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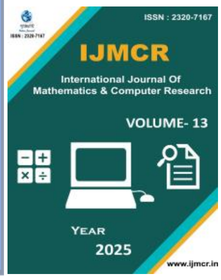

Design Worked Examples for Mathematical Problem Solving in Real-Life Contexts: A Focus on Learning the Pythagorean Theorem

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ARTICLE INFO	ABSTRACT
Published Online: **23 October 2025** Corresponding Author: **Yuni Apriani**	Students often struggle to solve mathematical problems related to real-life contexts, particularly in topics such as the Pythagorean theorem. Worked examples are an effective learning strategy for novice students, as they gradually guide learners through problem-solving steps while reducing extraneous cognitive load. This study aims to design worked examples for mathematical problem solving in real-life contexts on learning the Pythagorean theorem through three stages: analysis, design, and development. In the analysis stage, the material content, learning objectives, problem-solving indicators, and student characteristics were examined. The design stage involved selecting real-life contexts, such as staircases and electrical installations, relevant to applying the Pythagorean theorem. The worked examples were created with attention to cognitive load theory principles, specifically by minimizing split attention and redundancy effects. The development stage produced two worked example sets that systematically present problem-solving steps aligned with Polya's stages, along with paired problems sharing similar contexts and solution structures. Furthermore, this study outlines instructional steps for implementing worked examples in classroom settings. The resulting design is expected to enhance students' conceptual understanding of the Pythagorean theorem and support the development of their mathematical problem-solving abilities.

KEYWORDS: Worked examples; Mathematical problem solving; Pythagorean theorem

INTRODUCTION

Successful problem solving is inseparable from the design of the tasks given (Santos-Trigo, 2024; Liljedahl & Cai, 2021). According to Albay (2019), problem solving refers to mathematics tasks that have the potential to provide intellectual challenges to improve students' understanding and development of mathematics. Problem-solving as an important part of mathematics learning (Bradshaw & Hazell, 2017), requires learners to transfer knowledge in situations encountered (Polya, 1973; Schunk, 2012). This process involves interpreting information, planning, implementing and evaluating solutions logically, so that learners actively build mathematical ideas and take responsibility for the learning process. Therefore, teachers need to design assignments that provide space for students to deal directly with problems and design how to solve them (Colburn, 2000).

Mathematical problems associated with real-life situations are considered meaningful because they encourage students to understand the relationship between symbolic representations and reality (Verschaffel *et al*., 2020). Real-world-based math problems allow for more forms of collaboration, so assignment designs are recommended that allow for better integration between mathematical arguments and students' real-world experiences (Nieminen *et al*., 2022). However, contextual problems have their own complexity, especially in the linguistic, numerical, and implicit information complexities that learners must infer (Vessonen *et al*., 2024; Pongsakdi *et al*., 2020). In addition, difficulties in understanding contextual issues are also a common obstacle (Verschaffel *et al*., 2020). This makes contextual problems an appropriate and complex medium to facilitate mathematical problem solving.

Discovery-based learning or problem-solving can have drawbacks when students are asked to find solutions to complex problems (Renkl & Atkinson, 2010**).** Students' tend to use most of their working memory capacity to try out various possibilities at random, thus risking irrelevant cognitive overload. In such situations, direct instruction, for example through a worked example, is more recommended (Mwangi and Sweller, 1998; Newman & DeCaro, 2019;

Ashman *et al*., 2020; Chen *et al.*, 2023). Learning guided by *worked example* can facilitate students in gaining new understandings that support them to solve problems more effectively.

Worked example is a learning approach designed to help students understand problem solving in a more structured and cognitively light way (Sweller, 2019). This method models the problem-solving process by presenting the problem along with its complete solution steps, so that students do not have to spend their working memory capacity guessing or experimenting. This is very important, especially for beginner students who are still building basic knowledge schemes. By studying the systematic stages of solving, they can more easily recognize patterns, recall important information, and understand similar problem structures (Cooper, 1990). Worked example-based learning also helps in the process of internalizing knowledge because it provides a concrete representation of how a problem can be solved from start to finish.

The worked example has proven to be effective, especially for beginners who often experience confusion when faced directly with problems (Van Gog et al., 2011; Manson & Ayres, 2021). Students gain better understanding and learn faster when they first learn sample questions before being asked to solve the questions independently (Atkinson et al., 2003). However, the effectiveness of a worked example depends largely on the quality of the design. If the examples presented cause split attention and redundancy effects, then it can actually hinder understanding (Sweller, 2019). Therefore, a good worked example design must pay attention to the clarity of the presentation and the relevance of the information provided in order to really be able to support the optimal learning process.

Worked examples have been extensively researched, but their design can vary according to the topic and learning needs. This condition encourages the need for research that highlights the design of mathematical problem-solving in the context of real-life. Some researchers have developed worked example designs on various learning materials and needs, such as Retnowati & Fadlila designing worked examples in the combined wide learning of triangles and squares. Julianingsih et al. (2023) designed a worked example on trigonometry material by integrating the ARCS motivation model and Azizah & Retnowati (2017) designed a worked example on geometry and algebra material for visually impaired students using Braille letters. The material of the Pythagorean theorem is used in this study because the application of the material can be found in various contextual situations. However, this material is still poorly understood by students. This can be seen from the students' learning achievement in the phytagoras theorem material which is still unsatisfactory because only 41% of students achieve completeness (Ritonga & Hasibuan, 2022). Wulandari & Riajanto (2020) and Nurmayunita et al. (2024) states that most students have difficulties at each stage of solving problems in the material of the Pythagorean theorem.

This study aims to design worked examples for mathematical problem solving in real-life contexts on learning the Pythagorean theorem. Thus, this research is expected to contribute to the development of more effective learning designs through contextual working examples, as well as enrich literature studies in the application of the Pythagorean theorem at the primary and secondary education levels.

RESEARCH METHOD

This research aims to design a working example of mathematical problem solving with the context of daily life in the material of the Pythagorean theorem through three stages, namely analysis, design, and development. In the analysis stage, the researcher identifies the content of the material and the learning objectives of the Pythagorean theorem in the curriculum to ensure compatibility with the objectives of the development of the worked example. In addition, the researcher analyzed the indicators of mathematical problem-solving to ensure that each solution step in the worked example was in line with the stages of problem-solving thinking. The researcher also determined the characteristics of students as subjects who studied the material so that the design of the questions was in accordance with their level of cognitive development and experience. Furthermore, the design stage is focused on determining the context of the problem to be used and the design of the worked example to be developed. The development stage includes the preparation of a complete worked example followed by similar problem-solving questions for students to work on.

RESULTS AND DISCUSSION

This research produced worked examples design for solving mathematical problems in the material of the Pythagorean theorem. The following discussion will outline the results of each step during the development of the worked example.

a. Analysis

Chen et al. (2023) revealed that the worked example is useful for mathematical materials with high element interactivity. The Pythagorean theorem is one of the materials that fall into this category. In learning the material of the Pythagorean theorem, students are expected to be able to show the truth of the Pythagorean theorem and use it in solving problems, including the distance between two points in the Cartesian coordinate plane. This shows that understanding concepts and problem-solving related to this material must be developed flexibly and meaningfully. In relation to the prerequisite material, students have studied the material of ranked numbers, root form numbers, definitions and types of triangles, as well as the properties of triangles as a logical basis for understanding and applying the Pythagorean theorem correctly. The learning objectives of the Pythagorean theorem include analyzing some information to prove the Pythagorean theorem, making proof in the form of a scheme or procedure for the Pythagorean theorem formula, determining the length of the sides of a triangle using the Pythagorean theorem, comparing the sides of a special right triangle, finding the shape of the Pythagorean triple, solving problems in daily life related to the application of the Pythagorean theorem, and determining the distance between the two points in Cartesian coordinates. The working example in this study will be prepared with the main purpose of learning, namely determining the length of the sides of a triangle using the Pythagorean theorem and solving problems in daily life related to the application of the Pythagorean theorem.

The next step is to formulate a mathematical problem-solving indicator. Polya was one of the first to systematize problem-solving in mathematics consisting of: (1) understanding the problems, (2) devising a plan, (3) carrying out the plan, and (4) looking back (Polya, 1973; Voskoglou, 2021). Therefore, the mathematical problem-solving indicators used in this study are adjusted to Polya's steps as follows.

Table 1. Mathematical Problem-Solving Indicators

Number	Mathematical Problem Solving Steps	Indicator
1.	Understanding the problem	Identifying relevant information includes what is known and asked about the given problem.
2.	Devising a plan	Write down the formula or strategy used to solve the problem.
3.	Carrying out the plan	Carry out a strategy that has been planned with systematic procedures or calculations until a solution to the problem is obtained.
4.	Looking Back	Interpret solutions according to the context of the given problem.

The characteristics of phase D students, especially grade VIII students who are at the stage of formal operational thinking according to Piaget's theory, have begun to think about concrete experiences and think about them in a more abstract, idealistic and logical way (Marinda, 2020). This allows them to consider solutions to real events more wisely and develop structured problem-solving patterns (Anggraeni et al., 2024). Based on the results of the analysis, it was concluded that the development of a worked example design for mathematical problem solving with the context of daily life in the material of the Pythagorean theorem is a relevant step and in accordance with the needs of students. This is reinforced by Sweller (2019) who states that the worked example is recommended as an instructional design for beginner learners.

b. Design

The design stage begins with context analysis, which is to identify various phenomena that are close to the lives of students and relevant to the concept of the Pythagorean theorem. The context chosen is the use of stairs and the installation of electrical cables, as they represent real situations that are commonly encountered in the home, community or school environment. These two contexts can be visually easily represented in the form of a right triangle, making it easier for learners to connect concrete experiences with mathematical models. In addition, the choice of this context also considers the importance of meaningful learning, which is when students can see the relevance between mathematical concepts and real life.

The arrangement of worked examples in the material of the Pythagorean theorem needs to be carefully designed so as not to cause unfamiliar cognitive loads (Atkinson et al., 2003). Sweller (2019) revealed several factors that must be avoided, namely split attention and redundancy effects. The split-attention effect is the separation of students' attention by the presentation of two or more different or separate sources. According to Ayres & Cierniak (2012), split attention occurs when learners are required to divide their attention between two or more sources of information (e.g. text and diagrams) that have been separated spatially or temporally. If students have to divide their attention into many sources, it will increase the cognitive content of students. The redundancy effect occurs due to the excess of information caused by too many sources used, even though with just one source it can provide comprehensive information. The presence of information sources that do not contribute to schema acquisition or automation interferes with learning (Plass et al., 2010). Therefore, the design developed in this study will consider both of these things.

c. Development

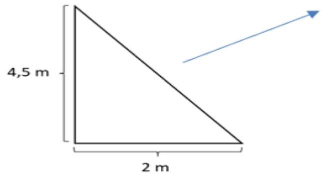
Worked example 1 is presented as follows.

The following is an example of how the Pythagorean theorem can be used to solve problems in real-life. Learn each step in order!

A builder will repair the windows on the 2nd floor of a house. **The height of the window is 4,5 meters** above ground level. In front of the window there is a **garden with a width of 2 meters**. **What is the minimum length of the stairs** needed so that the foot of the stairs does not damage the garden and the top end of the stairs can reach the window? — **Problem**

Solutions:

Known : Window height $= 4{,}5\ m$
Width of the garden $= 2\ m$
Asked : What is the minimum length of stairs needed so that the foot of the stairs does not damage the garden and the top end of the stairs can reach the window? — **Understanding the Problem**
Answer :



Looking for the slant with the Pythagorean theorem: — **Devising a Plan**

$$c^2 = a^2 + b^2$$
$$c^2 = (4{,}5)^2 + (2)^2$$
$$c^2 = 20{,}25 + 4$$
$$c^2 = 24{,}25$$
$$c = \sqrt{24{,}25}$$
$$\approx 4{,}92\ m$$

— **Carrying out the Plan**

So, the minimum length of the stairs needed so that the foot of the stairs does not damage the garden and the top end of the stairs can reach the window is 4,92 meters. — **Looking Back**

Figure 1. Worked Example 1

The worked example presented in the Pythagorean theorem material is systematically designed to reflect the steps of mathematical problem solving, starting from understanding the problem, devising a plan, carrying out the plan, to looking back the results obtained. As shown in Figure 1, each step in solving the problem is presented in a structured manner to help students identify important information, determine the right strategy, implement it sequentially, and interpret solutions according to the context of the given problem. The paired problems of the worked example shown in Figure 2 only lists the problem that must be solved, without any solution steps. The problems have a context and structure equivalent to the worked example, allowing students to apply the knowledge that has been gained and develop a deeper understanding through independent practice.

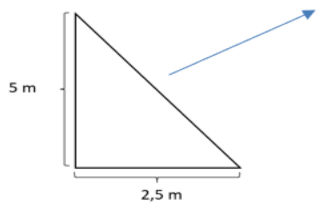
The following are problems in real-life related to the Pythagorean theorem. Work through the steps as in the previous example in order!

A school will install a large signage on the outer wall of the second floor of the main building. **The height of the wall where the signage will be installed is 5 meters** from the ground level. In front of the wall there is a **2,5-meter-wide line of ornamental plants**. **What is the minimum length of the stairs** needed so that the foot of the stairs does not damage the path of ornamental plants and the top end of the stairs can reach where the signage will be installed? — **Problem**

Figure 2. The Paired Worked Example 1

The expected answers for paired problem as problem-solving exercises are shown in Figure 3. Worked example and paired problem have a similar procedure. First, the problems are illustrated using pictures; second, the size of the sides is given; and third, the length of the oblique side is calculated using the Pythagorean theorem.

Known : The height of the wall where the signage will be installed $= 5\ m$
Width of ornamental plants $= 2{,}5\ m$
Asked : What is the minimum length of stairs needed so that the foot of the stairs does not damage the garden and the top end of the stairs can reach the window? — **Understanding the Problem**
Answer :



Looking for the slant with the Pythagorean theorem: — **Devising a Plan**

$$c^2 = a^2 + b^2$$
$$c^2 = (5)^2 + (2{,}5)^2$$
$$c^2 = 25 + 6{,}25$$
$$c^2 = 31{,}25$$
$$c = \sqrt{31{,}25}$$
$$\approx 5{,}59\ m$$

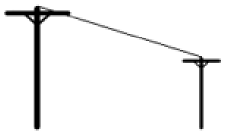
— **Carrying out the Plan**

So, the minimum length of the stairs needed so that the foot of the stairs does not damage the path of ornamental plants and the top end of the stairs can reach where the signage will be installed is 5,59 meters. — **Looking Back**

Figure 3. The Answer of the Paired Worked Example 1

Worked example 2 with the same learning objective, which is to determine the length of the sides of a triangle using the Pythagorean theorem and solve problems in real-life related to the application of the Pythagorean theorem is presented as follows.

The following is an example of how the Pythagorean theorem can be used to solve problems in real-life. Learn each step in order!



In a power grid installation project, there are two support poles with a **height of 17 meters and 10 meters respectively**. **The distance between the two poles is 24 meters**. A technician needs to install **electrical wires that run from the top of the higher pole to the top of the lower pole**. In anticipation of temperature changes that could lead to expansion or shrinkage of the cable, the technician decided to **add 8% of the minimum cable length** required. **What is the total length of the cable** to be prepared?

Problem

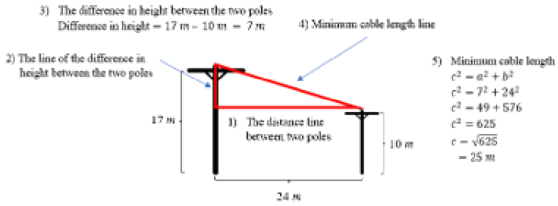
Solutions:

Known:
Height of the first electric pole = 17 meters
The height of the second electric pole = 10 meters
Distance between two power poles = 24 meters
The power cable is stretched from the top of the higher mast to the top of the lower mast
Additional cable length = 8% of min length

Asked:
The total length of the cable to be prepared?

Understanding the Problem

Answer:



6) Total length of cable required

$$\begin{aligned} \text{Total length} &= \textit{minimum cable length} + 8\% \times \textit{minimum cable length} \\ &= 25 + 8\% \times 25 \\ &= 25 + \frac{8}{100} \times 25 \\ &= 25 + 2 \\ &= 27\ m \end{aligned}$$

Devising a Plan & Carrying out the Plan

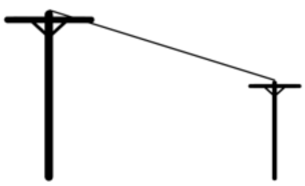
So, the total length of the cable that must be prepared is 27 m.

Looking Back

Figure 4. Worked Example 2

In the worked example 2 above, the researcher displays the image on the question as a form of visual representation of the given problem. These visual representations are used as tools to capture mathematical relationships and processes (van Garderen et al, 2021). The researcher pays attention to the principles of visual simplicity and coherence between text and images. It is expected that learners can engage with real-world problems, develop plans in response, justify mathematically through representations, and then evaluate and communicate solutions. Then, a pair problem from worked example 2 is presented in Figure 5 below.

The following are problems in real-life related to the Pythagorean theorem. Work through the steps as in the previous example in order!



In a power grid installation project, there are two support poles with a **height of 16 meters and 12 meters respectively**. **The distance between the two poles is 20 meters**. A technician needs to install **electrical wires that run from the top of the higher pole to the top of the lower pole**. In anticipation of temperature changes that could lead to cable expansion or shrinkage, technicians decided to **add 10% of the minimum cable length** required. **What is the total length of the cable** to be prepared?

Problem

Figure 5. The Paired Worked Example 2

Worked examples and pairing questions are designed to contain the context of real-life so that students are encouraged to transfer mathematical knowledge into practical situations. This kind of context helps students to be actively involved in the problem-solving process by applying the concepts and procedures they have learned into meaningful situations (Matty, 2016). Worked examples and paired problems are designed to guide students to take solution steps in accordance with mathematical problem-solving indicators, starting from identifying relevant information including what is known and asked about the given problem, writing down formulas or strategies used to solve problems, implementing strategies that have been planned with systematic procedures or calculations to obtain solutions from problems, and interpret solutions according to the context of the given problem. This series of processes is in line with the opinion of Parvaneh & Duncan (2021) who emphasize the importance of logical sequencing of ideas and evaluating strategies as strengthening problem-solving skills.

The application of mathematical problem solving requires theoretical support that considers the cognitive limitations of students in understanding new material. Cognitive load theory explains that human working memory capacity is limited when processing complex or unknown information, so an approach that is able to optimize cognitive load is needed to keep learning effective (Sweller et al, 2011). Cognitive load theory predicts that when students are asked to construct or figure out how to solve problems or perform complex tasks, cognitive effort most often overloads working memory and hinders learning for novice learners (Plass et al, 2010). Worked example is present as a strategy that can help teachers maximize student learning by optimizing their cognitive load (CESE, 2019). A worked example offers a systematic approach to a task or problem (Ayres, 2011).

Worked example is a problem that has been solved for students with each step fully explained (CESE, 2019). The worked example consists of the formulation of the problem, the steps of the solution, and the final solution itself (Schworm & Renkl, 2019; Renkl & Atkinson, 2010; Ayres & Sweller, 2013). Worked example as an instructional tool that provides a solution to a problem from an expert for students to learn (Atkinson et al., 2000). Furthermore, McLaren et al. (2016) revealed that the worked example is presented to students by displaying an example of solving a complete problem to be learned and then explained again by the students if possible.

Various studies show evidence of the effectiveness of working example learning. Yeo & Tzeng (2020) revealed that the worked example is an effective and efficient method to be easily processed in working memory in constructing schemas. The use of worked examples is suitable for complex learning materials and novice students (Rodiawati & Retnowati, 2019). Worked examples are very effective in helping students learn about new material (Rabaza & Hamilton, 2022). Hartmann et al. (2021) states that studying worked examples can be more beneficial than just problem solving. In line with that, Chen et al. (2023) states that the worked example has been shown to be effective in providing effective learning procedures compared to problem solving because the worked example group reported a lower level of cognitive load at each step, compared to the problem-solving group.

Studying the worked example is a not too difficult method to develop problem solving, but it is also prone to incurring irrelevant cognitive burdens when the worked example is poorly designed (Plass et al, 2010). Split attention and redundancy effects are two things that are of concern in this study. In the developed worked example design , split attention is minimized by bolstering important information on the problem to make it easier for students to recognize the core of the problem. All the steps of mathematical problem-solving are integrated into one non-separate page, so that learners do not need to move their gaze from one page to another. In addition, split attention is minimized by providing clear arrows to direct students attention from one element to another, especially between text and images that complement each other. The use of different colors to emphasize important elements in the image is also applied to make it easier for students to focus on key information that supports problem-solving. Meanwhile, the redundancy effect is minimized by designing problems that do not contain unnecessary information. This is so that students are not burdened by repetition or excess information that does not support understanding. The overall design aims to create an efficient and effective learning experience, taking into account important aspects in cognitive load theory.

The learning steps of the Pythagorean theorem material involve several stages according to Wittwer and Renkl (2010), namely: (1) students get a general explanation of the material, including the introduction of principles and concepts; (2) students learn concepts through the worked examples that have been provided; and (3) students solve similar problems to the worked examples that they have learned before. In the first stage, the teacher gives an initial explanation of the concept of the Pythagorean theorem, for example by introducing a right triangle and explaining that the square of the length of the oblique side (hypotenuse) is equal to the sum of the squares of the other two sides. This explanation can be accompanied by visual illustrations or real context to build the student's initial understanding. The second stage is carried out by presenting a working example with completion steps ranging from identifying known and questioned information, determining formulas or strategies, implementing strategies with appropriate procedures or calculations, and interpreting solutions according to the context of the given problem. These examples aim to show systematic solution steps so that students can recognize thinking patterns and problem-solving strategies. Furthermore, in the third stage, students are given the opportunity to solve similar questions independently. These questions are still in the same structure as the previous worked example, but without being accompanied by a solution, so students are required to actively apply their understanding. It is important to provide practice questions immediately after the presentation of examples so that the process of thinking and understanding concepts can be strengthened directly. After completing the pairs of questions, students were given an answer key to check the results of their work.

The implementation of learning with the worked example strategy in the Pythagorean theorem material needs to be designed by considering whether the solution of the sample problem is carried out individually or in groups. Retnowati et al. (2010) shows that worked examples with a high level of element interactivity are better studied individually than in groups. This is strengthened by the findings of Irwansyah & Retnowati (2019) who revealed that learning with the example worked strategy can reduce cognitive load more optimally when students learn individually, while the effectiveness of this strategy does not show a significant difference when compared to problem-solving learning when applied in groups. Therefore, teachers are advised to design individual student work example activities to maximize learning effectiveness.

CONCLUSION

Mathematical problems related to the context of real-life in the application of the Pythagorean theorem can be challenging for students, especially novice students. Worked examples are recommended as learning strategies that can reduce irrelevant cognitive load and help students understand completion steps in a more structured manner. The design of the worked example in this study is designed in harmony with the steps of problem solving according to Polya, namely understanding the problem, devising a plan, carrying out the plan, and looking back. The chosen context of real-life is the use of stairs and the installation of electrical cables. The worked example was designed with split attention and redundancy effects in mind. Split attention is minimized by bolstering important information on the problem, integrating all the steps of solving mathematical problems into one non-separate page, providing clear arrows to direct students' attention, and using different colors to emphasize important elements in the image. Redundancy effects are minimized by designing problems that do not contain unnecessary information. Furthermore, the learning steps of working example in the Pythagorean theorem material include: (1) students obtain a general explanation of the Pythagorean theorem, including an introduction to principles and concepts; (2) students learn concepts through the worked examples that have been provided; and (3) students solve similar problems to the worked examples that they have learned before.

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