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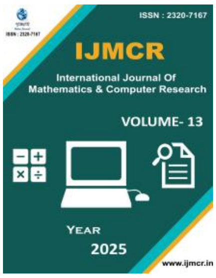

A Multi-Objective Assignment Problem Model to Optimize Task Allocation with the Help of Weight Sum Method

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ABSTRACT

This study addresses the Multi-Objective Assignment Problem (MOAP) involving five tasks and six team members with three distinct optimization objectives i.e. minimizing cost, time and risk. The weight sum method is applied to transform the multi-objective into a single objective model, reflecting priority-based preferences. The Hungarian method is used to derive optimal solutions for each priority cases. Result demonstrate that varying priorities significantly impact task allocation patterns and overall efficiency, Offering valuable insights for balanced decision-making in resource management.

KEYWORDS: Multi-Objective Assignment Problem, Weighted Sum Method, Hungarian Method, Optimization.

Subject Qualification Code: MSC 90C27, MSC 90B50

1. INTRODUCTION

A general assignment problem includes N tasks that must assign to N workers where each worker has the competence to do all tasks. The objective is to assign each task a proper worker so that the total resource spending to finish all tasks can be "minimized". Many researches focus on the above problems with multi-objective and try to provide better solution. Unfortunately, the provided approaches are difficult to execute in real situation.

Many researches have been developed to solve the assignment problem [1-9]. Most of the developed methods for the assignment problem consider only one-objective situation, such as (a) the minimum cost assignment problem, (b) the minimum finishing time assignment problem. The minimum cost assignment problem focuses on how to assign tasks to workers so that the total operation cost can be minimized. Such problems have been generally discussed and well developed in many operations research textbooks and relevant papers [1-7]. While the posterior problem may not focus on how to assign the tasks to workers so that the total operation time can be minimized, sometime it focus on how to minimize the finishing time of the last worker [8-9]. here are many researches also focus on the above problems and try to provide better solution. Using multiple-objective method, Geetha et al. [10] provide a solution for an assignment problem that minimizes both time and cost. Tsai et al. [11] try to solve a multi-objective decision making problem associated with cost, time, and quality by fuzzy concept. Unfortunately, the provided approaches deal only on the 2-objective assignment problem, or solve the problem by relatively complex method, which is difficult to execute in real situation.

To conquer above shortages, this work uses weight sum method to change a multiple-objective assignment problem into a single objective assignment problem. Moreover, the apply Hungarian method can handle non-quantification or quality situation. In the prerequisite that the tasks can be assigned effectively, we try to minimize the use of all kinds of resources and finish the assignment efficiently.

2. MULTI-OBJECTIVE ASSIGNMENT PROBLEM:

A Multi-Objective Assignment Problem (MOAP) is an extension of the classical assignment problem where multiple, often conflicting; objectives are optimized simultaneously. In this context, we are considering three objectives.

Objectives:

Let $o_{ij}^1, o_{ij}^2, o_{ij}^3, \cdots o_{ij}^r$ represent the cost coefficients associated with the r objectives for transporting from source i to destination j.

The multi-objective transportation problem can be stated as:

$$Min\ or\ Max\ Z_1 = \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^1 q_{ij}$$

$$Min\ or\ Max\ Z_2 = \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^2 q_{ij}$$

$$Min\ or\ Max\ Z_3 = \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^3 q_{ij}$$

$$\vdots \qquad \vdots$$

$$Min\ or\ Max\ Z_r = \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^r q_{ij}$$

Where$Z_1, Z_2, Z_3 \ldots\ldots Z_r$ are the objectives to be minimized or maximized.

Decision Variables:

Let q_{ij} represent the amount of goods transported from source i to destination j, where:

$i = 1, 2, \cdots, p$ (number of sources)

$j = 1, 2, \cdots, q$ (number of destinations)

Constraints:

1. ***Resources availability***:

$\sum_{j=1}^{q} q_{ij} = 1 \qquad \forall\ i = 1, 2, \cdots, p$

2. ***Activity requirement***:

$\sum_{i=1}^{p} q_{ij} = 1 \qquad \forall\ j = 1, 2, \cdots, q$

3. ***Non-Negativity Constraints***:

$q_{ij} = 0\ or\ 1 \qquad \forall\ i = 1, 2, \cdots, p;\ j = 1, 2, \cdots, q$

3. WEIGHTED SUM METHOD:

Since Multi-Objective Assignment Problem have multiple objectives, one way to handle them is to convert the multi-objective problem into a single-objective problem using weighted sum method. Solution Procedure for MOAP as follows:

Step 1: Firstly, we checked the our MOAP is balanced. i.e. number of row and column are equal or not. If number row and column are equal then move to step 3.

Step 2: If number of rows is not equal to number of columns, then add dummy rows or columns with cost 0, to make it a square matrix.

Step 3: Weights are assigned $w_1, w_{2,} w_{3,} \cdots w_r$ to each objective to reflect their relative importance according to requirement of customers. The weight must be satisfied the following condition:

$$w_1 + w_{2,} + \cdots + w_r = 1$$

i.e

$$\sum_{l=1}^{r} w_r = 1$$

Step 4: Multiple objective change into the single objective by using assign weights, which know as weighed objective function. The new objective function to be minimized or maximized i.e.

$$Z = w_1 . \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^1 q_{ij} + w_2 . \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^2 q_{ij} + w_3 . \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^3 q_{ij} + \cdots + w_r . \sum_{i=1}^{p} \sum_{j=1}^{q} o_{ij}^r q_{ij}$$

The constraints considered as standard transportation problem;

Resource availability: $\sum_{j=1}^{q} q_{ij} = 1 \qquad \forall\ i = 1, 2, \cdots, p$

Activity requirement: $\sum_{i=1}^{p} q_{ij} = 1 \qquad \forall\ j = 1, 2, \cdots, q$

Non-Negativity Constraints: $q_{ij} = 1\ or\ 0 \qquad \forall\ i = 1, 2, \cdots, p;\ j = 1, 2, \cdots, q$

Step 5: To solve single objective optimization problem, we are applying Hungarian Method to obtained optimal solution of the problem.

4. NUMERICAL PROBLEM:

A Software company wants to assign five tasks to six available team members. Optimize three objectives simultaneously: minimize time (hours), minimize cost (Rs Thousands) and minimize the delivery risk. Each member can take at most one task; one member builds three objective matrices with notations as follows:

Table 1: Task List

S No	Task Notation	Name of Task
1	T1	Requirement & Planning
2	T2	Data Collection & Cleaning
3	T3	Algorithm Development
4	T4	UI/Prototype
5	T5	Testing & Documentation

Table 2: Multi-Objective Assignment Problem

	Task	T1	T2	T3	T4	T5
M1	Cost	12	18	14	10	16
	Time	8	12	10	6	11
	Risk	6	8	7	9	7
M2	Cost	14	16	13	12	15
	Time	10	11	9	7	10

	Risk	7	7	8	8	7
M3	Cost	10	14	12	13	11
	Time	7	10	8	8	9
	Risk	9	8	8	7	9
M4	Cost	11	13	15	12	14
	Time	9	9	10	7	8
	Risk	8	9	6	8	7
M5	Cost	13	15	14	13	12
	Time	10	11	9	8	7
	Risk	7	7	8	9	6
M6	Cost	15	17	11	14	13
	Time	11	13	7	9	8
	Risk	6	6	9	7	8

According to step 1 all the objectives of problems are unbalanced i.e. they have same number of row and columns are not equal. Now according to Step 2 we going to add dummy column to convert our unbalanced problem into balanced problem and obtained following table 3 as follows

Table 3: Balanced Multi-Objective Assignment Problem

	Task	T1	T2	T3	T4	T5	T6
M1	Cost	12	18	14	10	16	0
	Time	8	12	10	6	11	0
	Risk	6	8	7	9	7	0
M2	Cost	14	16	13	12	15	0
	Time	10	11	9	7	10	0
	Risk	7	7	8	8	7	0
M3	Cost	10	14	12	13	11	0
	Time	7	10	8	8	9	0
	Risk	9	8	8	7	9	0
M4	Cost	11	13	15	12	14	0
	Time	9	9	10	7	8	0
	Risk	8	9	6	8	7	0
M5	Cost	13	15	14	13	12	0
	Time	10	11	9	8	7	0
	Risk	7	7	8	9	6	0
M6	Cost	15	17	11	14	13	0
	Time	11	13	7	9	8	0
	Risk	6	6	9	7	8	0

Now, in this sequence we are going to consider three different scenarios to change multiple objectives into a single objective to aligning with the established priority of objectives. Priority of the objectives are considered as follows

Table 4: Weightage of each objective according to priority

Priority	w_1	w_2	w_3
Case I	0.6	0.2	0.2
Case II	0.1	0.3	0.6
Case III	0.4	0.1	0.5

As mentioned all the weights of each case are satisfied by the equation (2.5). i.e.

$$\sum_{l=1}^{3} w_l = 1$$

As mentioned in Step 4, To change the multiple objectives of the above-mentioned tables into single objective assignment, we are going to assign weight to each objective according to the priority of condition which is defined in Table 4. We used the equation (2.6) as follows to combine all the three objectives

$$Z = w_1.\sum_{i=1}^{5}\sum_{j=1}^{5} o_{ij}^1 q_{ij} + w_2.\sum_{i=1}^{5}\sum_{j=1}^{5} o_{ij}^2 q_{ij} + w_3.\sum_{i=1}^{5}\sum_{j=1}^{5} o_{ij}^3 q_{ij}$$

With the help of the above equation, we are going to obtain a single objective function for all ranking functions as follows

Table 5: According to priority I assignment problem

	T1	T2	T3	T4	T5	T6
M1	10	14.8	11.8	9	13.2	0
M2	11.8	13.2	11.2	10.2	12.4	0
M3	9.2	12	10.4	10.8	10.2	0
M4	10	11.4	12.2	10.2	11.4	0
M5	11.2	12.6	11.8	11.2	9.8	0
M6	12.4	14	9.8	11.6	11	0

Table 6: According to priority II Assignment Problem

	T1	T2	T3	T4	T5	T6
M1	7.2	10.2	8.6	8.2	9.1	0
M2	8.6	9.1	8.8	8.1	8.7	0
M3	8.5	9.2	8.4	7.9	9.2	0
M4	8.6	9.4	8.1	8.1	8	0
M5	8.5	9	8.9	9.1	6.9	0
M6	8.4	9.2	8.6	8.3	8.5	0

Table 7: According to priority III Assignment Problem

	T1	T2	T3	T4	T5	T6
M1	8.6	12.4	10.1	9.1	11	0
M2	10.1	11	10.1	9.5	10.5	0
M3	9.2	10.6	9.6	9.5	9.8	0
M4	9.3	10.6	10	9.5	9.9	0
M5	9.7	10.6	10.5	10.5	8.5	0
M6	10.1	11.1	9.6	10	10	0

According to Step 5, we are going to apply Hungarian method to obtained optimal solution in Table No 5, 6, & 7 and then found the allocations of task and optimal solution as follows.

Table 8: According to priority I allocation table

	T1	T2	T3	T4	T5	T6
M1	0.8	3.4	2	**0**	3.4	0
M2	2.6	1.8	1.4	1.2	2.6	**0**
M3	**0**	0.6	0.6	1.8	0.4	0
M4	0.8	**0**	2.4	1.2	1.6	0
M5	2	1.2	2	2.2	**0**	0
M6	3.2	2.6	**0**	2.6	1.2	0

Now we obtained the optimal solution of priority I as

Table 8 (a): According to priority I Optimal Solution

Member	M1	M2	M3	M4	M5	M6	COST
Task	T4	T6	T1	T2	T5	T3	
Cost	9	0	9.2	11.4	9.8	9.8	49.2

Table 9: According to priority II allocation table

	T1	T2	T3	T4	T5	T6
M1	**0**	1.1	0.5	0.3	2.1	0
M2	1.4	**0**	0.7	0.2	1.7	**0**
M3	1.3	0.1	0.3	**0**	2.2	0
M4	1.4	0.3	**0**	0.2	1	0
M5	1.4	0	0.9	1.3	**0**	0.1
M6	1.2	0.1	0.5	0.4	1.5	**0**

Now we obtained the optimal solution of priority II as

Table 9 (a): According to priority I Optimal Solution

Work	A	B	C	D	E	F	COST
Task	T1	T2	T3	T4	T5	T6	
Cost	7.2	9.1	7.9	8.1	6.9	0	39.2

Table 10: According to priority III allocation table

	T1	T2	T3	T4	T5	T6
M1	**0**	2.2	0.9	0	2.9	0.4
M2	1.1	0.4	0.5	0	2	**0**
M3	0.2	0	0	**0**	1.3	0
M4	0.3	**0**	0.4	0	1.4	0
M5	0.7	0	0.9	1	**0**	0
M6	1.1	0.5	**0**	0.5	1.5	0

Now we obtained the optimal solution of priority III as

Table 10 (a): According to priority I Optimal Solution

Work	A	B	C	D	E	F	COST
Task	T1	T6	T4	T2	T5	T3	
Cost	8.6	0	9.5	10.6	8.5	9.6	46.8

5. RESULT AND DISCUSSION:

The weight sum approach effectively transformed three objectives into single-objective functions for computational efficiency. The Hungarian method generated optimal assignments under three priority scenarios. Results indicated that cost values varied across priorities 49.2, 39.2, and 46.8 demonstrating trade-offs between objectives. Allocation patterns showed that different task aligned with different members depending on priority. This highlights the flexibility of multi-objective optimization and its ability to balance time, cost and risk in complex decision-making processes.

6. CONCLUSION

The research highlights the effectiveness of integrating the weighted sum approach with the Hungarian method for solving multi-objective assignment problems. By assigning priorities across time, cost and risk, organizations can achieve flexible and efficient task allocation tailored to strategic needs. The results confirm that varying objective priorities produce different optimal allocations, emphasizing the importance of adaptable models in real-world decision-making. The framework contributes to better utilization of resources, improvement project performance and offers a practical foundation for future extensions in multi-criteria optimization.

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