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Design Worked Examples for Mathematical Literacy: A Focus on Learning the Pythagorean Theorem

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ABSTRACT

Mathematical literacy skills of Indonesian students are still below the OECD average (PISA 2022) , where students face difficulties relating their mathematical knowledge to real-life contexts. This indicates that learning is still dominated by a mechanistic approach. This research aims to design *worked examples* in mathematical literacy skills for solving contextual problems in the Pythagorean Theorem material. The research method used is design and development research with the ADDIE model (Analyze, Design, Development). The problem design was developed based on the three main processes of OECD mathematical literacy: *formulating*, *employing*, and *interpreting*. The results yielded a problem design that provides systematic and contextual solution steps , while minimizing cognitive load by avoiding *split-attention* and *redundancy effects*. This *worked example* strategy is recommended for novice learners because it serves as *scaffolding* that guides students to understand the reasoning process and application of concepts systematically in real-world situations

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INTRODUCTION

Mathematical literacy is one of the core competencies of the 21st century and serves as an important indicator of a country's educational quality. This ability is not merely related to calculation skills or understanding mathematical concepts, but rather emphasizes the ability to use mathematical knowledge reflectively to solve problems in various real-life contexts (OECD, 2023). According to Stacey & Turner (2015), mathematical literacy reflects the extent to which a person is able to interpret and apply mathematical ideas in situations encountered, whether in personal life, work, society, or the increasingly complex global world. Thus, mathematical literacy requires students to not only understand formulas or procedures, but also to be able to think critically, reason logically, and make decisions based on mathematical evidence.

The results of the 2022 Program for International Student Assessment (PISA) international survey show that the mathematical literacy level of Indonesian students is still below the average for OECD countries (OECD, 2023). This achievement indicates that students still face difficulties in their mathematics education in the context of everyday life. This is in line with the findings of Wijaya et al. (2016), which states that the majority of students in Indonesia experience difficulties in formulating, interpreting and applying mathematics when understanding contextual problems. Many students tend to only focus on procedures or formulas without understanding the meaning behind their application. This condition shows that mathematics learning in schools is still dominated by a mechanistic approach, which emphasizes algorithmic practice rather than application of concepts.

According to OECD (2023), mathematical literacy involves three main processes, namely formulating, employing, and interpreting mathematics in context. This process encourages students to understand real-world situations, convert them into mathematical models, use relevant concepts and procedures, and then interpret the results back into the context of everyday life. This ability does not arise spontaneously, but needs to be facilitated through learning design that is authentic, contextual, and supports conceptual understanding.

One strategy that can support the development of mathematical literacy is worked examples, namely the presentation of example problems complete with solution steps explained systematically. Worked examples are a learning approach designed to help students understand problem solving in a more structured and cognitively lighter way (Sweller, 2019). This method models the problem solving process by presenting the problem and the complete steps for solving it, so that students do not need to use up their working memory capacity to guess or try. This is very important, especially for beginner students who are still building basic knowledge schemes. By learning the systematic stages of solving, they can more easily recognize patterns, remember important information, and understand the structure of similar problems (Cooper, 1990). Worked example-based learning also helps in the process of internalizing knowledge because it provides a concrete representation of how a problem can be solved from start to finish.

Worked examples have proven to be effective, especially for beginners who often experience confusion when faced directly with questions (Van Gog et al., 2011; Manson & Ayres, 2021). Students gain better understanding and learn faster when they first study example problems before being asked to solve the problems independently (Atkinson et al., 2003). However, the effectiveness of worked examples really depends on the quality of the design. If the examples presented give rise to split attention and redundancy effects, then it can actually hinder understanding (Sweller, 2019). Therefore, a good worked example design must pay attention to the clarity of presentation and relevance of the information provided so that it is truly able to support an optimal learning process.

Worked examples have been widely researched, but their design can vary according to the topic and learning needs. This condition encourages the need for research that highlights worked example designs to improve mathematical literacy in the context of everyday life. Several researchers have developed worked example designs for various learning materials and needs, such as Retnowati & Fadlila who designed worked examples for learning the combined area of triangles and quadrilaterals. Julianingsih et al. (2023) designed worked examples on trigonometry material by integrating the ARCS motivation model, and Azizah & Retnowati (2017) designed worked examples on geometry and algebra material for students with visual disabilities using Braille. The Pythagorean Theorem material was used in this research because the application of this material can be found in various contextual situations that are relevant to mathematical literacy. However, this material is still poorly understood by students. This can be seen from students' learning achievements in the Pythagorean Theorem material which is still unsatisfactory because only 41% of students achieved completeness (Ritonga & Hasibuan, 2022). Wulandari & Riajanto (2020) and Nurmayunita et al. (2024) stated that the majority of students experience difficulties in relating mathematical concepts to real life contexts, including in the formulation, application and interpretation processes which are the main elements of mathematical literacy in the Pythagorean Theorem material.

This study aims to design worked examples for mathematical literacy on the Pythagorean Theorem. Thus, it is hoped that this research can contribute to the development of more effective learning designs through contextual worked examples, as well as enriching literature studies related to the application of the Pythagorean Theorem in developing mathematical literacy at primary and secondary education levels.

RESEARCH METHOD

This type of research is design and development research with models development of ADDIE (Analyze, Design, Development, Implementation, Evaluation). Study this uses three stages of the ADDIE model, namely analyze, design, development for designing and developing a product in the form of worked-based math problems example that facilitates students in mathematical literacy skills. At the analysis stage, Researchers identify related needs before designing and developing questions mathematics. Researchers analyzed relevant literature reviews to identify urgency from worked example-based mathematical literacy skills in theorem learning Pythagoras. Next, at the design stage, researchers designed products in the form of literacy questions mathematics based on worked examples. The designed math problems must meet indicators of mathematical literacy abilities and in accordance with the stages that students will work on. At the development stage, researchers arrange the product according to the design that has been prepared. The question design in the form of a prototype was validated by two mathematics education lecturers. Results validation is used as a basis for revising and perfecting the prototype, so that resulting in an effective question design in facilitating students' mathematical literacy skills.

RESULT AND DISCUSSION

This research produced worked examples design for mathematical literacy in the material of the Pythagorean theorem. The following discussion will outline the results of each step during the development of the worked example.

a. Analysis

Chen et al. (2023) revealed that worked examples are useful in mathematics material with high interactivity elements. The Pythagorean Theorem is one of the materials that falls into this category. In learning material on the Pythagorean Theorem, students are expected to be able to demonstrate the truth of the Pythagorean Theorem and use it in various contexts, including determining the distance between two points on the Cartesian coordinate plane. This shows that understanding of concepts and mathematical literacy skills related to this material must be developed flexibly and meaningfully. Regarding prerequisite material, students have studied exponent numbers, root form numbers, the meaning and types of triangles, as well as the properties of triangles as a logical basis for understanding and applying the Pythagorean Theorem correctly. The learning objectives of the Pythagorean Theorem include analyzing information to prove the theorem, creating a scheme or procedure to prove the formula, determining the length of the side of a triangle using the Pythagorean Theorem, comparing the sides of a special right triangle, finding the form of a Pythagorean triple, solving problems in everyday life related to its application. The worked examples in this research will be prepared with the main focus, namely helping students determine the length of the sides of a triangle using the Pythagorean Theorem and improving their mathematical literacy through solving contextual problems in everyday life.

Wijaya et al. (2015) explains that students will understand concepts more easily mathematics when given a context that is close to everyday life. However, research by Rafiepur and Faramarzpour (2023) shows that the majority of students still have difficulty connecting abstract concepts with their application in context real because learning tends to focus on formulas and final results. Based on these findings, it is necessary to design questions that can guide students to understand the concept thoroughly gradually through clear completion steps. Worked example approach is an effective strategy because it provides an example of a complete solution can help students reduce cognitive load and focus on mathematical thinking processes (Sweller, van Merriënboer, & Paas, 2019).

The next step is to formulate indicators of mathematical literacy. The OECD through PISA defines mathematical literacy as the ability to formulate, employ and interpret mathematics in various real life contexts (OECD, 2021). These three processes reflect how students understand situations, apply mathematical concepts and procedures, and give meaning to the results obtained. The mathematical literacy indicators used in this research are adjusted as follows

Table 1. Mathematical Literacy indicators

Aspect	Indicator
Formulate	Recognize aspects of a problem that are relevant toknown issues
Employ	Applying facts, tools, algorithmic rules and mathematical structures, When searching for solutions
Interpret	Interpret/evaluate the results of inward mathematical solutions real world context

The characteristics of phase D students, especially grade VIII students who are at the stage of formal operational thinking according to Piaget's theory, have begun to think about concrete experiences and think about them in a more abstract, idealistic and logical way (Marinda, 2020). This allows them to consider solutions to real events more wisely and develop structured problem-solving patterns (Anggraeni et al., 2024). Based on the results of the analysis, it was concluded that the development of a worked example design for mathematical problem solving with the context of daily life in the material of the Pythagorean theorem is a relevant step and in accordance with the needs of students. This is reinforced by Sweller (2019) who states that the worked example is recommended as an instructional design for beginner learners.

b. Design

The design stage, a prototype of a mathematical literacy ability instrument based on worked examples was developed for learning the Pythagorean Theorem. The aim of this stage is to produce an instrument design that is in accordance with mathematical literacy indicators, student characteristics and learning objectives. The design is prepared based on the results of the needs analysis and materials obtained in the previous stage.

The design stage begins with context analysis, which is to identify various phenomena that are close to the lives of students and relevant to the concept of the Pythagorean theorem. The selected contexts are the journey to school, the home environment and window repairs, because both represent real situations that are commonly found in a person's personal, social and work life These two contexts can be visually easily represented in the form of a right triangle, making it easier for learners to connect concrete experiences with mathematical models. In addition, the choice of this context also considers the importance of meaningful learning, which is when students can see the relevance between mathematical concepts and real life.

The arrangement of worked examples in the material of the Pythagorean theorem needs to be carefully designed so as not to cause unfamiliar cognitive loads (Atkinson et al., 2003). Sweller (2019) revealed several factors that must be avoided, namely split attention and redundancy effects. The splitattention effect is the separation of students' attention by the presentation of two or more different or separate sources. According to Ayres & Cierniak (2012), split attention occurs when learners are required to divide their attention between two or more sources of information (e.g. text and diagrams) that have been separated spatially or temporally. If students have to divide their attention into many sources, it will increase the cognitive content of students. The redundancy effect occurs due to the excess of information caused by too many sources used, even though with just one source it can provide comprehensive information. The presence of information sources that do not contribute to schema acquisition or automation interferes with learning (Plass et al., 2010). Therefore, the design developed in this study will consider both of these things.

The questions are developed according to the grid that has been prepared, validated and improved according to input suggestions from the validator. The validators consisted of two mathematics education lecturers. The suggestions given by the validator can be seen in Table 2 below.

Table 2. Expert Review Result

Validator	Suggestion and comments
Mathematics Education Lecturer	- The content actually needs to be improved, - The numbers chosen do not make sense in some cases - Command sentences need to be corrected

The results of the validator's suggestions and input are used to improve the design of the questions which are designed according to indicators of mathematical literacy ability. The question design was revised according to Table 2 and declared valid.

c. Development

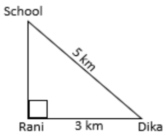
Worked example 1 is presented as follows.

Social Context
One morning, Rani and Dika planned to go to SMP Negeri 1 Depok together. Rani's house is to the north of Dika's house, and their school is to the west of Rani's house. **The distance between Rani's house and Dika's house is 3 km, while the distance from Dika's house to school via a sloped road is 5 km.** If Rani goes alone to school by taking a straight road from her house to school,

Determine the distance between Rani's house and school and the difference in the distance between Dika and Rani to school?

— **Problem**

Solution:
Known:
Rani's house is to the north of Dika's house, and the school is to the west of Rani's house. This forms a right triangle in Rani's house.
Distance Rani - Dika (vertical side, a) = 3 km.
Distance from Dika to school (hypotenuse, c) = 5 km.



— **Formulate**

- Calculate the distance from Rani to school (base side, b):
  $b^2 = c^2 - a^2$
  $b^2 = 5^2 - 3^2$
  $b^2 = 25 - 9$
  $b^2 = 16$

Distance from Rani to school = b = $\sqrt{16}$ = 4km
- Distance Difference

Distance traveled by Dika = Slant distance = 5 km
Distance traveled by Rani = Straight distance = 4 km
Distance difference = 5 km - 4 km = 1 km

— **Employ**

So, the distance from Rani's house to school is 4 km and the difference in distance traveled when Rani takes a straight road compared to when she takes a slanted road is 1 km.

— **Interpret**

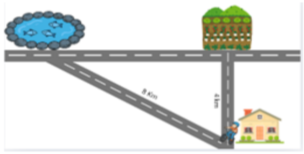
Figure 1. Worked Example 1

The worked examples presented in the Pythagorean Theorem material are designed systematically to develop students' mathematical literacy skills, especially in understanding, use, and interpret mathematical concepts in real-life contexts. Every example worked examples reflect the stages of the mathematical literacy process, namely formulating problem, use mathematical concepts and procedures, and interpret completion results. As shown in Figure 1, each step solutions are presented in a structured manner to help students identify important information from the problem context, constructing appropriate mathematical models, using Pythagorean Theorem formula to find solutions and interpret the results again to the original situation.

In addition to the complete example, a pair problem is also included as shown in Figure 2, which has a similar context and structure but without the solution steps shown. This question serves to train students to apply the mathematical literacy process independently based on the thinking patterns that have been exemplified in the worked examples. Through this activity, students not only learn to solve problems procedurally, but also develop the ability to understand meaning, reason mathematically, and relate calculation results to relevant real-life situations. Thus, the worked example design in this instrument acts as a learning tool that guides students towards more reflective and meaningful mathematical literacy.

The following presents a contextual situation in everyday life that is related to the application of the Pythagorean Theorem. Solve this problem by following the solution steps as exemplified previously in a coherent and systematic manner.

Personal Context
One afternoon, Rafi cycled around his house. **He left the house for a small garden 2 km to the west, then turned south to a fish pond 5 km away.** Rafi wanted to know the distance if he cycled directly from the house to the fish pond without passing through the small garden
.



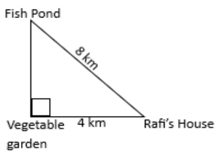
Determine the distance between the vegetable garden and the fish pond and the difference in the distance Rafi traveled to the fish pond and the vegetable garden?

— **Problem**

Figure 2. The Paired Worked Example 1

The expected answers from paired questions as mathematical literacy training are shown in the personal context picture of Rafi cycling to the vegetable garden and fish pond. Worked examples and problem pairs are developed through similar procedures. The work step begins with presenting the context in the form of an illustrative image, then giving the dimensions of the sides of the triangle, and ending with calculating the length of the hypotenuse using the Pythagorean Theorem to determine the direct distance between two points in a real situation.

Known:
Rafi left the house for the vegetable garden 4 km to the north,
The distance from the house to the fish pond via a sloping road is 8 km.
This position forms a right triangle in the vegetable garden



— **Formulate**

- Calculate the distance from the Vegetable Garden to the Fish Pond (base side, b):

$b^2 = c^2 - a^2$
$b^2 = 8^2 - 4^2$
$b^2 = 64 - 16$
$b^2 = 48$
$b = \sqrt{48} = 6.9$ km

- Distance Difference:

Distance traveled through the vegetable garden = 4 km + 6.9 km = 10.9 km
Direct distance (sloping road) = 8 km
Distance difference = 10.9 km – 8 km = 2.9 km

— **Employ**

So, the distance between the vegetable garden and the fish pond is 6.9 km, and the difference in the distance Rafi travels if he cycles through the garden compared to going straight to the fish pond is 2.9 km.

— **Interpret**

Figure 3. The Answer of the Paired Worked Example 1

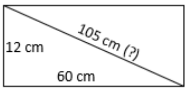
Worked example 2 with different learning objectives, namely determining the length of the side of a triangle using the triple Pythagorean theorem and solving problems in everyday life related to the application of the Pythagorean theorem, is presented as follows.

Occupational Context
In the world of construction, carpenters often use mathematical principles to ensure the correct shape and size of a structure. One principle that is often used is the Pythagorean Theorem to ensure right angles in window frames, doors or walls so that buildings are sturdy and symmetrical.

A carpenter is repairing a window that appears to be installed incorrectly. To make sure the window was rectangular, he took measurements. **The measurement results show that the window height is 80 cm, the window width is 60 cm, and the diagonal length corresponding to the measured height and width of the window is 105 cm.** Is the window really rectangular? explain!

— Problem

Solution:
Known:
Information that can be used:
Window height (t) = 80 cm
Window width (l) = 60 cm
The measured diagonal length (d) = 105 cm



— Formulate

To be a perfect rectangle, the diagonal formed by the height and width must conform to the Pythagorean Theorem.
$d^2 = t^2 + l^2$
$d^2 = 80^2 + 60^2$
$d^2 = 6400 + 3600$
$d^2 = 1000$
$d = \sqrt{1000} = 100\ cm$

— Employ

The diagonal measured by the carpenter is 105 cm, even though the diagonal should be 100 cm. Because 105 cm ≠ 100 cm, the window is not truly rectangular

— Interpret

Figure 4. Worked Example 2

In worked example 2 above, the researcher displays an image on the problem as a form of visual representation of the problem given. This visual representation is used as a tool to capture mathematical relationships and processes (van Garderen et al, 2021). Researchers pay attention to the principles of visual simplicity and coherence between text and images. It is expected that students can engage with real world problems, develop plans in response, justify them mathematically through representations, then evaluate and communicate solutions. Then, paired questions from worked example 2 are presented in Figure 5 below.

The following are problems in everyday life related to the Pythagorean theorem. Follow the steps in the previous example in sequence!

Occupational Context
In the world of construction, carpenters often use mathematical principles to ensure the correct shape and size of a structure. One principle that is often used is the Pythagorean Theorem to ensure right angles in window frames, doors or walls so that buildings are sturdy and symmetrical.

A carpenter is repairing a window that appears to be installed incorrectly. To make sure the window was rectangular, he took measurements. **The measurement results show that the window height is 120 cm, the window width is 90 cm, and the diagonal length corresponding to the measured height and width of the window is 160 cm.** Is the window really rectangular? explain!

— Problem

Figure 5. The Paired Worked Example 2

Worked examples and paired questions are designed to contain everyday life contexts so that students are encouraged to connect and transfer their mathematical knowledge into real situations. A context like this helps students be actively involved in the process of developing mathematical literacy, namely the ability to use mathematical concepts and reasoning to understand, assess and solve problems in everyday life (Matty, 2016). These worked examples and paired questions guide students through stages of literate thinking, starting from identifying information that is relevant to the context of the problem (what is known and asked), writing down appropriate mathematical models or solution strategies, carrying out calculation steps systematically, to interpreting the results according to their contextual meaning. This series of processes is in line with the views of Parvaneh & Duncan (2021) who emphasize the importance of organizing ideas logically and evaluating strategies as part of strengthening reflective thinking and reasoning skills in developing students' mathematical literacy.

The application of mathematical literacy requires theoretical support that considers students' cognitive limitations in understanding new information and concepts. Cognitive load theory explains that human working memory capacity is limited in processing complex or unknown information, so an approach is needed that can optimize cognitive load so that learning takes place effectively (Sweller et al., 2011). This theory predicts that when students are asked to connect mathematical concepts with real life contexts, high cognitive effort can burden working memory and hinder the mathematical literacy process, especially for novice students (Plass et al., 2010).

The worked example approach exists as a strategy that helps teachers facilitate the development of mathematical literacy by gradually guiding students to understand the relationship between concepts, procedures and real contexts (CESE, 2019). Worked examples provide a systematic approach to a task or contextual problem (Ayres, 2011). Worked examples in the context of mathematical literacy are examples of questions that show the application of mathematical concepts to real world situations, where each step in solving them is explained explicitly (CESE, 2019). This material includes problem formulation, mathematical steps, and interpretation of results in a meaningful context (Schworm & Renkl, 2019; Renkl & Atkinson, 2010; Ayres & Sweller, 2013). Thus, worked examples function as instructional tools that guide students to understand how mathematics is used in decision making and solving contextual problems (Atkinson et al., 2000).

Various studies support the effectiveness of worked examples in improving mathematical literacy skills. Yee & Tzeng (2020) stated that worked examples help students process information in working memory more efficiently, so that they are able to relate concepts to the context of everyday life. Worked examples are also effective for students who are still in the early stages of developing mathematical literacy because they provide a structured model of mathematical thinking (Rodawati & Retnowati, 2019). Hartmann et al. (2021) emphasized that learning through worked examples is more useful than simply solving contextual problems independently, because it provides opportunities for students to reason and reflect on the use of mathematics in social and scientific contexts. Similar findings were stated by Chen et al. (2023) which shows that the systematic use of worked examples reduces cognitive load and increases students' ability to interpret mathematical results meaningfully.

In the design of worked example-based mathematics literacy learning, attention to aspects of split attention and redundancy effects is important (Plass et al., 2010). Teachers need to ensure that the presentation of information is not separated between text, symbols and visual context, so that students can understand mathematical meaning without excessive cognitive load. Preparing effective worked examples can start from simple to complex contexts, with explicit explanations of the steps so that students are able to independently interpret the results of solutions in real life situations.

The steps for learning worked examples in the context of mathematical literacy include several stages that lead students to understand the application of mathematical concepts in real life. Referring to the stages described by Wittwer and Renkl (2010), learning activities can begin by providing a general explanation of the basic concepts that will be used, including an introduction to mathematical principles and relationships between concepts that are relevant to the context of the situation. At this stage, the teacher helps students understand the mathematical meaning of a statement or contextual phenomenon, for example the relationship between length, distance or geometric shapes in everyday life, to build initial awareness of the mathematical literacy concept that will be studied.

The second stage is carried out by presenting worked examples which show the steps for applying the concept in a real context, starting from identifying information that is known and asked, determining an appropriate strategy, carrying out calculations or mathematical representations, to interpreting the results in the context of the problem. This stage aims to ensure that students are able to follow the mathematical thinking process logically and systematically, and understand how calculation results can be used to explain or solve contextual problems.

In the next stage, students are given the opportunity to work on similar questions independently, but still in a structure that resembles the previous worked example. This question aims to train students' ability to apply mathematical reasoning in new contexts without losing the meaning of the examples they have studied. Presenting advanced questions is important to strengthen the ability to interpret and communicate mathematical ideas contextually. After completing the questions, students are given feedback or answer keys so they can reflect on their understanding and correct inappropriate thinking strategies.

The implementation of worked example-based learning on mathematical literacy material is designed by considering whether the activity is carried out individually or in groups. Retnowati et al. (2010) shows that learning worked examples in groups provides more optimal results because it allows discussion between students to interpret the mathematical context. Irwansyah & Retnowati (2019) also emphasized that worked example activities in mathematical literacy reduce students' cognitive load, especially when they work collaboratively to understand the meaning and application of concepts. This shows that the integration of worked examples in mathematical literacy learning not only helps understand procedures, but also improves reasoning abilities and relates mathematics to everyday life.

This research does not include the trial phase of implementing the worked example design directly in the classroom environment. A comprehensive evaluation of the quality of the design, both in terms of clarity of presentation and its effectiveness in supporting students' mathematical literacy, has also not been carried out. Therefore, it is recommended that future research continue all stages of the development of the ADDIE model in order to perfect the worked example design that has been created. In addition, it is hoped that the results of this research can be used flexibly according to the needs and characteristics of learning in the field. It is hoped that these findings will be able to make a real contribution to improving the quality of the learning process and strengthening students' mathematical literacy.

CONCLUSION

Mathematical problems related to everyday life contexts in the application of the Pythagorean theorem often become obstacles for students, especially for those who are still in the early stages of understanding the concept. Therefore, a mathematical literacy strategy based on worked examples is recommended as an approach that can reduce irrelevant cognitive load while helping students understand the reasoning process and the steps for solving it in a structured manner. The worked example design in this research was developed by referring to the stages of mathematical literacy, namely: understanding the context and information in the problem, formulating a solution strategy, applying the strategy through calculations or mathematical reasoning, and interpreting the results according to the real life context. The context used in this design includes situations of using stairs and installing electrical cables to show the connection between Pythagorean concepts and their application in the real world. In addition, the principle of split attention is minimized by highlighting important information visually through thickening the text, combining all mathematical literacy steps in one integrated display, and using arrows and different colors to direct students' attention to the main elements. Meanwhile, redundancy effects are reduced by ensuring that each problem only contains information that is relevant to the context being analyzed. The stages of working example-based mathematical literacy learning on the topic of the Pythagorean theorem include: (1) students gain an initial understanding of the concepts and principles of the Pythagorean theorem; (2) students learn examples of mathematical literacy accompanied by complete completion steps (worked examples); and (3) students practice working on similar contextual problems to strengthen their ability to apply mathematical reasoning to real situations.

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