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Effect of Flexural Rigidity on Dynamic Response to Moving Load of Damped Shear Beam Resting on Vlasov Foundation

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ABSTRACT
This research investigates the effect of flexural rigidity on dynamic response to moving load of damped shear beam resting on an Vlasov foundation when subjected to moving load traveling at a constant velocity. The governing equations are coupled second-order partial differential equations. The finite Fourier series method was employed to transform the coupled second-order partial differential equations into a set of coupled second-order ordinary differential equations. The resulting simplified equations that characterize the motion of the beam-load system were then solved using Laplace transformation alongside the convolution theory to derive the solutions. Extensive analyses were performed to assess the impact of flexural rigidity on the transverse displacement and rotation of damped shear beams of different lengths when subjected to the moving load traversing at a constant velocity. Furthermore, the research investigates how flexural rigidity affects the critical velocities of the vibrating system. The results indicate that both the transverse displacement and rotation of the beam significantly decrease as flexural rigidity increases. Additionally, it was observed that an increase in flexural rigidity correlates with a rise in critical velocity, suggesting a more stable dynamic system. From a practical standpoint, these findings clearly demonstrate that flexural rigidity plays a crucial role in enhancing the dynamic stability of the beam under the influence of the moving load.

KEYWORDS: Flexural Rigidity, Moving Load, Shear Beam, Critical velocity, Vlasov Foundation

I. INTRODUCTION

The analysis of dynamic response of elastic structures subjected to moving loads remains research of considerable interest owing to the extensive applications of interactions between structures and moving loads in fields such as Civil, Mechanical, Aerospace, and Structural Engineering, among others. In structural dynamics, moving loads are characterized as loads that change position over time on a structure. These dynamic loads can generate vibrations and alter stress within the structure, thereby requiring thorough analysis and design [1-12].

Practical instances of vibrations induced by moving loads encompass those experienced in the interactions between vehicles and bridges, human interactions with footbridges, dynamics of railway tracks, cranes functioning on rails, and processes involving high-speed machining. Thus, a thorough comprehension of the interactions between structures and moving loads is vital for the design of safe and efficient structures in real-world applications. Consequently, a significant volume of literature has been devoted to tackling problems related to moving loads. The vibrational response of beams subjected to moving loads has been extensively explored in [13-17]. Elastic structures, particularly elastic beams, which are defined by their capacity to undergo both bending and shear deformations, are commonly utilized in structural designs due to their adaptability and ability to endure various forms of loading [18]. When loads are applied to a structure, the dynamic behavior of the beams is influenced by various factors, such as the velocity of the load, the material and geometric characteristics of the beam, and the boundary conditions, which are essential for comprehending and forecasting their behavior under different loading scenarios. Furthermore, external factors like damping, interactions with the foundation, and environmental conditions can add complexity to the response.

A beam resting on a Vlasov foundation and subjected to a traveling load presents a particularly intriguing case due to the complex interactions among the traveling load, the internal properties of the beam, and the support characteristics of the foundation. An undamped system is presumed to oscillate freely with a constant amplitude indefinitely. However, this assumption does not hold true in practice; any system possessing mass and elasticity is capable of oscillation, resulting in energy dissipation from the system. Conversely, incorporating damping into dynamic analysis significantly improves the realism of the models. Damping is of paramount importance in structural and construction engineering, as it protects systems from excessive vibrations and potential damage, thereby ensuring smooth and efficient operation.

Notable researcher in this field includes Crandall [19], who explored the significance of damping in specific contexts where minimal damping is critical in affecting a system's dynamic behavior. Mousa and Reza [20] introduced a novel method for the free vibrational synthesis of a cracked cantilever beam with a breaking crack, taking into account the effects of distributed structural damping. Robin and Rana [21] examined the vibrations of isotropic and orthotropic damped plates with varying thicknesses that rest on a foundation.

Furthermore, Famuagun [22] explored the impact of damping coefficients on the dynamic behavior of Rayleigh beams that are supported by a bi-parametric elastic foundation when traversed by moving distributed masses. Similarly, Rayleigh [23] proposed proportional damping models, while Caughey and O'Kelly [24] formulated more comprehensive theories. Contemporary research, such as that conducted by Liu et al. [25], emphasizes the utilization of viscous and hysteretic damping models in beams, and Ogunbamike [26] assesses the influence of a simply supported beam subjected to partially distributed loads, incorporating damping due to resistance against transverse displacement. Recently, Ogunbamike [27] investigated the effects of viscous damping and strain resistance damping. He employed generalized finite integral transform and Struble asymptotic techniques to address the beam problem. It is important to note that, damping does not only affect natural frequencies but also governs the amplitude decay of oscillations, which is essential for applications in seismic and vibration isolation systems. The integration of damping mechanisms, which may stem from material characteristics, structural interfaces, or external devices, into these beams facilitates energy dissipation during dynamic loading, a critical factor for minimizing vibrations and improving structural stability [28, 29].

Moreover, the response of beams under dynamic loads on elastic foundations, such as Vlasov foundations, becomes intricate and necessitates sophisticated analytical methods. The Vlasov foundation model is widely recognized for accounting for the continuity of surface displacement beyond the load area. This model introduces an additional foundation constant, the shear modulus G, in conjunction with the foundation stiffness K. The addition of the shear modulus significantly enhances the precision and dependability of the analysis. Importantly, the dynamics of moving loads on bi-parametric elastic foundations have been extensively studied [30-34]. Also, Ogunbamike and Oni [35] conducted an investigation into the dynamic characteristics of a non-prismatic Rayleigh beam with general boundary conditions, which is supported by a Vlasov elastic foundation and subjected to partially distributed moving masses with varying velocities. They utilized the Generalized Galerkin method to derive closed-form solutions for this type of dynamic problem. Rajib et al. [36] examined the dynamic response of a beam under both moving loads and moving masses, supported by a Pasternak foundation.

It is important to note that, despite extensive research focused on the dynamic behavior of beams subjected to moving loads, studies concerning damped shear beams have been largely limited in the existing literature. The shear beam theory is a fundamental approach for analyzing beams subjected to dynamic loads. In contrast to the Euler-Bernoulli beam theory, which assumes pure bending, the shear beam model accounts for transverse shear deformations, rendering it more appropriate for short and thick beams. Timoshenko [37] was the first to introduce the concept of shear deformation in beam theory, establishing the foundation for advanced dynamic analyses. These foundational theories are essential for understanding the behavior of beams in dynamic scenarios, particularly when they are supported on elastic foundations.

The literature concerning shear beam models has remained sparse until recently, when a significant achievement was made by [38], who investigated the dynamic response of a damped shear beam resting on a bi-parametric elastic foundation while traversed by a moving load traveling at a constant velocity. Likewise, Ajijola [39] analyzed the transverse displacement and rotation of an axially prestressed damped shear beam supported by a Vlasov foundation when subjected to a moving load. In a similar manner, Ajijola O. O, Ogunbamike O. K., and Adedowole A . [40] explored the dynamic behavior of a damped shear beam resting on a Vlasov foundation. Recently, Ajijola [41] examined the influence of axial force on transverse displacement and rotation of an elastically supported damped shear beam subjected to a moving load. Similarly, in a more recent study, Ajijola [42] explored the influence of the damping coefficient on transverse displacement and rotation under a moving load of an elastically supported prestressed shear beam.

In the existing literature, the influence of flexural rigidity has been rarely considered. However, such effect can have a considerable impact on the dynamic response of structures. Flexural rigidity which is defined as the product of the modulus of elasticity and the moment of inertia, is a critical property that governs a beam's response to loading. The primary function of flexural rigidity is to provide resistance within the structures, ensuring durability and the effective operation of the dynamic system when subjected to heavy or dynamic loads. In the field of civil engineering, flexural rigidity is paramount when designing any type of structure. It is one of the essential parameters that affect how structural components such as beams, slabs, and planks react to applied loads [43]. Flexural rigidity has a direct impact on transverse displacement, rotation, bending stress, stability, and the natural frequency of structures.

Consequently, considering the significant influence of flexural rigidity on the dynamic behavior of beams and other similar structural elements, this present study therefore critically investigates the effect of flexural rigidity on the transverse displacement, rotation and critical velocity of a simply supported damped shear beam when traversed by a moving load traveling at a constant velocity.

II PROBLEM STATEMENT

The governing equations that describe the dynamic response of a damped shear beam resting on an Vlasov foundation when subjected to moving load traveling at a constant velocity are based on the following assumptions:

(i) The material exhibits linear elasticity, and the beam is homogeneous at any cross-section (prismatic).

(ii) The x-y plane is designated as the principal plane.

(iii) There exists a beam axis that experiences neither extension nor contraction, with the x-axis located along this neutral axis.

(iv) After bending, plane sections remain plane but are no longer perpendicular to the longitudinal axis.

(v) The influence of shear deformation is taken into account.

(vi) The beam is simply supported with pin-pin ends.

(vii) The moving load applied is concentrated.

(viii) The parameters related to damping, prestressing and the foundation are all constants.

III. MATHEMATICAL MODEL

The governing equations of motion describing the transverse displacement $W(x,t)$ and rotation $\varphi(x,t)$ of a shear beam when subjected to a moving load traveling at a constant velocity are formulated as coupled second order partial differential equations given by

$$\mu\frac{\partial^2 W(x,t)}{\partial t^2}+\frac{\partial}{\partial x}\left[K^*G^*A\left(\varphi(x,t)-\frac{\partial W(x,t)}{\partial x}\right)\right]+F(x,t)=P(x,t) \quad (1)$$

$$\frac{\partial}{\partial x}\left(FR\frac{\partial\varphi(x,t)}{\partial x}\right)-K^*G^*A\left(\varphi(x,t)-\frac{\partial V(x,t)}{\partial x}\right)=0. \quad (2)$$

where μ is the mass per unit length of the beam, K^* is the shear correction factor, G^* is the shear parameter of the beam, A is the cross-sectional area of the beam, E is the Young modulus of elasticity of the beam material, I is the moment of inertia, $FR=\mathrm{EI}$ is the flexural stiffness / rigidity, x is the spatial coordinate, t is the time coordinate, $F(x,t)$ is the foundation reaction and $P(x,t)$ is the moving load acting on the beam per unit length.

The relationship between the foundation reaction $F(x,t)$ and lateral deflection $W(x,t)$ is given by

$$F(x,t)=KV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2} \quad (3)$$

where K and G are two parameters of the foundation model. Specifically, K is the Foundation Stiffness and G is the Shear Modulus.

In this study, it is assumed that the load function $P(x,t)$ is given in the form

$$P(x,t)=P_0\delta(x-ct). \quad (4)$$

$\delta(\cdot)$ is the well-known Dirac delta function with the property.

$$\int_b^a \partial(x-ct)f(x)dx=\{\ 0,\ \ for\ ct<a<b,\ f(ct),\ for\ a<ct<b,\ 1,\ for\ a<b<ct. \quad (5)$$

It is remarked here that the beam under consideration is assumed to have simple support at both ends $x=0$ and $x=L$. Thus, boundary conditions are given as

$$W(0,t)=W(L,t)=0,\ \frac{\partial W(0,t)}{\partial x}=\frac{\partial W(L,t)}{\partial x}=0 \quad (6)$$

$$\varphi(0,t)=\varphi(L,t)=0,\ \frac{\partial\varphi(0,t)}{\partial x}=\frac{\partial\varphi(L,t)}{\partial x}=0 \quad (7)$$

and the initial conditions are given as

$$W(0,x)=0=\frac{\partial W(x,0)}{\partial t},\quad \varphi(0,x)=0=\frac{\partial\varphi(x,0)}{\partial t} \quad (8)$$

Now, introducing damping and axially prestressed parameters and in view of (3) and (4) after some simplifications and re-arrangements, equation (1) and (2) become

$$\frac{\partial}{\partial x}\left[K^*G^*A\left(\varphi(x,t)-\frac{\partial W(x,t)}{\partial x}\right)\right]+\mu\frac{\partial^2 W(x,t)}{\partial t^2}-N_0\frac{\partial^2 W(x,t)}{\partial x^2}-C_a\frac{\partial W(x,t)}{\partial t}+KW(x,t)-G\frac{\partial^2 W(x,t)}{\partial x^2}=P_0\delta(x-ct) \quad (9)$$

$$\frac{\partial}{\partial x}\left(FR\frac{\partial\varphi(x,t)}{\partial x}\right)-K^*G^*A\left(\varphi(x,t)-\frac{\partial W(x,t)}{\partial x}\right)=0 \quad (10)$$

Where N_0 is the axial force and C_a is the coefficient of viscous damping per unit length of the beam. (2)

Hence, (9) and (10) are the second order partial differential equations governing the flexural motion of an elastically supported damped shear beam when subjected to a moving load traveling at a constant velocity.

IV. SOLUTION PROCEDURE

The shear beam examined in this study is both finite and uniform. To derive the analytical solution for the initial boundary value problem presented in equations (9) and (10), we utilize the finite Fourier transformation method in conjunction with the Laplace Transform techniques. Consequently, we present the following definitions.

Definition 1: The finite Fourier sine transform $\phi_0(n,t)$ of a function $\phi(x,t)$ is defined as

$$\phi_0(n,t) = \int_0^l \phi(x,t)\, sin\, \frac{n\pi x}{l}\, dx \qquad (11)$$

and the inverse transform is

$$\phi(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} \phi_0(n,t)\, sin\, sin\, \frac{n\pi x}{l}\, dx. \qquad (12)$$

Definition 2: The finite Fourier cosine transform $\gamma_0(n,t)$ of a function $\gamma(x,t)$ is defined as

$$\gamma_0(n,t) = \int_0^l \gamma(x,t)\, cos\, \frac{n\pi x}{l}\, dx \qquad (13)$$

and the inverse transform is

$$\gamma(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} \gamma_0(n,t) cos\, \frac{n\pi x}{l}\, dx. \qquad (14)$$

Thus, applying (11) and (13) to the governing equations (9) and (10) respectively, in conjunction with the Dirac delta function property in (5), we obtain

$$\frac{\partial^2 W(n,t)}{\partial t^2} + \mu_1 \frac{\partial W(n,t)}{\partial t} + \mu_2 W(n,t) - \mu_3 \frac{\partial \varphi(n,t)}{\partial x} = Q_1\, sin\, \theta_n\, t \qquad (15)$$

And

$$\varphi(n,t) = \mu_0 W(n,t) \qquad (16)$$

where

$$\mu_1 = -\frac{C_a}{\mu}, \quad \mu_2 = \left(\frac{n\pi}{\mu L}\right)^2 (N_0 + G) - \frac{K}{\mu}, \quad \mu_6 = \frac{n\pi}{L} K^* G^* A$$

$$\mu_3 = \left(\frac{n\pi}{\mu L}\right) K^* G^* A, \quad \mu_7 = FR\left(\frac{n\pi}{L}\right)^2 + K^* G^* A$$

$$Q_1 = \frac{P_0}{\mu}, \quad \theta_n = \frac{n\pi c}{L}, \quad \mu_0 = \frac{\mu_6}{\mu_7}$$

Now putting (16) into (15), we have

$$\frac{\partial^2 W(n,t)}{\partial t^2} + \mu_1 \frac{\partial W(n,t)}{\partial t} + \mu_2 W(n,t) - \mu_3 \frac{\partial}{\partial x}\left(\mu_0 W(n,t)\right) = Q_1\, sin\, \theta_n\, t \qquad (17)$$

The term involving the derivative with respect to x in (17) vanishes as $V(n,t)$ is a function of t alone and after some simplifications and re-arrangements, we obtain

$$W_{tt}(n,t) + \mu_1 W_t(n,t) + \mu_4 W(n,t) = Q_1\, sin\, \theta_n\, t \qquad (18)$$

where

$\mu_4 = \mu_2 - \mu_0\mu_3$

Next, we subject (18) to Laplace transformation

$$\mathcal{L}(f(t)) = F(s) = \int_0^{\infty} f(t)\, e^{-st} dt \qquad (19)$$

where s is the Laplace parameter. In view of (19), (18) becomes

$$s^2\widetilde{W}(n,s) + \mu_1 s\widetilde{W}(n,s) + \mu_4\widetilde{W}(n,s) = Q_1\left[\frac{\theta_n}{s^2+\theta_n^{\,2}}\right]. \qquad (20)$$

After simplification and rearrangement, we obtain the simple algebraic equation given by

$$\widetilde{W}(n,s) = Q_1\left[\frac{1}{s^2+\mu_1 s+\mu_4}\right]\left[\frac{\theta_n}{s^2+\theta_n^{\,2}}\right] \qquad (21)$$

which is further simplified to give

$$\widetilde{W}(n,s) = Q_1\left[\frac{1}{(s+\mu_5)^2+\beta^2}\right]\left[\frac{\theta_n}{s^2+\theta_n^{\,2}}\right] \qquad (12) \qquad (22)$$

Where

$$\beta^2 = \mu_4 - (\mu_5)^2, \quad \mu_5 = \left(\frac{\mu_1}{2}\right)$$

At this juncture, in order to obtain the Laplace inversions of (22), we let

$$F(s) = \left[\frac{1}{(s+\alpha_5)^2+\beta^2}\right] \qquad (16) \qquad (23)$$

And

$$G(s) = \left[\frac{\theta_n}{s^2+\theta_n^{\,2}}\right] \qquad (24)$$

so that the Laplace inversion of (22) is the convolution of (23) and (24) defined by

$$F(s) * G(s) = \int_0^t f(t-u)g(u)du. \qquad (17) \qquad (25)$$

Noting that

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{p}\exp\,(-\mu_5 t)\sin(\beta t) \qquad (26)$$

and

$$\mathcal{L}^{-1}[G(s)] = \sin(\theta_n t) \qquad (27)$$

Now using (23) and (24) in (25), (22) becomes

$$W(n,t) = \frac{Q_1 e^{-\mu_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[\beta e^{\mu_5 t} sin\,\theta_n\, t - \theta_n sin\,\beta\, t] + \varphi_0[\beta e^{\mu_5 t} sin\,\theta_n\, t + \theta_n sin\,\beta\, t] - \mu_1\beta\theta_n[e^{\mu_5 t} cos\,\theta_n\, t - cos\,\beta\, t]\} \qquad (18) \qquad (28)$$

where,

$$\varphi_1 = (\beta+\theta_n)^2, \quad \varphi_2 = (\beta-\theta_n)^2, \quad \varphi_0 = -(\mu_5)^2$$

Thus, in view of (12), one obtains

$$W(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{Q_1 e^{-\mu_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[\beta e^{\mu_5 t} \sin\theta_n t - \theta_n \sin\beta t] + \varphi_0[\beta e^{\mu_5 t} \sin\theta_n t + \theta_n \sin\beta t] - \mu_1\beta\theta_n[e^{\mu_5 t}\cos\theta_n t - \cos\beta t]\}\sin\frac{n\pi x}{l} \quad (29)$$

which represents the transverse displacement of a damped shear beam resting on an Vlasov foundation when subjected to moving load traveling at a constant velocity.

Now, using (28) in (16), we have

$$\varphi(n,t) = \frac{\mu_0 Q_1 e^{-\mu_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[\beta e^{\mu_5 t} \sin\theta_n t - \theta_n \sin\beta t] + \varphi_0[\beta e^{\mu_5 t} \sin\theta_n t + \theta_n \sin\beta t] - \mu_1\beta\theta_n[e^{\mu_5 t}\cos\theta_n t - \cos\beta t]\}. \quad (30)$$

Similarly, in view of (14), one obtains

$$\varphi(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{\mu_0 Q_1 e^{-\mu_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[e^{\mu_5 t} \sin\theta_n t - \theta_n \sin\beta t] + \varphi_0[\beta e^{\mu_5 t} \sin\theta_n t + \theta_n \sin\beta t] - \mu_1\beta\theta_n[e^{\mu_5 t}\cos\theta_n t - \cos\beta t]\}\cos\frac{n\pi x}{l} \quad (31)$$

which represents the rotation of a damped shear beam resting on an Vlasov foundation when subjected to moving load traveling at a constant velocity.

V. NUMERICAL SIMULATION AND DISCUSSION OF RESULT

The uniform damped shear beam of lengths (L) = 12.5m, 25.00m, 37.50m and 50.00m respectively are considered in order to illustrate the analysis presented in this study. The Young modulus of elasticity $E = 2.10924 \times 10^9 Kg/m$, moment of inertia I = 2.87698×10^{-3}, $\pi = 22/7$, the damping coefficient $Ca = 3000$, axial force $No = 4e3$, Foundation Stiffness $K = 4e3$, Shear Modulus $G = 4e3$ and the mass per unit length of the beam $\mu = 2758.291 kg/m$. The values of Flexural Rigidity FR are varied between 0 *and* $900000000\ Nm^2$.

In this present study, two special cases of effect of flexural rigidity FR on dynamic response of a simply supported damped shear beam under the action of moving load were investigated. The cases are termed;

1. the effect of flexural rigidity FR on transverse displacement and rotation of a damped shear beam when the lengths of the beam (L) = 12.5m, 25.00m, 37.50m and 50.00m respectively and
2. the effect of flexural rigidity FR on critical velocity.

The transverse displacement W and rotation φ of the beam are calculated and plotted against time t for various values of flexural rigidity FR. The results are shown on the various graphs given below.

Figures 1 to 8 describe the transverse displacement and rotation of a simply supported damped shear beam under the action of moving load travelling at constant velocity for various values of Flexural Rigidity FR when the lengths of the Shear beam (L) are 12.5m, 25.00m, 37.50m and 50.00m respectively and for the fixed values of other parameters. It is clearly evident from figures 1 to 8 that as the Flexural Rigidity FR value increases, there is a noticeable decrease in both the transverse displacement and rotation of the beam. In practical applications, this suggests that an increase in the Flexural Rigidity FR value enhances the stiffness of the beam, counteracting the bending effects and significantly reducing the transverse displacement and rotation of the vibrating beam. Consequently, the beam becomes more rigid and stable, enabling it to resist lateral deflection and intense vibrations. Hence, the presence of flexural rigidity FR contributes to the overall rigidity of the beam system.

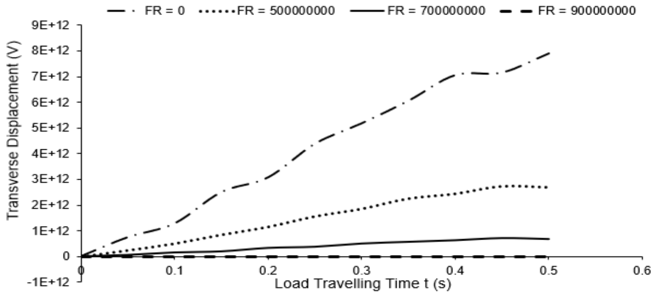

Figure 1: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 12.5$ and for fixed values of $No = 4e3$, $G = 4e3$, $c = 8.128$ and $C_a = 3000$

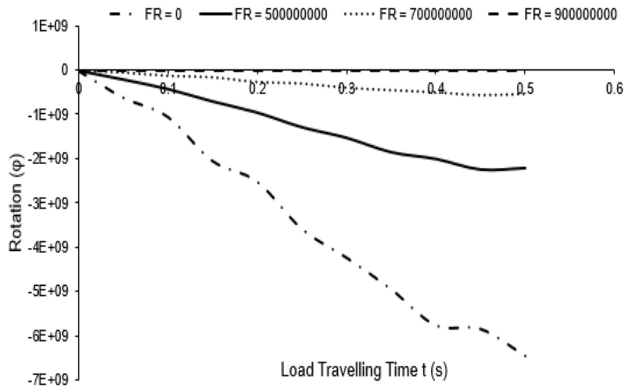

Figure 2: Rotation of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 12.5$ and for fixed values of $No = 4e3$, $G = 4e3$, $c = 8.128$ and $C_a = 3000$

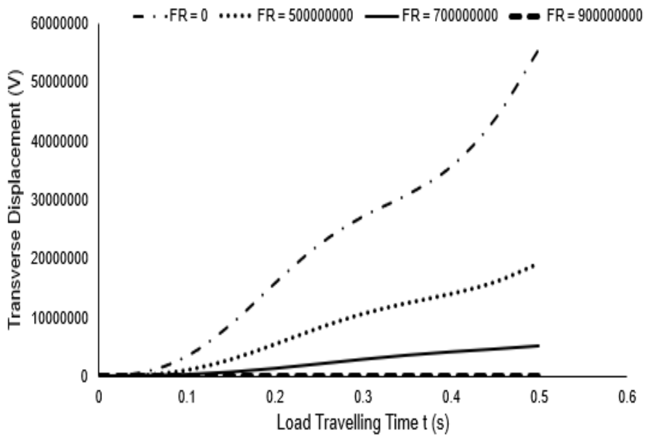

Figure 3: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 25.00$ and for fixed values of $No = 4e3,\ G = 4e3,\ c = 8.128$ and $C_a = 3000$

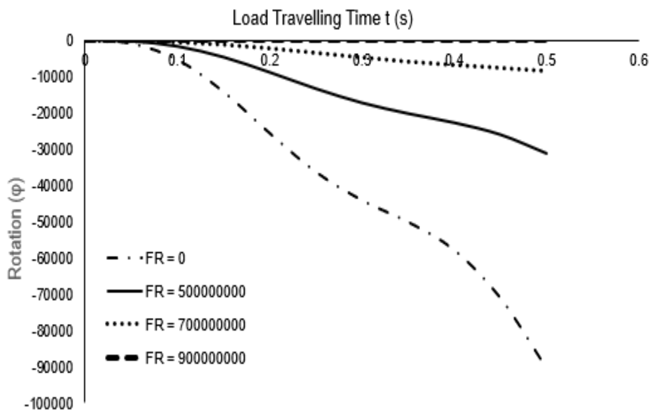

Figure 4: Rotation of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 25.00$ and for fixed values of $No = 4e3,\ G = 4e3,\ c = 8.128$ and $C_a = 3000$

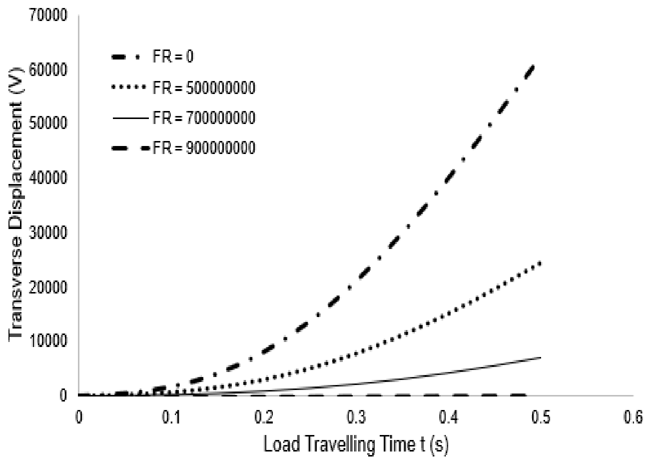

Figure 5: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 37.50$ and for fixed values of $No = 4e3,\ G = 4e3,\ c = 8.128$ and $C_a = 3000$

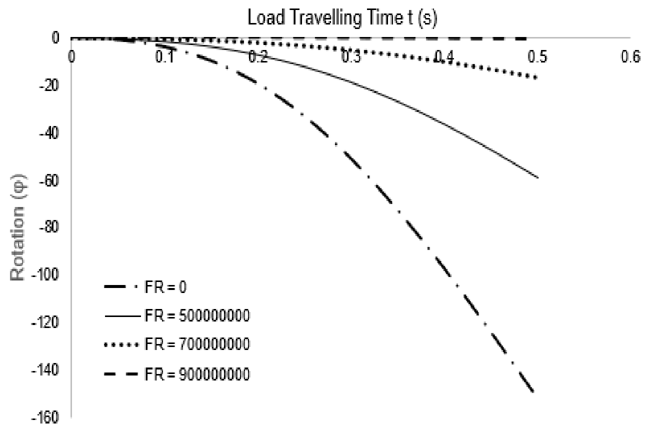

Figure 6: Rotation of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 37.50$ and for fixed values of $No = 4e3,\ G = 4e3,\ c = 8.128$ and $C_a = 3000$

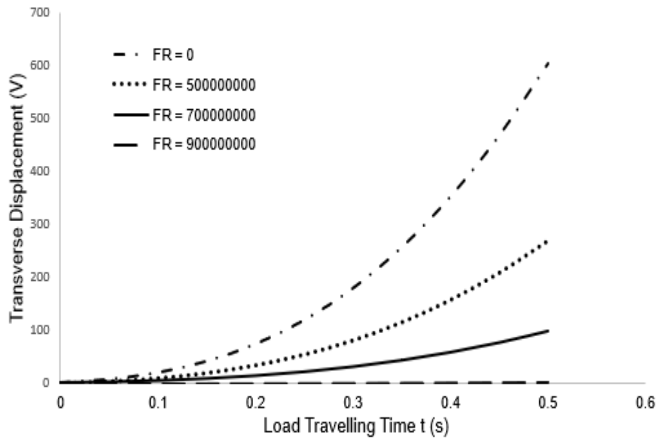

Figure 7: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 50.00$ and for fixed values of $No = 4e3,\ G = 4e3,\ c = 8.128$ and $C_a = 3000$

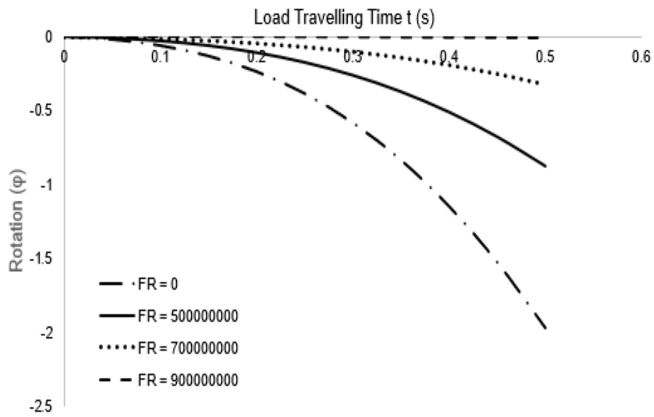

Figure 8: Rotation of a simply supported damped shear beam under the action of moving load for various values of FR when the beam length $(L) = 50.00$and for fixed values of $No = 4e3$, $G = 4e3$, $c = 8.128$ and $C_a = 3000$

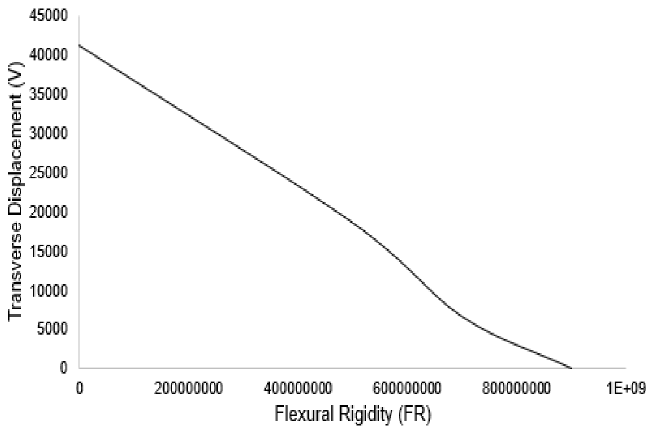

Figure 9: Variation of the Transverse displacement (V) against Flexural Rigidity (FR)

Figure 9 depicts the variation of the Transverse displacement (V) against Flexural Rigidity (FR) of a simply supported damped shear beam under the action of moving load travelling at constant velocity. The graph is monotonic decreasing. This is evident from the graph, as the Flexural Rigidity (FR 0 to approximately $9 \times 10^9\ Nm^2$, Transverse displacement (V) drops from approximately 42,000 down to 0. From practical stand point damped shear beam deflects less under the same load. Similarly, from the graph, three interesting regimes are visible. They are namely.

(i) **nearly linear decline** which ranges from 0 to $6 \times 10^8\ Nm^2$. Each increment in FR gives about the same reduction in Transverse displacement (V). This is denotes the classic elastic-bending behavior.

(ii) A **steeper knee** which ranges from $6 \times 10^8\ Nm^2$ to $7.5 \times 10^8\ Nm^2$**.** Here, small increases in FR give a large Transverse displacement (V). This is where stiffness starts to dominate other compliances like joint/ support flexibility, shear effects, or proximity to a resonance.

(iii) **Diminishing returns** which ranges from $7.5 \times 10^8\ Nm^2$ to $9 \times 10^9\ Nm^2$**.** Further FR at this point adds only small improvement because other factors like damping, boundary compliance, load path limit the Transverse displacement (V). The curve tends toward zero.

Consequently, increasing flexural rigidity strongly suppresses the transverse displacement (V) of a simply supported damped shear beam under the action of moving load. The plot shows a near-linear reduction at low Flexural Rigidity (FR, a sharp improvement around the knee as stiffness begins to dominate, and then diminishing returns as other limitations take over.

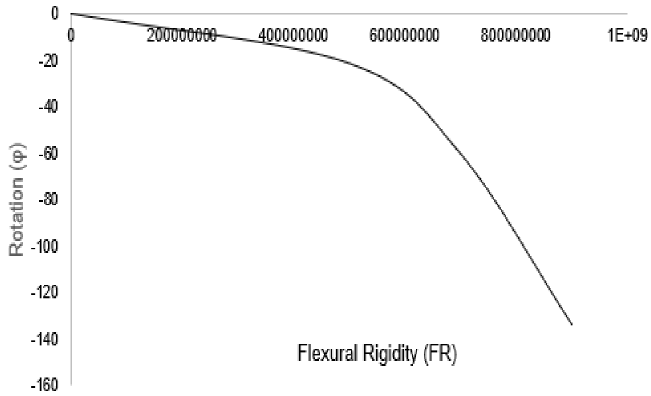

Figure 10: Variation of the Rotation against Flexural Rigidity (FR)

Figure 10 illustrates the variation of the Rotation (φ) against Flexural Rigidity (FR) of a simply supported damped shear beam under the action of moving load travelling at constant velocity. This graph shows that rotation **(φ)** becomes increasingly negative as flexural rigidity (FR) increases**.** At low FR, rotation is small and nearly constant, but at higher FR, rotation drops sharply, showing a nonlinear effect. This suggests that stiffening the structure alters how loads are distributed.

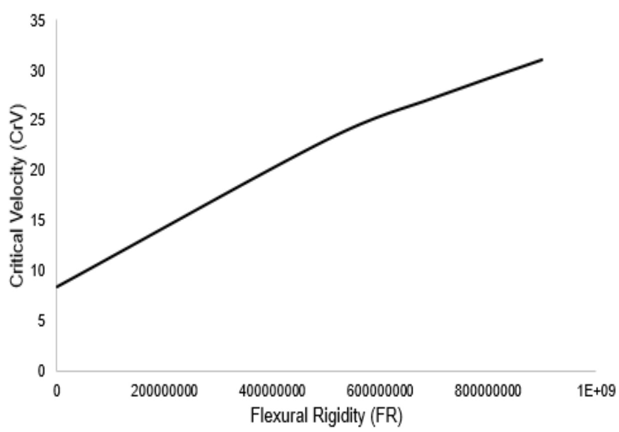

Figure 11: Variation of the Critical Velocity (CrV) against Flexural Rigidity (FR)

Figure 11 shows the variation of the Critical Velocity (CrV) against Flexural Rigidity (FR) of a simply supported damped shear beam under the action of moving load travelling at constant velocity. From the graph, the following interesting trends are obtained.

(i) **Positive relationship:** The curve shows that as **FR increases, Critical Velocity also increases**.
- At **FR = 0**, CRV = 10 Nm^2
- At **FR** = 4 × 10^8 Nm^2, CRV = 20 Nm^2.
- At **FR** = 9 × 10^8 Nm^2, CRV =31 Nm^2.

(ii) **Shape of curve:**
- **0** to **6** × $\mathbf{10^8}$ $\mathbf{Nm^2}$: Almost linear increase.
- **Beyond 6** × $\mathbf{10^8}$ $\mathbf{Nm^2}$**:** Growth rate slows down (curve flattens). This indicates **diminishing returns** — each additional stiffness increment gives smaller improvement in CRV.
- **Beyond 6×** $\mathbf{10^8}$ $\mathbf{Nm^2}$**:** Growth rate slows down (curve flattens). This indicates **diminishing returns** — each additional stiffness increment gives smaller improvement in CRV.

Also, it is vivid from the graph that for various values of Flexural Rigidity FR and for the fixed values of other parameters, the higher the value of the Flexural Rigidity FR, the higher the critical velocity of the beam. In practical terms, increase in Flexural Rigidity FR reduces the peak amplitude of resonance, where the beam's natural frequency matches the excitation frequency. Consequently, Flexural Rigidity FR contributes immensely to the dynamic stability of the beam system, mitigating the risk of resonance that may result in structural failure and thus, safeguarding the safety of the structure's occupants.

VI. CONCLUSION

This paper presents the effect of flexural rigidity on dynamic response to moving load of damped shear beam resting on an Vlasov foundation when subjected to moving load traveling at a constant velocity. A solution approach utilizing finite Fourier transform techniques, Laplace transformation, and convolution theory is applied to derive the solution for the coupled second-order partial differential equations that define the dynamics of the beam-load system. Comprehensive analyses are conducted to examine the impact of Flexural Rigidity FR on the transverse displacement and rotation of damped shear beams of different lengths when subjected to a moving load. Additionally, the study explores how Flexural Rigidity FR affects the critical velocities of the vibrating system. The generated graphs clearly illustrate that Flexural Rigidity FR significantly enhances the stability of the beam under the moving load. The findings reveal that both the transverse displacement and rotation of the beam substantially decrease with an increase in Flexural Rigidity FR. Moreover, it is noted that higher values of Flexural Rigidity FR are associated with higher critical velocities, indicating a more stable dynamic system. Therefore, in the design of engineering structures such as bridges, pipelines, railway tracks, aerospace components, railway bridges, overhead cranes, cableways, and tunnels, the significant influence of flexural rigidity on critical velocity and dynamic stability must be considered to ensure the safety, reliability, and efficiency of the design.

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