International Journal of Mathematics and Computer Research
ISSN: 2320-7167
Volume 13 Issue 11 November 2025, Page no. – 5860-5869
Index Copernicus ICV: 57.55, Impact Factor: 8.615
DOI: 10.47191/ijmcr/v13i11.04

Problem Design on Rational Number Material Oriented to the Mathematical Communication Skills of Junior High School Students

Diah Nur Hidayati[1], Ali Mahmudi[2]
[1,2]Department of Mathematics Education, Yogyakarta State University, Indonesia

ARTICLE INFO	ABSTRACT
Published Online: **11 November 2025**	This study was motivated by the low level of mathematical communication skills among junior high school students, as reflected in Indonesia's PISA 2022 results, particularly in expressing mathematical ideas and reasoning in written form. The aim of this research was to develop problem designs on rational number material oriented toward improving students' mathematical communication skills. The research method employed was Research and Development (R&D) using the ADDIE model, limited to the Development stage. The stages included analyzing students' characteristics and learning needs, formulating learning objectives and indicators of mathematical communication, and developing four initial contextual problem designs. The results of the study were four prototype problems based on real-world contexts: time management, financial literacy, temperature change, and cooking activities, which required students to represent data, perform mathematical calculations, and interpret results logically. The developed problems served as an initial design that had not yet been validated or implemented, thus requiring further research through implementation and evaluation stages to assess their effectiveness in enhancing students' mathematical communication skills in junior high school.
Corresponding Author: **Diah Nur Hidayati**	

KEYWORDS: ADDIE; mathematical communication skills; problem construction; problem design; rational numbers

I. INTRODUCTION

The 2022 Programme for International Student Assessment (PISA) results revealed that Indonesian students' average score in mathematical literacy was only 366, far below the OECD average of 472, placing Indonesia 69th out of 81 participating countries (Avvisati & Ilizaliturri, 2023). The evidence of Indonesia's low performance, with an average score of 366, can be seen in Figure 1. Indonesian students performed below the OECD average across all three domains: mathematics, reading, and science. Only 18% of Indonesian students reached at least Level 2 proficiency in mathematics, compared to 69% on average across OECD countries. At this level, students are expected to recognize and represent simple situations mathematically, such as comparing distances or converting prices between currencies. Conversely, almost no Indonesian students reached Level 5 or 6, indicating top performance in mathematics (OECD average: 9%). In contrast, countries such as Singapore (41%), Chinese Taipei (32%), and Japan (23%) showed significantly higher proportions of high-achieving students (OECD, 2023).

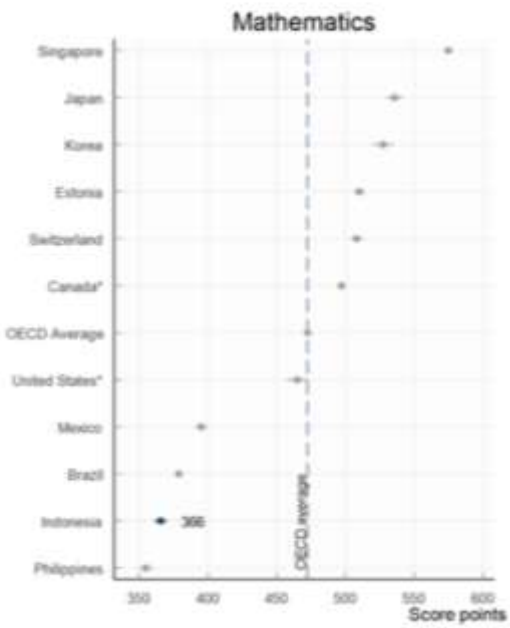

Figure 1. Indonesia's Mean Performance in Mathematics in PISA 2022

These findings highlight Indonesian students' weak mathematical literacy, particularly their difficulties in communicating mathematical ideas, interpreting problems logically, and evaluating mathematical arguments coherently (Chasanah et al., 2020). International studies have emphasized that mathematical communication skills play a crucial role in supporting reasoning and representational competence (Lenz et al., 2024; Planas & Pimm, 2024). Therefore, developing learning approaches that explicitly strengthen students' mathematical communication is an urgent priority to enhance both their conceptual understanding and overall mathematical literacy.

The lack of mathematical communication skills is closely related to instructional approaches that emphasize final answers rather than the process of reasoning and communication itself (MacKay et al., 2023; T.-T. Nguyen et al., 2024; Zhang et al., 2025). In meaningful mathematics learning, the process of expressing ideas and constructing logical arguments plays a vital role in conceptual understanding (NCTM, 2000; H. T. M. Nguyen et al., 2025). Recent research shows that problem-based learning (PBL) effectively fosters communication skills by engaging students in exploring, discussing, and reflecting on mathematical ideas collaboratively (Cevikbas & Kaiser, 2021; Mirna et al., 2023; Tong et al., 2021). Thus, instructional models that focus on contextual problem solving and collaboration can serve as strategic approaches to strengthen students' mathematical communication.

Problem design is an effective strategy to cultivate students' mathematical communication abilities. Problem-based tasks encourage students to analyze contextual situations, select relevant strategies, and communicate their reasoning and conclusions both orally and in writing (Hidayati et al., 2020; Sulastri, 2023). Moreover, well-constructed problem designs promote mathematical interaction among students, encouraging them to justify their ideas and explain problem-solving processes systematically (Cartwright, 2020). International research supports this notion, emphasizing that tasks promoting multiple representations and mathematical justification enhance students' conceptual engagement and discourse (Canogullari & Radmehr, 2025; Johnson, 2022).

Rational numbers are among the mathematical topics that particularly benefit from contextual problem design. Many students find this topic abstract because it is often taught procedurally without connection to real-world contexts (Ezaki et al., 2024; Fajri et al., 2025). Recent studies show that integrating multiple representations of rational numbers significantly improves conceptual understanding and mathematical performance (Braithwaite & Siegler, 2021; Schiller & Siegler, 2023). Real-life contexts such as recipes, trade, and measurement tasks can make rational number learning more meaningful and communicative (McMullen et al., 2022; Rosenberg-Lee et al., 2023). Therefore, contextual problem design within the topic of rational numbers is essential for promoting both mathematical communication and conceptual comprehension.

Although numerous studies have developed instructional media or learning tools, few have focused specifically on the design of context-based problems to assess or foster students' mathematical communication (Toalongo et al., 2022; Van der Wal et al., 2023). Musdi et al. (2024) developed a mathematical communication assessment tool in algebra, while Hidayat and Aripin (2023) designed a scientific-based E-worksheet to enhance students' communication skills. However, contextual problem ideas in rational number topics remain limited and require further development to help teachers vary instructional tasks that support mathematical communication. Thus, this study focuses on developing a problem design oriented toward junior high school students' mathematical communication skills on rational number material using the first three stages of the ADDIE model: analysis, design, and development (Branch, 2009).

This study is expected to provide both theoretical and practical contributions to 21st-century mathematics education. Theoretically, it enriches the body of knowledge regarding contextual problem design as a means of enhancing students' mathematical communication. Practically, the developed problem designs can be used by teachers as alternative learning materials or assessment tools that support problem-based instruction. Furthermore, this study serves as a foundation for future research focusing on the implementation and evaluation stages to measure the effectiveness of the developed designs in improving students' mathematical communication skills. Therefore, this research marks an initial step toward building mathematics learning that is more communicative, reflective, and contextually meaningful in junior high schools.

II. RESEARCH METHOD

This study employs a Research and Development (R&D) approach using the ADDIE model (Analysis, Design, Development, Implementation, and Evaluation) as proposed by Branch (2009). However, the scope of this study is limited to the first three stages, namely Analysis, Design, and Development, while the Implementation and Evaluation stages are reserved for further research. The ADDIE model was chosen for its systematic yet flexible structure in developing educational products. In this case, a set of problem designs on rational numbers oriented toward students' mathematical communication skills at the junior high school level.

Analysis Stage

The analysis stage aims to identify learning needs, student characteristics, and essential components of rational number learning as the basis for developing problem designs. This stage consists of four main activities:

1. Determining student characteristics, including cognitive level, learning preferences, and prior experience in mathematical problem solving.
2. Analyzing the content of rational number material to identify key concepts relevant to real-life contexts, such as fractions, decimals, and percentages.
3. Analyzing learning objectives, by reviewing the curriculum and expected learning outcomes related to rational numbers.
4. Analyzing mathematical communication indicators, to define specific aspects to be developed, such as expressing mathematical ideas, using appropriate representations, and providing logical reasoning.

Results of this stage are data on student characteristics, content analysis, learning objectives, and indicators of mathematical communication.

Design Stage

The design stage focuses on developing the initial draft of problem designs based on the previous analysis. The aim of this stage is to produce an initial draft of contextual problem designs on rational numbers oriented toward mathematical communication skills. The main activities include:

1. Determining real-life themes or contexts that are relatable to students' daily experiences, such as time management, financial literacy, temperature changes, and cooking activities.
2. Defining mathematical procedures that guide students in solving the problems, including steps for computation, data representation, and conclusion formulation.
3. Presenting the content of the problems in a contextual narrative designed to encourage critical thinking, discussion, and logical communication of mathematical ideas.

This stage has results on initial drafts of problem designs containing real-life contexts, mathematical procedures, and problem narratives.

Development Stage

The development stage aims to produce an initial product in the form of rational number problem designs ready for future validation. The main activities are as follows:

1. Developing problem designs based on the results of analysis and the real-life contexts previously determined.
2. Constructing four contextual problem designs on rational numbers that integrate mathematical communication skills into the problem-solving process.

This stage resulted in an initial product consisting of four contextual problem designs, which have been systematically developed but have not yet undergone expert validation or field testing. These later stages will be carried out in subsequent research to evaluate the effectiveness and practicality of the developed designs. The first three stages (Analysis, Design, and Development) can be seen Figure 2 below.

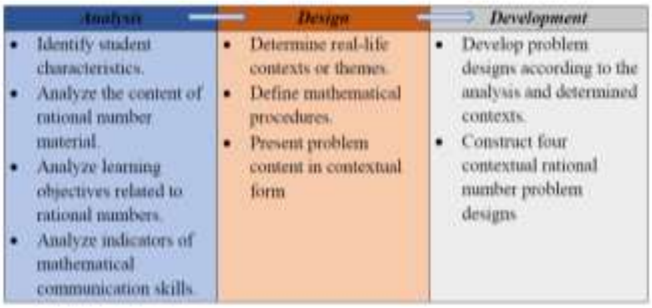

Figure 2. Research Stages Based on the ADDIE Model

III. RESULTS AND DISCUSSION

This study produced problem designs developed based on indicators of mathematical communication skills. The results of each step in the process of designing these problems are discussed as follows.

Analysis

The first step in the analysis stage was to determine the characteristics of the students. A preliminary study was conducted through interviews with a seventh-grade mathematics teacher at a public junior high school in Yogyakarta. The subjects of this study were seventh-grade students. Their characteristics indicated that understanding of rational number concepts was still limited, particularly in connecting these concepts to real-life situations. Furthermore, the prerequisite knowledge students should possess before learning rational numbers includes understanding the concept of integers and their arithmetic operations.

The interview results revealed that although some students had an initial understanding of rational number operations, they often encountered difficulties in expressing and communicating their ideas systematically. Therefore, it is necessary to design problems that connect mathematical concepts to familiar, real-life contexts such as time management, financial expenditure, temperature changes, and food distribution. These contexts are expected to effectively activate students' communication skills through writing, symbols, and other visual representations.

Next, the content of the rational number material was analyzed, which includes fractions, decimals, and the basic operations of addition, subtraction, multiplication, and division. This material is crucial as it forms the foundation for understanding more advanced mathematical concepts. The analysis revealed that students often experience difficulties in converting between different forms of rational numbers and in understanding the contextual meaning of their use in daily life.

The learning objectives for the rational number material in this study are that students are expected to:

1. Express rational numbers in fractional and decimal forms.
2. Compare rational numbers.
3. Perform addition, subtraction, multiplication, and division of rational numbers in real-life contexts.

4. Solve contextual problems involving rational numbers.

The final step of the analysis stage was to examine the indicators of mathematical communication skills. To assess students' mathematical communication skills, clear and measurable indicators are required. Several experts have proposed relevant indicators, like Baroody (Baroody, 1993) emphasized two important reasons why communication should be a central focus in mathematics learning:

1. Mathematics is not only a tool for thinking, discovering patterns, or solving problems, but also a valuable language for clearly, precisely, and concisely communicating ideas.
2. Mathematics learning is a social activity in which interaction between teachers and students plays a vital role in developing students' mathematical potential.

According to the National Council of Teachers of Mathematics (NCTM, 2000), the indicators for assessing students' mathematical communication skills include:

1. Organizing and connecting mathematical thinking through communication.
2. Communicating mathematical ideas coherently to peers, teachers, and other.
3. Analyzing and evaluating others' mathematical thinking and strategies.
4. Using mathematical language to express ideas precisely.

Meanwhile, Sumarmo (Sumarmo, 2006) defined the indicators of mathematical communication skills as students' abilities to:

1. Represent real objects, pictures, and diagrams into mathematical ideas or symbols.
2. Eexplain mathematical ideas, situations, and relationships verbally or in writing using real objects, diagrams, graphs, or algebraic expressions.
3. Eexpress daily life situations in mathematical language or symbols and construct mathematical models of such events.
4. Listen, discuss, and write about mathematics.
5. Construct conjectures, formulate arguments, definitions, and generalizations.
6. Restate a mathematical statement or passage in their own words.

Recent developments in mathematics education emphasize that mathematical communication is not merely the ability to express solutions but also the capacity to reason, represent, and construct meaning through multiple modes: symbolic, verbal, and visual. Contemporary works such as Halai and Clarkson (2016) and Webb and Webb (2016) highlight that dialogic interaction and language use play a crucial role in developing reasoning and communication, particularly in multilingual classrooms. Likewise, Greefrath et al. (2016) emphasize that communication is an integral process within mathematical modeling, where students represent real-world situations mathematically and interpret the outcomes coherently. Empirical studies also support this multidimensional view of communication; for instance, Cartwright (2020) analyzed students' communicative behavior in collaborative tasks, while Toalongo et al. (2022) developed and validated a rubric assessing representational and communicative competencies in mathematical modeling.

Based on these perspectives, this study limits the scope of communication to written mathematical communication. Students are considered to have achieved good mathematical communication skills when they meet the following indicators:

1. Accurately express situations, data, or information from problems or real-life events in tables, figures, or number lines.
2. Correctly use mathematical notations, symbols, or formulas and carry out calculations accurately.
3. Interpret and communicate the results or solutions of mathematical problems effectively.

These indicators, adapted from foundational frameworks proposed by Baroody (1993), NCTM (2000), and Sumarmo (Sumarmo, 2006), are further strengthened by recent international perspectives that position communication as a bridge between problem solving, reasoning, and understanding in mathematics.

Design

The initial step in the design stage was to determine real-life contextual themes related to rational numbers. The contexts were selected based on everyday problems experienced by students so that they would find the problems relevant and be motivated to solve them. The blueprint of the rational number problem designs oriented toward mathematical communication skills developed in this study is presented in the Table 1.

Table 1. Blueprint of Rational Number Problem Design

Problem Contexts	Learning Objectives	Indicators	Problem Items
1. Time Management 2. Financial Literacy in Buying and Selling Activities 3. Temperature Changes in	▪ Express rational numbers in fractional and decimal forms ▪ Compare rational numbers ▪ Perform addition, subtraction, multiplication, and division of	1. Accurately express situations, data, or information from problems or real-life events in appropriate tables, diagrams, or number lines.	1a, 2a, 3a
		2. Use correct notations, symbols, or	1b, 2b, 3b, 4a

Daily Life 4. Fraction Conversion and Distribution in Cooking Activities	rational numbers in real-life contexts ▪ Solve contextual problems involving rational numbers	formulas and solve problems through accurate computational processes. 3. Interpret and communicate the results or solutions of mathematical problems effectively.	1c, 2c, 3c, 4b

Then the next step is to determine the mathematical procedures and to present the content of each problem that has been designed. The detailed problem statements will be provided in the development stage. Below are the concise mathematical procedures for solving each contextual problem.

Problem 1:
1. Convert Fika's and Rina's race times into decimal form.
2. Represent the times on a number line.
3. Compare the times to determine who is the fastest.

Problem 2:
1. Calculate the price of one slice of watermelon and one slice of melon.
2. Create a comparison table showing the fractional portion and the full-fruit price.
3. Compare the total cost with the amount of money the buyer has.

Problem 3:
1. Calculate the temperature difference.
2. Represent the temperature change on a number line (including negative and positive values).
3. Interpret the meaning of the temperature change in a real-life context.

Problem 4:
1. Determine how many muffins can be made by dividing the total batter by the amount required per mold.
2. Interpret the result of the division (including any remainder) in context.

Development

In the development stage, the contexts that had been designed were further developed into contextual problem designs aimed at assessing students' mathematical communication skills. The problems were constructed to be as closely related as possible to students' real-life experiences so that learning would become more meaningful, engaging their cognitive processes and promoting active mathematical communication. Table 2 are the four developed problem designs along with an analysis of their communicative potential:

Table 2. Rational Number Problem Design

Problem 1. Comparing Swimming Race Times (Context: Time Management)

After two full weeks of training, Fika and Rina participated in a 100-meter school swimming competition. The event was attended by all seventh-grade students. Fika swam very quickly, recording a time of 0.85 minutes, while Rina, who initially fell behind but caught up near the end, finished the race in $\frac{7}{8}$ minutes. Both students were confused because their physical education teacher reported their times in different forms. To determine who was faster, they needed to understand both fractional and decimal representations and visualize their results.

a. Convert Fika's and Rina's swimming times into equivalent decimal forms.
b. Represent their swimming times on a number line.
c. Based on your number line, who reached the finish line faster? Explain your reasoning.

Analysis:

The first problem integrates rational numbers in both fractional and decimal forms. Students are expected to:

- Convert time units from fractions to decimals.
- Use a number line representation to compare two rational numbers.
- Explain the time difference using logical and precise mathematical language.

This problem reflects the indicators of mathematical communication skills, particularly the ability to represent problem information visually and draw conclusions from the comparison of rational numbers.

Problem 2. Buying Fruit Slices at a Traditional Market (Context: Financial Literacy in Buying and Selling Activities)

Mrs. Narti sells fresh fruit at a traditional market. To attract customers, she sells the fruit in sliced portions. That day, she offered watermelon and melon in slices as follows:

- Each slice of watermelon is sold for $\frac{5}{8}$ of the whole-fruit price of Rp32,000.
- Each slice of melon is sold for $\frac{3}{5}$ of the whole-fruit price of Rp25,000.

A customer wants to buy one slice of each fruit but only has Rp30,000. The customer asks Mrs. Narti whether the money is enough to buy both slices.

a. Calculate the price of one slice of watermelon and one slice of melon.
b. Create a table showing the comparison between the price of each fruit slice and the full-fruit price.
c. Based on the table, determine whether the customer has enough money to buy both fruits. Explain your reasoning.

Analysis:
The second problem develops students' competence in using fractions and decimals within a financial literacy context. Students are expected to:

- Convert fractions into decimal form.
- Calculate monetary values based on fractions and percentages.
- Present data in a table and write a mathematical conclusion based on the results.

This problem trains students' mathematical communication skills in organizing financial information and expressing conclusions clearly based on numerical reasoning.

Problem 3. Temperature Changes During Camping in the Highlands (Context: Temperature Changes in Daily Life)

Arya went camping with his scout group in the Dieng Highlands. When he woke up at 4 a.m., he felt very cold. He recorded the temperature using his digital thermometer, which showed $-\frac{4}{5}$°C. Later, at 10 a.m., he measured the temperature again, and it had risen to 4°C, making the air feel slightly warmer. Arya became curious about how much the temperature had changed from 4 a.m. to 10 a.m. He also wanted to illustrate this change in his camping activity report.

a. Calculate the difference in temperature between 4 a.m. and 10 a.m.
b. Represent the temperature change on a number line showing both negative and positive values.
c. Explain the meaning of this temperature change in real-life terms and interpret it mathematically.

Analysis:
The third problem requires students to:

- Convert fractional and negative numbers into decimal form.
- Interpret the difference between two numbers located on opposite sides of the number line.
- Explain the meaning of negative values in daily life using mathematical reasoning.

This problem explicitly develops students' mathematical communication skills in using symbols and visual representations to explain real-world situations logically and meaningfully.

Problem 4. Dividing Muffin Batter (Context: Fraction Conversion and Division in Cooking Activities)

Mrs. Rina made $4\frac{1}{2}$ kilograms of muffin batter. Each muffin mold requires $\frac{1}{10}$ kilogram of batter.

a. How many muffins can be made by Mrs. Rina?
b. Is there any batter left after filling all the molds? Explain your reasoning.

Analysis:
The final problem sharpens students' ability to:

- Perform division involving whole numbers and fractions.
- Interpret the result of division in terms of whole numbers and remainders.
- Explain the meaning of the result and the remainder in a real-life context through written expression.

This problem integrates mathematical modeling, rational number computation, and mathematical communication, particularly in explaining non-whole-number results within a contextual situation.

DISCUSSION

The problem designs developed across four real-life contexts were constructed to promote conceptual understanding of rational numbers while simultaneously enhancing junior high school students' mathematical communication skills. The first problem presents a swimming race time comparison between Fika and Rina. In this problem, students are guided to compare 0.85 minutes and $\frac{7}{8}$ minutes (or 0.875 minutes), requiring them to convert between decimals and fractions and visualize the results on a number line. This design aims to strengthen students' ability to compare rational numbers both visually and symbolically, aligning with the findings of Morano et al. (2019), who stated that visual representations such as number lines help students (especially those with learning difficulties) grasp the concept of comparing decimals and fractions more concretely. Through this process, students learn to articulate their reasoning in written and visual forms, which enhances their mathematical communication fluency.

The second problem introduces basic financial literacy through the context of purchasing fruit slices. In this design, students are challenged to calculate fractional values of product prices and interpret the results in relation to a limited amount of money. The objective is to build the understanding that fractions are not merely abstract symbols but quantities that carry real meaning in everyday life. This approach aligns with Brown (2019), who emphasized the importance of understanding rational numbers in meaningful contexts. Such contextual reasoning encourages students to explain their problem-solving strategies, connecting mathematical symbols to everyday interpretations.

The third problem, which focuses on temperature changes, serves as a bridge for understanding negative numbers and their operations. In this task, students are asked to determine the difference between an initial temperature of $-\frac{4}{5}$°C (or –0.8°C) and a final temperature of 4°C. The computation would be 4 – (–0.8) = 4.8°C, is designed to help students conceptualize subtraction involving negative numbers using number line support. This design reflects the study of González-Forte et al. (2023), which demonstrated that visual representations are effective in addressing misconceptions caused by integer bias in the operation of negative and rational numbers. By interpreting and explaining their reasoning with reference to the number line, students strengthen both conceptual understanding and written mathematical justification.

The fourth problem centers on the division of mixed fractions in the context of dividing muffin batter. Students are expected to determine that $4\frac{1}{2} \div \frac{1}{10}$ = 45 muffins, resulting in a whole number without remainder. This design encourages students to use pictorial or fractional visual models as verification tools, supporting the argument of Pittalis (2025) that contextual learning activities reinforce students' conceptual schema of rational number operations. This form of verification promotes students' ability to communicate mathematical arguments coherently and justify the logic behind their solutions.

Mathematical communication ability is one of the key indicators of success in mathematics learning. NCTM emphasized that communication is not limited to presenting answers but also includes representing ideas, using correct notation, and providing logical and convincing reasoning (Jahangiri et al., 2022; Saleh Haji, 2019). This approach aligns with MacKay et al. (2023), who found that consistent visual structures such as number lines enhance rational number processing in both children and adults, as evidenced through eye-tracking studies. This finding reinforces that visual tools are not merely learning aids but essential components of mathematical communication. Therefore, designing learning tasks that integrate visualization and explanation simultaneously supports the dual goals of understanding and expressing mathematical reasoning.

Furthermore, mastering rational numbers is a crucial foundation for learning basic statistics. When students explore concepts such as proportion, relative frequency, and averages, they must handle numerical representations in fractions, decimals, and percentages. Students' difficulties in statistics often stem from challenges in converting and comparing fractions, decimals, and percentages, which directly affects the accuracy of computing means, medians, modes, and proportions (Kärki et al., 2022). As Pittalis (2025) also noted, rational understanding developed through concrete contexts helps students manage quantitative information effectively in statistics. Hence, developing communication-oriented rational number understanding at an early stage contributes directly to students' later competence in statistical reasoning.

Consistent with Schoenfeld (2020), students' mathematical thinking develops optimally when they are presented with authentic problems that require justification and reflection on their solutions. The use of real-world contexts as starting points, term didactical phenomenology, helps students move from informal models toward formal concepts (Rojo et al., 2023). Challenging problems stimulate active engagement, enabling students to construct understanding and communicate it effectively (Lee & Jung, 2025; Wijnia et al., 2024). In this sense, communication serves not only as a product of learning but as a process that deepens students' reflective and conceptual engagement with mathematics.

The four developed problems thus represent functional designs that reflect mathematical communication competence. Each problem contains a degree of cognitive complexity requiring students to transform representations, perform calculations, and construct logical interpretations. Therefore, the development of these problem designs not only demonstrates mastery of rational number concepts but also serves as a medium for strengthening 21st-century skills, particularly students' mathematical communication abilities.

IV. CONCLUSION

This study produced four rational number problem designs developed through the ADD (Analysis, Design, Development) stages, oriented toward improving junior high school students' mathematical communication skills. Each problem was designed within real-life contexts relevant to students' experiences, such as time management, financial spending, temperature changes, and dough distribution. These four problems reflect indicators of mathematical communication through activities such as expressing information in visual representations (tables, diagrams, number lines), using mathematical notations or symbols accurately, and formulating logical interpretations of results.

The developed problem designs aim to address students' difficulties in understanding rational numbers, which are often taught abstractly. By connecting the material to real-life contexts, students are encouraged to think critically, engage in discussions, and express their understanding mathematically in written form. The problems are not only oriented toward obtaining final answers but also emphasize the process of reasoning, argumentation, and students' ability to explain their thinking in writing. Moreover, rational numbers are closely related to basic statistics, particularly in data representation, proportion calculation, and result interpretation.

In general, this study provides practical contributions for teachers and mathematics educators by offering a variety of rational number problems oriented toward enhancing students' mathematical communication skills. These problem designs can serve as alternative assessment instruments as well as meaningful learning materials that connect mathematical concepts to real-world situations. Further research is recommended to continue the complete ADDIE development cycle, testing and implementing these problem designs systematically in classroom settings, and quantitatively measuring their impact on students' mathematical communication performance. Such follow-up studies are expected to provide deeper insights into the effectiveness, practicality, and scalability of contextual problem-based learning in mathematics education.

REFERENCES

1. Avvisati, F., & Ilizaliturri, R. (2023, December 5). *PISA 2022 Results (Volume I and II) - Country Notes: Indonesia*. Https://Www.Oecd.Org/En/Publications/Pisa-2022-Results-Volume-i-and-Ii-Country-Notes_ed6fbcc5-En/Indonesia_c2e1ae0e-En.Html.
2. Baroody, A. J. (1993). *Problem solving, reasoning, and communicating, K-8 : helping children think mathematically*. Merrill.
3. Braithwaite, D. W., & Siegler, R. S. (2021). Putting fractions together. *Journal of Educational Psychology*, *113*(3), 556–571. https://doi.org/10.1037/edu0000477
4. Branch, R. M. (2009). *Instructional Design: The ADDIE Approach*. Springer US. https://doi.org/10.1007/978-0-387-09506-6
5. Brown, B. (2019). Rational number understanding: The big picture, not the essence. *South African Journal of Childhood Education*, *9*(1). https://doi.org/10.4102/sajce.v9i1.561
6. Canogullari, A., & Radmehr, F. (2025). Task design principles in mathematics education: a literature review. *International Journal of Mathematical Education in Science and Technology*, 1–33. https://doi.org/10.1080/0020739X.2025.2457365
7. Cartwright, K. (2020). Analyzing students' communication and representation of mathematical fluency during group tasks. *The Journal of Mathematical Behavior*, *60*, 100821. https://doi.org/10.1016/j.jmathb.2020.100821
8. Cevikbas, M., & Kaiser, G. (2021). A Systematic Review on Task Design in Dynamic and Interactive Mathematics Learning Environments (DIMLEs). *Mathematics*, *9*(4), 399. https://doi.org/10.3390/math9040399
9. Chasanah, C., Riyadi, R., & Usodo, B. (2020). The Effectiveness of Learning Models on Written Mathematical Communication Skills Viewed from Students' Cognitive Styles. *European Journal of Educational Research*, *volume-9-2020*(volume-9-issue-3-july-2020), 979–994. https://doi.org/10.12973/eu-jer.9.3.979
10. Ezaki, J., Li, J., & Copur-Gencturk, Y. (2024). Teachers' Knowledge of Fractions, Ratios, and Proportional Relationships: the Relationship Between Two Theoretically Connected Content Areas. *International Journal of Science and Mathematics Education*, *22*(2), 235–255. https://doi.org/10.1007/s10763-023-10372-z
11. Fajri, H. M., Marini, A., & Suyono. (2025). Trends and patterns in realistic mathematics education research in elementary schools: A bibliometric approach. *Social Sciences & Humanities Open*, *12*, 101730. https://doi.org/10.1016/j.ssaho.2025.101730 González
12. Forte, J. M., Fernández, C., Van Hoof, J., & Van Dooren, W. (2023). Incorrect Ways of Thinking About the Size of Fractions. *International Journal of Science and Mathematics Education*, *21*(7), 2005–2025. https://doi.org/10.1007/s10763-022-10338-7
13. Greefrath, G., & Vorhölter, K. (2016). *Teaching and Learning Mathematical Modelling*. Springer International Publishing. https://doi.org/10.1007/978-3-319-45004-9
14. Halai, A., & Clarkson, P. (2016). *Teaching and Learning Mathematics in Multilingual Classrooms* (A. Halai & P. Clarkson, Eds.). SensePublishers. https://doi.org/10.1007/978-94-6300-229-5
15. Hidayat, W., & Aripin, U. (2023). How To Develop An E-Lkpd With A Scientific Approach To Achieving Students' Mathematical Communication Abilities? *Infinity Journal*, *12*(1), 85. https://doi.org/10.22460/infinity.v12i1.p85-100
16. Hidayati, Abidin, Z., & Ansari, B. I. (2020). Improving students' mathematical communication skills and learning interest through problem based learning model. *Journal of Physics: Conference Series*, *1460*(1), 012047.

https://doi.org/10.1088/1742-6596/1460/1/012047

17. Jahangiri, J., Oxman, V., & Stupel, M. (2022). Testing the NCTM 2020 Standards Using Rigorous Mathematics and Multiple Solutions to a Single Geometric Problem. *Resonance*, *27*(6), 1061–1077. https://doi.org/10.1007/s12045-022-1397-z
18. Johnson, H. L. (2022). Task design for graphs: rethink multiple representations with variation theory. *Mathematical Thinking and Learning*, *24*(2), 91–98. https://doi.org/10.1080/10986065.2020.1824056
19. Kärki, T., McMullen, J., & Lehtinen, E. (2022). Improving rational number knowledge using the NanoRoboMath digital game. *Educational Studies in Mathematics*, *110*(1), 101–123. https://doi.org/10.1007/s10649-021-10120-6
20. Lee, A., & Jung, E. (2025). University students' subjective experiences with problem-based learning and associated generic skills. *Frontiers in Psychology*, *16*. https://doi.org/10.3389/fpsyg.2025.1618997
21. Lenz, K., Reinhold, F., & Wittmann, G. (2024). Transitions between conceptual and procedural knowledge profiles. Patterns in understanding fractions and indicators for individual differences. *Learning and Individual Differences*, *116*, 102548. https://doi.org/10.1016/j.lindif.2024.102548
22. MacKay, K. J., Germeys, F., Van Dooren, W., Verschaffel, L., & Luwel, K. (2023). The structure of the notation system in adults' number line estimation: An eye-tracking study. *Quarterly Journal of Experimental Psychology*, *76*(3), 538–553. https://doi.org/10.1177/17470218221094577
23. McMullen, J., Hannula-Sormunen, M. M., Lehtinen, E., & Siegler, R. S. (2022). Predicting adaptive expertise with rational number arithmetic. *British Journal of Educational Psychology*, *92*(2), 688–706. https://doi.org/10.1111/bjep.12471
24. Mirna, M., Yulanda, S., Martin, S. N., Jamaris, J., & Solfema, S. (2023). Analisis Peningkatan Kemampuan Komunikasi Matematis Siswa dengan Penerapan Model Problem Based Learning. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, *7*(1), 645–657. https://doi.org/10.31004/cendekia.v7i1.1922
25. Morano, S., Riccomini, P. J., & Lee, J. (2019). Accuracy of Area Model and Number Line Representations of Fractions for Students with Learning Disabilities. *Learning Disabilities Research & Practice*, *34*(3), 133–143. https://doi.org/10.1111/ldrp.12197
26. Musdi, E., Syaputra, H., Arnellis, & Harisman, Y. (2024). Students' mathematics communication behavior: Assessment tools and their application. *Journal on Mathematics Education*, *15*(1), 317–338. https://doi.org/10.22342/jme.v15i1.pp317-338
27. NCTM. (2000). *Principles and standards for school mathematics*. Https://Www.Nctm.Org/Standards-and-Positions/Principles-and-Standards/.
28. Nguyen, H. T. M., Nguyen, G. T. C., Nguyen, B. N., Truong, D. T., & Thai, L. T. H. (2025). Mathematical Communication Skills for High School Students: A Comprehensive Review of Present Practices, Challenges, and Prospective Advancements. *TEM Journal*, 2349–2360. https://doi.org/10.18421/TEM143-40
29. Nguyen, T.-T., Nguyen, N. D., Trinh, T. P. T., Tong, D. H., Uyen, B. P., & Han, N. N. (2024). Vietnamese sixth graders' mathematical communication competency developed by teaching fraction topics using the 5E model. *Heliyon*, *10*(20), e39440. https://doi.org/10.1016/j.heliyon.2024.e39440
30. OECD. (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*. OECD. https://doi.org/10.1787/53f23881-en
31. Pittalis, M. (2025). An empirically validated rational number sense framework. *Mathematics Education Research Journal*, *37*(1), 125–160. https://doi.org/10.1007/s13394-024-00484-2
32. Planas, N., & Pimm, D. (2024). Mathematics education research on language and on communication including some distinctions: Where are we now? *ZDM – Mathematics Education*, *56*(1), 127–139. https://doi.org/10.1007/s11858-023-01497-0
33. Rojo, M., King, S., Gersib, J., & Bryant, D. P. (2023). Rational Number Interventions for Students With Mathematics Difficulties: A Meta-Analysis. *Remedial and Special Education*, *44*(3), 225–238. https://doi.org/10.1177/07419325221105520
34. Rosenberg-Lee, M., Varma, S., Cole, M. W., & Abreu-Mendoza, R. A. (2023). Competing numerical magnitude codes in decimal comparison: Whole number and rational number distance both impact performance. *Cognition*, *241*, 105608. https://doi.org/10.1016/j.cognition.2023.105608
35. Saleh Haji. (2019). NCTM's Principles and Standards for Developing Conceptual Understanding in Mathematics. *Journal of Research in Mathematics Trends and Technology*, *1*(2), 56–65. https://doi.org/10.32734/jormtt.v1i2.2836
36. Schiller, L. K., & Siegler, R. S. (2023). Integrated knowledge of rational number notations predicts children's math achievement and understanding of numerical magnitudes. *Cognitive Development*, *68*, 101380. https://doi.org/10.1016/j.cogdev.2023.101380
37. Schoenfeld, A. H. (2020). Reframing teacher knowledge: a research and development agenda. *ZDM*, *52*(2), 359–376. https://doi.org/10.1007/s11858-019-01057-5

38. Sulastri, E. (2023). Mathematical communication skills of junior high school students based on statistics. *Jurnal Inovasi Pembelajaran Matematika: PowerMathEdu*, *2*(3), 373–384. https://doi.org/10.31980/pme.v2i3.1770
39. Sumarmo. (2006). *Penilaian Pembelajaran Matematika*. Refika Aditama
40. Toalongo, X., Trelles, C., & Alsina, Á. (2022). Design, Construction and Validation of a Rubric to Evaluate Mathematical Modelling in School Education. *Mathematics*, *10*(24), 4662. https://doi.org/10.3390/math10244662
41. Tong, D. H., Uyen, B. P., & Van Anh Quoc, N. (2021). The improvement of 10th students' mathematical communication skills through learning ellipse topics. *Heliyon*, *7*(11), e08282. https://doi.org/10.1016/j.heliyon.2021.e08282
42. Van der Wal, N. J., Bakker, A., & Drijvers, P. (2023). Designing an instrument to measure the development of techno-mathematical literacies in an innovative mathematics course for future engineers in STEM education. *ZDM –Mathematics Education*, *55*(7), 1243–1254. https://doi.org/10.1007/s11858-023-01507-1
43. Webb, L., & Webb, P. (2016). Developing Mathematical Reasoning in English Second-Language Classrooms Based on Dialogic Practices. In *Teaching and Learning Mathematics in Multilingual Classrooms* (pp. 195–209). SensePublishers. https://doi.org/10.1007/978-94-6300-229-5_13
44. Wijnia, L., Noordzij, G., Arends, L. R., Rikers, R. M. J. P., & Loyens, S. M. M. (2024). The Effects of Problem-Based, Project-Based, and Case-Based Learning on Students' Motivation: a Meta-Analysis. *Educational Psychology Review*, *36*(1), 29. https://doi.org/10.1007/s10648-024-09864-3
45. Zhang, Y., Xin, J. J., Yu, Z., Liu, Y., Zhao, W., Li, N., Li, Y., & Chen, G. (2025). Enhancing Preservice Teachers' Use of Dialogic Teaching and Dynamic Visualizations in Mathematics Classes: Bridging the Knowing–Doing Gap. *International Journal of Science and Mathematics Education*. https://doi.org/10.1007/s10763-025-10558-7