative distribution function of the hypothetical distribution
d) Test Criteria
   On the test criteria, the distribution is rejected if $D \geq D_{N;\alpha}$ or if $P\_value < \alpha$. Table D (is the critical value α) of *the Kolmogorov Smirnov table* for the double-sided test (Daniel, 1989).

Weibull Distribution

Probability Density Function (pdf) from weibull with the parameters k and λ as follows (Walpole and Myers, 1995).

$$f(x;\lambda,k) = \begin{cases} k\lambda x^{k-1}e^{-\lambda x^k} & ,x > 0 \\ 0 & ,others \end{cases} \quad (4)$$

Gaussian Inverse Distribution

Probability Density Function (pdf) of Gaussian inverse distribution which has the parameters of μ dan λ that is (De Jong and Heller, 2008):

$$f(x) = \sqrt{\frac{\lambda}{2\pi x^3}} exp\left[-\frac{\lambda(x-\mu)^2}{2x\mu^2}\right] \quad (5)$$

Gamma Distribution

The gamma distribution has α and β as its parameter, with the probability density function can be expressed as follow:

$$f(x,\alpha,\beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} exp\left(-\frac{x}{\beta}\right) \quad (6)$$

where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}dx$.

Notation in these circumstances, namely $X \sim Gamma(\alpha,\beta)$.

General Queue System Model (G/G/c): (GD/∞/∞)

Calculation of the performance measure of the queue model system (G/G/c):(GD/∞/∞) is done with the following equation (Kakiay, 2004):

$$L_q = \left(\frac{\rho r^c}{c!\,(1-\rho)^2}\right) P_o \frac{\mu^2 v(t) + \lambda^2 v(t')}{2} \quad (7)$$

where:

$v(t) = \left(\frac{1}{\mu^2}\right)^2$, $v(t') = \left(\frac{1}{\lambda^2}\right)^2$, $L_s = L_q + r$,

$W_q = \frac{L_q}{\lambda}$, and $W_s = W_q + \frac{1}{\mu}$.

Bayesian Method

The Bayesian method is a technique required to combine prior and current information (samples). In the Bayesian method, the initial information of the parameter is called the prior distribution, while the current information is obtained from the sample distribution, and then the likelihood function is sought. The result of combining the two distributions will get posterior information, which is the likelihood function of the sample distribution and the initial information (prior distribution). The result of the merger is expressed as the posterior distribution (Yendra and Noviandi, 2015).

1. Prior Distribution

The probability distribution that represents the initial knowledge of a person about the parameter values before the sample data is collected is called the prior distribution. The prior distribution is categorised into 2 categories based on the shape of its likelihood function, namely conjugate and non-conjugate. As for determining the prior distribution pattern, it is divided into 2 unequal forms (non-informative prior distribution and informative prior distribution).

2. Jefrey's Method

The following is the formula for the prior non-informative calculation using Jefrey's method (Berger, 1980):

$$f(\lambda) = \sqrt{[\,\mathrm{I}\,(\lambda)]} \quad (8)$$

where

$I\,(\lambda) = -E_0[\frac{\partial^2 \log f(x,\lambda)}{\partial^2 \lambda}]$

3. Likelihood Function

The likelihood function is information obtained about the probability density function from the sample distribution called the likelihood function (Bain and Engelhardt, 1992).

$$L(\lambda) = \prod_{i=1}^{n} f(y_i|\lambda) \quad (9)$$

4. Posterior Distribution

The posterior distribution is a function of the conditional density λ. If the observed value x is known, the posterior density function of the continuous random variable can be formulated as follow (Soejati da Soebanar, 1988).

$$f(\lambda|x) = \frac{f(\lambda)f(x|\lambda)}{\sum_{-\infty}^{\infty} f(\lambda)\, f(x|\lambda)d\lambda} \quad (10)$$

III. RESEARCH METHODS

Research Data and Variables

This study uses primary data**.** The data collection was conducted during two weeks, from 27 December 2022 to 9 January 2023 at 08:00 - 18:00. The location of the study chosen was the Adi Soemamo-Surakarta International Airport Apron. The following are the variables in this study:

- Time data between regular aircraft arrivals at airport service facilities
- Regular aircraft service time data at airport service facilities.

Stages of Data Analysis

The data analysis process is carried out with various *statistical software*, namely *Microsoft Excel* 2019, *SPSS Statistics, Easyfit*, and R-Studio (GUI R). The following are the steps for analysis:

1) Initiating research to obtain information (primary data/secondary data) needed to solve problems.
2) Processing the time data between arrivals and service time obtained by the researcher from primary data.
3) Calculating *the steady state* $\rho = \frac{\lambda}{c\mu}$, ρ refers to the state in which the queue system has reached a long-term equilibrium. Data that meets the steady state condition ($\rho < 1$) can be continued in the next stage. If the stable conditions are not met, the number of services or service times must be increased.
4) The poisson distribution fit test with *Kolmogorv Smirnov* used *SPSS Statistics Software* for sample data on the number of arrival rates and the number of service times. If the hypothesis is that the distribution of the number of arrival rates and the number of service times pvalue > 0.05 (accepted) then the data follow the poisson distribution. If the pvalue < 0.05 (rejected), another distribution fit test is performed with *the EasyFit Software* to obtain the appropriate distribution.
5) Performed an exponential distribution fit test with *Kolmogorv Smirnov* using *SPSS Statistics Software* for sample data of the time between arrival and service time. If the hypothesis on the distribution of time between arrival and service time pvalue > 0.05 (accepted), then it follows the exponential distribution. If the pvalue < 0.05

(rejected), another distribution fit test is performed with *the EasyFit Software* to obtain the appropriate distribution.

6) After obtaining the distribution of the *EasyFit* ranking session, a likelihood function is performed to estimate the paramater.
7) Then do the Jeffrey method to get a non-informative prior.
8) After that, the posterior distribution is calculated using the probability sample distribution and the prior non-informative distribution.
9) The posterior distribution has been obtained, it can determine the queue model that corresponds to the posterior distribution.
10) Then an analysis of the performance measures of the queue system will be carried out, namely the average time of customers waiting in the queue (Wq), the average number of customers waiting in the queue (Lq), the average waiting time for each customer in the system (Ws) and the average number of customers in the system (Ls).
11) Interpreting the results of data analysis and calculations
12) Provide decisions and conclusions as a result of discussions regarding aircraft services at Adi Soemarmo Airport comprehensively.

IV. RESULTS AND DISCUSSION

Description of the Aircraft Queue System at Adi Soemarmo Airport

The queue system at Adi-Soemarmo Airport is a *Multichannel-Single Phase* queue system which usually consists of only one section and has 3 aprons that operate as scheduled flight services. Planes entering through the runway at Adi-Sumarmo International Airport will then be parked on a predetermined apron. When entering the *taxiway* (the connection between the runway and the parking lot), the aircraft will *be taxed in* to the aircraft parking guided by the *marshaller* in this event the aircraft is declared to be coming. The service starts when the plane is *blocked on* and the service is finished when the plane *is blocked off.*

Steady State Condition

Based on the research data obtained, the average of inter-arrival time (λ) is 59.67368 per aircraft, and the average of service time is (μ) is 41.04630 minutes per aircraft. Therefore, the steady-state value of 3 services (Airport Apron) can be obtained as follows:

$$\rho = \frac{\lambda}{c\mu} = \frac{59.6736}{3\ x\ 41.0463} = 0.4846\ ,\ \rho < 1$$

From these results it can be seen that the value of the level of usefulness of airport service facilities is less than one. Thus, it can be concluded that the aircraft queuing system has fulfilled a steady state, namely the average of inter-arrival time of aircraft does not exceed the average of service time. This means that the queue system has reached stability and can be used to determine the performance measures of the queue system.

Distribution Fit Test

In this study, the *Kolmogorov Smirnov* test is used to test the suitability of the appropriate distribution, so that it can obtain an exponential distribution of inter-arrival time and service time.

Table 1. The results of Kolmogorov-Smirnov Test

Variable	D_{stat}	D_{table}	Sig.	Decision
Inter-arrival time	0.159	0.14027	0.017	Reject H_0
Service time	0.460	0.13087	0.000	Reject H_0

The results of *the Kolmogorov-Smirnov* test of the sample data inter-arrival time and service time are not exponentially distributed, so further tests must be carried out to determine the suitable distribution of the ranking results using *EasyFit software.* The results of hypothesis testing are presented in Table 2.

Table 2. The results of Kolmogorov-Smirnov Test based on EasyFit Software

Variable	D_{stat}	D_{table}	Sig.	Conclusion
Inter-arrival time	0.1088	0.14027	0.1382	Data fits a Weibull distribution
Service time	0.1129	0.13087	0.1307	Data fits an Invers Gaussian distribution

Bayesian Method

Based on the Kolmogorov Smirnov test, the general distribution of inter-arrival time and service time was obtained, namely the weibull distribution and the inverse gaussian distribution. These two pieces of information are used as prior information in determining the distribution of arrival times and service times using the Bayesian approach. Furthermore, with additional information from empirical data, the likelihood function and posterior distribution will be formed.

1. Non-Informative Prior

The prior non-informative value uses jeffrey's method as follows:

a. Non-Informative Prior Gaussian Inverse Distribution

$$f(x;\lambda,\mu) = \sqrt{\frac{\lambda}{2\pi x^3}}\, exp\left[-\frac{\lambda(x-\mu)^2}{2x\mu^2}\right]$$

$$log f(x;\lambda,\mu) = -\frac{1}{2}log2\pi - \frac{1}{2}log x^3 + \frac{1}{2}log\lambda - \left(\frac{\lambda(x-\mu)^2}{2x\mu^2}\right)$$

$$\frac{\partial \log f(x;\lambda,\mu)}{\partial\lambda} = \frac{1}{2\lambda} + \left(\frac{(x-\mu)^2}{2x\mu^2}\right)$$

$$\frac{\partial^2 \log f(x;\lambda,\mu)}{\partial\lambda^2} = -\frac{1}{2\lambda^2}$$

$$I(\lambda) = -E\left[\frac{\partial^2 \log f(x;\lambda;\mu)}{\partial\lambda^2}\right] = -E\left[-\frac{1}{2\lambda^2}\right] = \frac{1}{2\lambda^2}$$

$$\mathrm{f}(\lambda) = \sqrt{\mathrm{I}(\lambda)} = \sqrt{\frac{1}{2\lambda^2}} = \frac{1}{\sqrt{2}\lambda}$$

Hence the non-informative prior for the Inverse Gaussian distribution is $f(\lambda) = \frac{1}{\sqrt{2}\lambda}$.

b. Non-Informative Prior Weibull Distribution

$$f(x;\lambda,k) = k\lambda x^{k-1} exp[-\lambda x^k]$$

$$\log f\,(x;\lambda,k) = \log k + \log\lambda + k\,logx - logx - \lambda x^k$$

$$\frac{\partial^2 \log f(x;\lambda,k)}{\partial\lambda} = \frac{1}{\lambda} - x^k$$

$$\frac{\partial^2 \log f(x;\lambda,k)}{\partial\lambda^2} = -\frac{1}{\lambda^2}$$

$$I(\lambda) = -E\left[\frac{\partial^2 \log f(x;\lambda:k)}{\partial\lambda^2}\right] = -E\left[-\frac{1}{\lambda^2}\right] = \frac{1}{\lambda^2}$$

$$\mathrm{f}(\lambda) = \sqrt{\mathrm{I}(\lambda)} = \sqrt{\frac{1}{\lambda^2}} = \frac{1}{\lambda}$$

Hence the non-informative prior for the Weibull distribution is $f(\lambda) = \frac{1}{\lambda}$.

2. Likelihood Function
   a. Likelihood of Weibull Distribution

$$L(\lambda,k) = \prod_{i=1}^{n} f(x;\lambda;k)$$

$$= \prod_{i=1}^{n} k\lambda x_i^{k-1}\exp[-\lambda x_i^k]$$

$$= (k\lambda)^n exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]\prod_{i=1}^{n}(x_i)^{k-1}$$

   b. Likelihood of Inverse Gaussian Distribution

$$L(\lambda,\mu) = \prod_{t=1}^{n} f(x;\lambda;\mu)$$

$$= \prod_{i=1}^{n}\sqrt{\frac{\lambda}{2\pi x_i^3}}\,exp\left[-\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]$$

$$= (2\pi)^{-\frac{n}{2}}(\lambda)^{\frac{n}{2}} exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]\prod_{i=1}^{n}(x_i^3)^{\frac{1}{2}}$$

3. Posterior Distibustion

Posterior distribution is obtained by substituting the likelihood function of the sample data distribution and the non-informative prior value of the prior distribution.

a. Likelihood Function of Weibull Sample Distribution and Non Informative Prior of Gaussian Inverse Distribution

$$f(\lambda|x) = \frac{L(\lambda,k)f(\lambda)}{\int_{-\infty}^{\infty} L(\lambda,k)f(\lambda)d\lambda}$$

$$f(\lambda|x) = \frac{k^n\lambda^n exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]\prod_{i=1}^{n}(x_i)^{k-1}\left(\frac{1}{\lambda\sqrt{2}}\right)}{\int_0^{\infty} k^n\lambda^n exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]\prod_{i=1}^{n}(x_i)^{k-1}\left(\frac{1}{\lambda\sqrt{2}}\right)d\lambda}$$

$$f(\lambda|x) = \frac{\frac{1}{\sqrt{2}}k^n\lambda^{n-1}exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]\prod_{i=1}^{n}(x_i)^{k-1}}{\int_0^{\infty}\frac{1}{\sqrt{2}}k^n\lambda^{n-1}exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]\prod_{i=1}^{n}(x_i)^{k-1}d\lambda}$$

$$f(\lambda|x) = \frac{\lambda^{n-1}exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]}{\int_0^{\infty}\lambda^{n-1}exp\left[-\lambda\sum_{i=1}^{n}x_i^k\right]d\lambda}$$

$$f(\lambda|x) = \frac{\lambda^{n-1}exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]}{\left(\sum_{i=1}^{n}x_i^k\right)\int_0^{\infty}\lambda^{n-1}exp[-\lambda]d\lambda}$$

$$f(\lambda|x) = \frac{\lambda^{n-1}exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]}{\left(\sum_{i=1}^{n}x_i^k\right)\Gamma(n)}$$

$$= \lambda^{n-1}exp\left[-\sum_{i=1}^{n}\lambda x_i^k\right]\left(-\sum_{i=1}^{n}x_i^k\right)^{-1}[\Gamma(n)]^{-1}$$

$$f(\lambda,k|x)\sim GAM\left(n,\left(\sum_{i=1}^{n}x_i^k\right)\right)$$

Therefore, the posterior distribution generated is the Gamma distribution with parameter values $\alpha = n$ and $\beta = \left(\sum_{i=1}^{n}x_i^k\right)$.

b. Likelihood Function of Inverse Gaussian Sample Distribution and Non-Informative Prior Weibull Distribution

The posterior distribution of λ is obtained by substituting the likelihood function of the inverse Gaussian distribution and the non-informative prior value of the Weibull distribution that has been obtained as follows:

$$f(\lambda|x) = \frac{L(\lambda,\mu)f(\lambda)}{\int_{-\infty}^{\infty}L(\lambda,\mu)f(\lambda)d\lambda}$$

$$f(\lambda|x) = \frac{(2\pi x_i^3)^{-\frac{n}{2}}(\lambda)^{\frac{n}{2}}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]\left(\frac{1}{\lambda}\right)}{\int_0^{\infty}(2\pi x_i^3)^{-\frac{n}{2}}(\lambda)^{\frac{n}{2}}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]\left(\frac{1}{\lambda}\right)d\lambda}$$

$$f(\lambda|x) = \frac{(2\pi x_i^3)^{-\frac{n}{2}}(\lambda)^{\frac{n}{2}-1}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]}{\int_0^{\infty}(2\pi x_i^3)^{-\frac{n}{2}}(\lambda)^{\frac{n}{2}-1}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]d\lambda}$$

$$f(\lambda|x) = \frac{(\lambda)^{\frac{n}{2}-1}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]}{\int_0^{\infty}(\lambda)^{\frac{n}{2}-1}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]d\lambda}$$

$$f(\lambda|x) = \frac{(\lambda)^{\frac{n}{2}-1}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]}{\left(\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2x_i\mu^2}\right)\int_0^{\infty}\lambda^{\frac{n}{2}-1}exp[-\lambda]\,d\lambda}$$

$$f(\lambda|x) = \frac{(\lambda)^{\frac{n}{2}-1}exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]}{\left(\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2x_i\mu^2}\right)\Gamma\left(\frac{n}{2}\right)}$$

$$f(\lambda|x) = (\lambda)^{\frac{n}{2}-1} exp\left[-\sum_{i=1}^{n}\frac{\lambda(x_i-\mu)^2}{2x_i\mu^2}\right]\left(\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2x_i\mu^2}\right)^{-1}\left[\Gamma\left(\frac{n}{2}\right)\right]^{-1}$$

$$f(\lambda|x) \sim GAM\left(\frac{n}{2},\left(\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2x_i\mu^2}\right)\right)$$

Therefore, the posterior distribution generated is the Gamma distribution with parameter values $\alpha = \frac{n}{2}$ and $\beta = \left(\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2x_i\mu^2}\right)$.

Queue System Model

Based on the results of the analysis of steady-state conditions and posterior distribution of prior data and sample data of inter-arrival time and aircraft service time at Adi Soemarmo-Surakarta International Airport, it can be determined that the queue system model in the aircraft service system at Adi Soemarmo-Surakarta International Airport is the queue system model (GAMMA/GAMMA/3): (GD/∞/∞). The model is a queuing system model with Gamma-distributed inter-arrival time and Gamma-distributed service time, with the number of service systems as many as 3 service facilities. GD is the queuing discipline used, which is first-come-first-served (FIFO), as well as the number of customer capacities that come and unlimited call sources.

Performance Measures of Queue Systems at Airports

The following are the output results of system performance measures in the airplane queue with GUI R (*shiny*) software, which can be seen in Table 3.

Table 3. Airplane Queue System Performance Measures

Parameter Model			Performance		
C	λ	μ	Ls	Lq	Ws
3	60.085	41.046	1.570	0.106	0.0261
			Wq	P_0	Pr
			0.0017	0.219	0.7805

Based on Table 3, it can be seen that the performance measures of the aircraft queuing system at Adi Soemarmo-Surakarta International Airport are as follows:

1. The number of service facilities provided for aircraft services is 3 service facilities.
2. The average inter-arrivals time of arriving aircraft is 60.085 minutes. That is, within 60.085 minutes there is 1 aircraft that arrives at the aircraft queue system.
3. The average service time of aircraft served is 41.046 minutes. That is, within 41.046 minutes there is 1 aircraft served in the aircraft queue system.
4. The expected number of aircraft in the queue system is Ls = 1,570 aircraft. This means that on average there are 3 aircraft in the queue and being served for 120 minutes (2 hours).
5. The number of aircraft in the queue system is Lq = 0.106 aircraft. This means that on average there is 1 aircraft waiting in the queue before being served for 600 minutes (10 hours).
6. The estimated waiting time in the system is Ws = 0.0261 of 60 minutes. This means that the time it takes for 1 aircraft to wait in the queue and be served is 1.566 minutes.
7. The estimated waiting time in the queue is Wq = 0.0017 out of 60 minutes. This means that the time it takes for 1 aircraft to wait in the queue before being served is 0.102 minutes.
8. The probability that the service will be busy serving the aircraft is Pr = 0.7805, which means that the queue system has a 78.05% chance of being busy. This figure indicates that the service will be busy serving aircraft for 78.05% of the time.
9. The probability that the line is idle is Po = 0.219, which means the chance of the queue system not being busy is 21.9%.
10. Based on the value of the system performance measures obtained, it can be said that the aircraft service at Adi Soemarmo-Surakarta International Airport is good.

The results of this study provide information about the aircraft service system at Adi Soemarmo-Surakarta International Airport. Based on the results of the study, it shows that the aircraft queuing system is in good condition. This can be seen from the ability of service facilities that are able to meet customer needs, so there is no need for additional service facilities or additional aircraft lanes.

V. CONCLUSION

Thus, from the results of the data analysis and discussion that has been obtained, it can be concluded that there are changes or additional information on the aircraft service system at Adi Soemarmo-Surakarta International Airport over time. Based on the posterior distribution for the time between arrivals and service time, the aircraft queue model at Adi Soemarmo-Surakarta International Airport is (GAMMA/GAMMA/3): (GD/∞/∞). At the service at Adi Soemarmo-Surakarta International Airport, a queuing system model is obtained with a Gamma-distributed inter-arrival time and a Gamma-distributed service time. There are 3 service facilities (servers), the GD queuing discipline used is first-come-first-served (FIFO), the number of customer capacities that come is unlimited, and the source of the call is unlimited. Based on the perfomance system measure, it shows that the aircraft queue system at Adi Soemarmo International Airport has a condition that is considered good, This is seen from the ability of service facilities that are able to meet customer needs, so there is no need for additional service facilities or terminals.

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