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Optimization of Fuzzy Transportation Problems using Triangular Fuzzy Numbers and Ranking Techniques

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ABSTRACT
This study addresses a fuzzy transportation problem model with triangular fuzzy numbers and transformed onto crisp number with the help of three ranking methods: Pascal Triangular, Sub Interval and Magnitude Ranking Methods. Here feasible and optimal solutions are derived using Russell's Approximation Method and the MODI methods respectively. Comparative analysis reveals significant variations in costeffectiveness across ranking techniques. The research highlights the effectiveness of different approaches in handling fuzziness and provides insights into selecting the most suitable method for achieving optimal decisionmaking in transportation problems under uncertainty.

KEYWORDS: Fuzzy Transportation Problem, Triangular Numbers, Ranking function
Subject Qualification Code: MSC 90B06, MSC 90C70

1. INTRODUCTION

Transportation issue which illustrates the linear programming problem which is proposed by Hitchcock in 1941. In which the emergence of randomness and imprecision in the job is unavoidable due to certain unanticipated occurrences. In actual word circumstance occasionally the decision maker cannot offer their response either yes or no, then the new term arrives is called hazy. The word fuzzy which is invented by Lotfi Zade (1965) at University of California in Berkley. In other phrase we may state that anything which is not clear of a picture containing forms that do not have any distinct edge. By Lotfi Zade was the originator to construct the fuzzy sets in order to mathematically portray imprecision or vagueness in everyday life

According to study conducted by M. K. Purushothkumar, M. Ananthanarayanan and S. Dhanasekar [5] presented a technique of Fuzzy zero suffix algorithms to tackle totally fuzzy transportation issues. Roy etal [7] worked on fuzzy transportation issue utilizing zero point approach with ranking of trapezoidal fuzzy number. They identify the best solution for the transportation system by taking into account the closed, bounded, and non-empty feasible region of the transportation problem using fuzzy trapezoidal numbers. The problem is solved in two stages: in the first stage, a fuzzy transportation problem is made into a clear system through the application of a ranking of trapezoidal numbers. In the second step, the zero-point method is utilized to deal with the crisp transportation issue, and the North-West Corner method is examined for comparison. Based on the findings of a study by Nagastiti et al. [6] that compared the theories of the zero point method and the zero suffix method in determining the optimal solution, it is possible to draw the conclusion that the process of using the zero suffix method is more efficient in determining an optimal solution in 6 steps than the process of using the zero point method in 9 steps. A. Edward Samuel [2] proposed an improved zero pint method for the transportation problems. A. Edward Samuel and M. Venkatachalapathy [1] investigate a technique for enhancing the modified zero pointapproach for imbalanced fuzzy transportation issues. T. Karthy and K, Ganesan [12] suggested a new approach named by increased zero point method for the trapezoidal fuzzy transportation issues. P. Jayaraman and R. Jahairhussian [3] solve a fuzzy optimum transportation issue by increased zero suffix using Robust ranking approach. When the inventory and the interest of hubs, as well as the limit and cost edges, are addressing fuzzy numbers, the authors of this study proposed a positioning procedure for the purpose of solving the fuzzy transportation problem. This procedure assumes that the fuzzy interest and supply are in the form of triangular and trapezoidal fuzzy numbers. The authors' goal was to determine the least expensive method of transporting specific goods through a capacitated network. L. Kane et al. [4] proposed a two-step strategy for defining a fuzzy transportation problem that used polyhedral and hexagonal fuzzy numbers. Rubeelamary S. and Sivaranjani S. [10] find a fuzzified ranking of incident fuzzy numbers that can be utilized to predict a fuzzy min-max transportation problem. In order to find the best options, they executed their similar assessments of the proposed system with computation, such as Russell's Method, the Northern Parts Approach, the Least Cost Technique, and Vogel's Estimation Techniques.

This allowed them to determine which algorithms provided the most accurate results.

2. PRELIMINARIES

2.1 Fuzzy Set: Let *X* be a space point, with a generic element of *X* denoted by *x* Thus *X* = *{X}* A fuzzy set A in *X* is characterized by a membership function $f_A(X)$ which associated with each point in *X* a real number in the interval [0, 1], with the values of $f_A(x)$ at a representing the "grade of membership" of x in A. Thus, nearer the value of $f_A(x)$ to unity the higher grade of membership of *x* in A.

2.2 Fuzzy Transportation Problem: The Fuzzy Transportation problem is one of the special kinds of Fuzzy linear programming problems. A Fuzzy transportation problem is a transportation problem in which the transportation costs. Supply and demands quantities are in Fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal.

Let a transportation problem with *m* Fuzzy origins and *n* Fuzzy destinations.

Let $C_{ij} = \left(c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}\right)$ be the transportation cost of one unit of the product from Fuzzy origins i[th] to Fuzzy destination j[th].

$a_i = \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}\right)$ be the quantity of commodity available at Fuzzy origin i,

$b_j = \left(b_j^{(1)}, b_j^{(2)}, b_j^{(3)}\right)$ be the quantity of commodity needed at Fuzzy destination j

$X_{ij} = \left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}\right)$ is quantity transported from i[th] Fuzzy origins to j[th] Fuzzy destination.

The linear programming model representing the Fuzzy transportation is given by

Minimize = $Z = \sum_{i=1}^{m} \quad \sum_{j=1}^{n} \quad \left(c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}\right)\left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}\right)$

Subject to the constraint

$$\sum_{j=1}^{n} \quad \left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}\right) = \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}\right) \quad \textit{for } i = 1,2,....,m$$

$$\sum_{j=1}^{n} \quad \left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}\right) = \left(b_j^{(1)}, b_j^{(2)}, b_j^{(3)}\right) \quad \textit{for } j = 1,2,....,n$$

$$\left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right) \geq 0$$

The given Fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^{m} \quad \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right) = \sum_{j=1}^{n} \quad \left(b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right)$$

i.e if the total Fuzzy capacity is equal to the total Fuzzy demand.

2.3 Triangular Fuzzy Numbers: A number $\tilde{A}$ is a triangular fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3)$. where a_1, a_2, a_3 are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given

$$\mu_{\tilde{A}}(x) = \{0 \qquad for\ a < x_1 \ \frac{x - a_1}{a_2 - a_1} \qquad for\ a_1 \leq x \leq a_2 \ \frac{a_3 - x}{a_3 - a_2} \qquad for\ a_2 \leq x \leq a_3 \ 0 \qquad for\ x > a_3$$

By using min and max, we have an alternative expression for the proceeding equation:

triangle (x; a,b,c) = max $\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}, \right)\right)$

2.4 Arithmetic Operation of Fuzzy Number: The following are the four operations that can be performed on triangular fuzzy numbers: let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

- **Addition:** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- **Subtraction:** $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
- **Multiplication:** $\tilde{A} \times \tilde{B} = ((a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2,$
$(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$
- **Division:** $\frac{\tilde{A}}{\tilde{B}} = ((\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}), \frac{a_2}{b_2}, max(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}))$

2.5 Ranking Method for Triangular Fuzzy Number

Fuzzy number ranking techniques are essential for problem-solving, optimization, and decision-making in unpredictable situations. Ranking fuzzy numbers aids in comparing and prioritizing options in domains such as artificial intelligence, economics, and engineering since they reflect imprecise or ambiguous data. To maintain accuracy and consistency, different fuzzy number types—such as triangular, trapezoidal, and Gaussian—need particular ranking strategies. α-cut approaches, distance-based ranking, and centroid-based ranking are common techniques. By lowering ambiguity, an efficient ranking technique improves decision-making processes by offering a trustworthy means of sorting fuzzy numbers. For real-world problem-solving in uncertain circumstances, proper ranking is crucial because it improves computing efficiency, guarantees realistic comparisons, and supports systems like assessing risks, selecting suppliers, and multi-criteria decision-making.

Here are several commonly used ranking methods for triangular fuzzy numbers (a_1, a_2, a_3) to change into crisp numbers:

2.5.1 Pascal Triangular Method [7]: This technique utilizes the coefficients form pascal's triangular to assign graded weights to the triangular fuzzy number (TFN) components during the defuzzification process, thereby facilitating a systematic and balanced approach to fuzzy numbers. The Pascal triangular graded mean $A = (a_1, a_2, a_3)$ is defined as

$$P(a_1, a_2, a_3) = \frac{a_1+2a_2+a_3}{4} \quad (2.1)$$

2.5.2 Sub Interval Average[11]: By dividing a triangle fuzzy number into smaller intervals and averaging their midpoints for defuzzification, the Sub Interval Average technique improves decision-making accuracy by capturing interval variability. The formula $A = (a_1, a_2, a_3)$ is stated as follows

$$R(a_1, a_2, a_3) = \frac{4(a_1,a_2,a_3)}{12} \quad (2.2)$$

2.5.3 Magnitude Ranking: [7] A mathematical equation employing the centroid distance to determine the magnitude of TFNs. This guarantees an interpretable and consistent ranking for fuzzy decision analysis and is displayed as follows:

$$Mag\ (a_1, a_2, a_3) = \frac{1}{2}\int_0^1 \quad (a_3 + 3a_1 - a_2)rdr \quad (2.3)$$

2.7 Russell's Approximation Method:

The revised simplex algorithm can be expressed in the following steps:

Step 1: Convert Triangular Fuzzy numbers into crisp fuzzy numbers by Ranking Function

Step 2: If Transportation Problem is balanced i.e. sum of supply and sum of demand are equal move to step 4 to obtained feasible solution. Otherwise go to step 3.

Step 3: If Transportation problem is unbalanced then we are adding a dummy row and column to convert the unbalanced problem to a balanced problem.

Step 4: For each source row still under consideration, determine its $\underline{U}_i$(largest cost in row *i*).

Step 5: For each destination column still under consideration, determine its $\underline{V}_j$(largest cost in column *j*).

Step 6: for each variable, calculate $\Delta_{ij} = c_{ij} - (\underline{U}_i + \underline{V}_j)$

Step 7: Select the variable having the most negative Δ value, break ties arbitrarily.

Step 8: Allocate as much as possible. Eliminate necessary cells from consideration. Return to Step-4.

Step 9: Apply Modi Method to optimal solution.

3. NUMERICAL PROBLEM

In the section to demonstrate the proposed method we are considering the two numerical problems in which one of them is balanced fuzzy transportation problem and second is unbalanced transportation problem.

Table 1: Fuzzy Transportation Problem

	S1	S2	S3	S4	S5	**Demand**
D1	(6,10,14)	(9,14,19)	(10,12,15)	(10,14,19)	(5,10,16)	**(10,12,15)**
D2	(7,10,14)	(8,12,15)	(4,6,12)	(3,5,8)	(9,14,20)	**(9,14,19)**

D3	(4,8,12)	(6,11,17)	(9,13,16)	(3,6,11)	(9,13,19)	**(7,11,16)**
D4	(3,6,12)	(8,11,14)	(6,10,13)	(3,5,10)	(3,5,11)	**(4,9,14)**
Supply	**(8,11,14)**	**(3,7,11)**	**(5,7,11)**	**(7,9,11)**	**(9,12,15)**	

According to step 1 we are going to apply pascal triangular, sub interval and magnitude ranking methods to change the fuzzy triangular number into the crisp number, then obtained following table

Table 2: Transportation Problem after Pascal Triangular Method

	S1	S2	S3	S4	S5	**Demand**
D1	10.00	14.00	12.25	14.25	10.25	**12.25**
D2	10.25	11.75	7.00	5.25	14.25	**14.00**
D3	8.00	11.25	12.75	6.50	13.50	**11.25**
D4	6.75	11.00	9.75	5.75	6.00	**9.00**
Supply	**11.00**	**7.00**	**7.50**	**9.00**	**12.00**	

Table 3: Transportation Problem after Sub Interval Method

	S1	S2	S3	S4	S5	**Demand**
D1	10.00	14.00	12.33	14.33	10.33	**12.33**
D2	10.33	11.67	7.33	5.33	14.33	**14.00**
D3	8.00	11.33	12.67	6.67	13.67	**11.33**
D4	7.00	11.00	9.67	6.00	6.33	**9.00**
Supply	**11.00**	**7.00**	**7.67**	**9.00**	**12.00**	

Table 4: Transportation Problem after Magnitude Ranking

	S1	S2	S3	S4	S5	**Demand**
D1	5.50	8.00	8.25	8.75	5.25	**8.25**
D2	6.25	6.75	4.50	3.00	8.25	**8.00**
D3	4.00	6.00	7.50	3.50	8.25	**6.50**
D4	3.75	6.75	5.25	3.50	3.75	**4.25**
Supply	**6.75**	**3.25**	**4.75**	**5.75**	**7.50**	

From Step 2 our transportation problem is balanced, so move to step 4 to obtained feasible solution, after the implementation of Step 4 to Step 8 we obtained feasible solution for both ranking formulas as follows

Table 5: Feasible Solution of Transportation Problem

S No	Ranking Method	Feasible Solution
1	Pascal Triangular Method	Rs 377.88
2	Sub Interval Method	Rs 386.01
3	Magnitude Ranking	Rs 124.5

Now according to Step 9 we are going to apply MODI method to obtained the optimal of our problem, then obtained the optimal solution as follows

Table 6: Optimal Solution of Transportation Problem

S No	Ranking Method	Feasible Solution
1	Pascal Triangular Method	Rs 372.88
2	Sub Interval Method	Rs 381.61
3	Magnitude Ranking	Rs 119.81

4. RESULT AND DISCUSSION

The feasible and optimal solution obtained highlight the variability among ranking methods. As per table no 5 pascal Triangular Method produced a feasible solution of Rs 377.88, Sub Interval Method Rs 386.01 and Magnitude Ranking Rs 124.5 Similarly, Table No 6 shows the optimal solutions of Rs 372.88, Rs 381.61 and Rs 119.81 respectively. Results indicate that while Magnitude Ranking yields the lowest cost, Pascal Triangular and Sub Interval Methods ensure more stable and practical outcomes. Thus, method selection significantly influences decision-making efficiency in fuzzy transportation problems.

5. CONCLUSION

The comparative analysis demonstrates that the choice of ranking method greatly affects both feasible and optimal transportation costs. Although the Magnitude Ranking method delivers the lowest numerical values, Pascal Triangular and Sub Interval methods provide more reliable and consistent results for real-world applications. This research concludes that integrating fuzzy set theory with appropriate ranking and optimization techniques can significantly enhance decision-making under uncertainty. Future studies may explore hybrid approaches combining ranking methods to balance cost-effectiveness and stability in solving complex transportation problems.

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