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A GENERALIZED USEFUL R-NORM INFORMATION MEASURE AND ITS NOISELESS CODING THEOREM

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ARTICLE INFO | **ABSTRACT**

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In this paper, we propose a generalized class of useful information measures based on two parameters $R\ and\ \beta$, incorporating a utility function. This measure extends the useful $R - Norm$ information measure and includes several existing entropies as special cases. We derive a noiseless coding theorem associated with this generalized measure and establish tight bounds on the useful mean codeword length. An example is provided to demonstrate its applicability in decision-making and data compression problems. The flexibility of this measure offers potential for wider application in communication systems.

Key words: R-Norm information measure, Useful information, Useful mean codeword length, Coding theorem etc.

1. Introduction

Information theory provides foundational tools for analyzing communication systems. Shannon's entropy laid the groundwork for quantifying information, but real-world scenarios involving vagueness or imprecision motivated the development of useful information measures (UIM). Building upon these, Belis and Guiasu[1] incorporated utility into the entropy function, giving rise to the concept of useful information.

In recent years, several parametric forms of entropy such as Rényi, Tsallis, and Sharma Mittal have been studied to enhance adaptability. Among them, the useful $R - Norm$ information measure offers a qualitative-quantitative framework for evaluating information content. However, there remains a need to generalize this further to accommodate multiple parameters and utility-based encoding.

This paper proposes a two-parameter extension and explores its theoretical properties and practical implications through coding theorems and bounds. Specifically, we develop a noiseless coding theorem corresponding to the proposed measure and derive bounds on the useful mean codeword length. The results demonstrate that the generalized measure provides a comprehensive framework suitable for decision-making systems and data encoding processes involving uncertainty and utility prioritization.

If $D (D \geq 2)$ code alphabets are utilised to encode a finite collection of N source symbols

$X = \{x_1, x_2, \dots, x_n\}$ with probabilities $P = \{p_1, p_2, \dots \dots, p_n\}$, then there exists a completely decipherable code with lengths $\{l_1, l_2, \dots \dots, l_n\}$ if and only if

$$\sum_{i=1}^{n} D^{-l_i} \leq 1. \tag{1.1}$$

According to Feinstein [4], (1.1) has been defined to be Kraft inequality. Shannon's coding theorem for noiseless channels (Feinstein [4]) provides a lower bound on L in terms of Shannon [9] entropy for a code that fulfils (1.1) if $L = \sum_{i=1}^{n} p_i l_i$ is the average codeword length. For example, if and only if $l_i = -\log p_i \; for \; all \; i = 1,2,3,..,n$. Then

$$H(P) \leq L \tag{1.2}$$

In terms of its subjective quantity, Belis and Guiasu [1] noted that the probability distribution P over the source symbols X does not entirely specify the source of information. It is also reasonable to presume that the initial words and symbols are given weights based on their significance or usefulness to the observer.

Let u_i be the utility or significance of outcome x_i, and let $U = \{u_1, u_2, \dots \dots, u_n\}$ be the set that comprises positive real numbers. In most cases, the utility is not impacted by the likelihood that the source symbol x_i, or p_i, will be decoded.

Belis and Guiasu [1] introduced the following measure of information:

$$H(P,U) = -\sum_{i=1}^{n} u_i p_i \log p_i \tag{1.3}$$

Guiasu and Picard [5] introduced the useful mean length of the code

$$L(P,U) = \frac{\sum_{i=1}^{n} u_i p_i l_i}{\sum_{i=1}^{n} u_i p_i} \tag{1.4}$$

The following upper and lower limits for the cost measure (1.4) were subsequently established by Longo [8], who study (1.4) as the average communication cost of the letters x_i's is.

$$H(P,U) \leq L(P,U) < H(P,U) + 1 \tag{1.5}$$

where

$$H(P,U) = \frac{\sum_{i=1}^{n} u_i p_i log p_i}{\sum_{i=1}^{n} u_i p_i} \tag{1.6}$$

According to Guiasu and Picard [5], (1.6) is a "useful" information measure for insufficient distributions. Bhaker and Hooda [2] also described this via mean value representation. We determine their boundaries and define a generalised cost measure of (1.4). We develop a significant noiseless coding theory for sources with utilities in this communication. The theory offers a tool for power distributions and generalises the findings of Longo [8], Gurdial and Pessow [6], Hooda and Bhaker [7], and Singh [11].

NEW WORK

2. CODING THEOREM

Let $P = \{p_1, p_2, \dots .., p_n\}$ be a discrete probability distribution such that $p_i > 0 \; and \; \sum_{i=1}^{n} p_i = 1$, and let $U = \{u_1, u_2, \dots .., u_n\}$ be a set of utility values associated with each symbol.

We define the generalized useful $R - Norm$ average code length of type β.

$$L_R^{\beta}(P,U) = \frac{R}{R-1}\left(1 - \frac{\sum_{i=1}^{n} u_i^{\beta} p_i^{\beta} D^{-l_i \frac{(R-1)}{R}}}{\sum_{i=1}^{n} (u_i p_i)^{\beta}}\right)$$

and we also define the useful $R - Norm$ information measure as

$$H_R^{\beta}(P,U) = \frac{R}{R-1}\left[- \left(\frac{\sum_{i=1}^{n} u_i^{\beta} p_i^{R+\beta-1}}{\sum_{i=1}^{n} (u_i p_i)^{\beta}}\right)^{\frac{1}{R}}\right],$$

$$R \neq 1, \beta > 0$$

Proof: Hooda and Bhakar [7] stablised the following "useful information measures" using mean values:

$$H(P,U) = -\sum_{i \in N} \frac{u_i p_i log p_i}{u_i p_i} \tag{2.1}$$

and

$$H_{\alpha}(P,U) = \frac{1}{1-\alpha} \log \sum_{i \in N} \frac{u_i p_i^{\alpha}}{u_i p_i} \tag{2.2}$$

The following "useful" information measures have been characterised with new mean values:

$$H_\alpha(P,U) = \frac{1}{1-\alpha}\log\left[\sum_{i\in N} u_i p_i^\alpha \Big/ \sum_{i\in N} u_i p_i\right] \tag{2.3}$$

Additionally, Mahajan and Kumar [13] defined a useful information-generating function as follows:

$$\mathrm{I(P,U,t)} = -\sum_{i\in \boldsymbol{N}} (u_i p_i)^t \Big/ \sum_{i\in N} u_i p_i \tag{2.4}$$

where t is a real or complex variable and $P = \{p_1, p_2, \ldots, p_n\}$ and $U = \{u_1, u_2, \ldots, u_n\}$ are the probability and utility distributions, respectively.

We recognise that the following gives the weighted mean of u_i *and* P_i:

$$\frac{\sum_{i=1}^n u_i p_i}{\sum_{i-1}^n u_i} \tag{2.5}$$

We obtain a new weighted mean of order $1-\alpha$ as follows if we replace u_i with weights $(u_i p_i)^{\beta_i}$ and p_i of order $\alpha - 1$.

$$M_{\alpha,\beta}(P,U) = \left[\sum_{i=1}^n (u_i p_i)^{\beta_i} {p_i}^{\alpha-1} \Big/ \sum_{i=1}^n (u_i p_i)^{\beta_i}\right]^{\frac{1}{\alpha-1}}$$

i.e.

$$M_{\alpha,\beta}(P,U) = \left[\sum_{i=1}^n {u_i}^{\beta_i} {p_i}^{\alpha+\beta_i-1} \Big/ \sum_{i=1}^n (u_i p_i)^{\beta_i}\right]^{\frac{1}{\alpha-1}};$$

$$\alpha \ge 0, \alpha \ne 1, \beta_i \ge 1 \tag{2.6}$$

The generalised useful information generating function for this is provided by

$$I_{\alpha,\beta}(P,U,t) = [M_{\alpha,\beta_i}(P,U)]^{-t}$$

From (2.6) we get,

$$I_{\alpha,\beta}(P,U,t) = \left[\sum_{i=1}^n {u_i}^{\beta_i} {p_i}^{\alpha+\beta_i-1} \Big/ \sum_{i=1}^n (u_i p_i)^{\beta_i}\right]^{\frac{-t}{\alpha-1}} \tag{2.7}$$

in which t is a complex or real variable.

When we differentiate equation (2.7) with regard to t at t=0, respectively, we obtain

$$H_\alpha^{\beta_i}(P,U) = \frac{1}{1-\alpha}\log\left[\sum_{i=1}^n u_i^{\beta_i} p_i^{\alpha+\beta_i-1} \Big/ \sum_{i=1}^n (u_i p_i)^{\beta_i}\right] \tag{2.8}$$

This is the type β_i and order α generalised useful information measure.

Now,

Assume a prefix code such that

$$p_i = D^{-l_i} \Rightarrow l_i = -\log_D p_i$$

Then we get

$$p_i^{\alpha-1} = D^{-(\alpha-1)l_i}$$

Substituting in (2.8)

$$H_\alpha(P,U) = \frac{1}{1-\alpha}\left[\log_D\left(\sum_{i=1}^n u_i^\beta D^{-(\alpha-1)l_i}\right) - \log_D\left(\sum_{i=1}^n u_i^\beta D^{-l_i}\right)\right]$$

Now, define a new function for generalized useful average codeword length

$$L_\alpha^\beta = \frac{\alpha}{1-\alpha}\left[\log_D\left(\frac{\sum_{i=1}^n u_i^\beta D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}}{\sum_{i=1}^n u_i}\right)\right]$$

$$L_\alpha^\beta = \frac{\alpha}{1-\alpha}\left[\log_D\left(\frac{\sum_{i=1}^n u_i^\beta p_i^\beta D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)\right] \tag{2.9}$$

Although this isn't necessarily the case, it is considered in the construction of the cost measure (1.4) that the cost is a function that is linear of code length. Usually the cost acts as a logarithm of the lengths of the codewords. These kinds of functions are commonly seen in economic models of growth and market stability. Therefore, it may occasionally be more desirable to select a code that reduces a monotonic function,

$$C = \sum_{i=1}^n u_i^\beta p_i^\beta D^{l_i\left(\frac{R-1}{R}\right)} \tag{2.10}$$

Where the cost-related parameters are $R > 0(\ne 1), \beta > 0$.

To make this paper's result more equivalent to the standard noiseless coding theorem, rather than minimising (2.10), we shall minimize

$$L_R^\beta(P,U) = \frac{R}{R-1}\left(1 - \frac{\sum_{i=1}^n u_i^\beta p_i^\beta D^{-l_i\frac{(R-1)}{R}}}{\sum_{i=1}^n (u_i p_i)^\beta}\right) \tag{2.11}$$

and generalized useful *R-norm* information measure of type β as

$$H_R^{\beta}(P,U) = \frac{R}{R-1}\left[1 - \left(\frac{\sum_{i=1}^{n} u_i^{\beta} p_i^{R+\beta-1}}{\sum_{i=1}^{n}(u_i p_i)^{\beta}}\right)^{\frac{1}{R}}\right], \quad R \neq 1, \beta > 0 \qquad (2.12)$$

This measure generalizes the useful *R-norm* information measure and includes the following as special cases for equation (2.11) and (2.12).

From equation (2.11)

$$L_R^{\beta}(P,U) = \frac{R}{R-1}\left(1 - \frac{\sum_{i=1}^{n} u_i^{\beta} p_i^{\beta} D^{-l_i\frac{(R-1)}{R}}}{\sum_{i=1}^{n}(u_i p_i)^{\beta}}\right)$$

Equation (2.11) as the useful $R - Norm$ average code length of type β and includes the following as special cases

Special cases

1) When $u_i = 1 \; and \; \beta = 1$, it reduces to $R - Norm$ entropy studied by Boekee and Lubbe [3].

2) When $\beta = 1, u_i = 1 \; and \; R \to 1$ then (2.11) reduces to optimal code length by Shannon [9].

Next, from equation (2.12) we have,

$$H_R^{\beta}(P,U) = \frac{R}{R-1}\left[1 - \left(\frac{\sum_{i=1}^{n} u_i^{\beta} p_i^{R+\beta-1}}{\sum_{i=1}^{n}(u_i p_i)^{\beta}}\right)^{\frac{1}{R}}\right], \quad R \neq 1, \beta > 0$$

Special cases

1).When $\beta = 1$, it reduces to the useful $R - Norm$ entropy studied by Singh, Kumar and Tuteja [11].

2).When $R \to 1$, it approaches a logarithmic form, analogous to the Shannon-type entropy [9].

3).When $u_i = 1 \; and \; \beta = 1$, it reduces to $R - Norm$ entropy studied by Boekee and Lubbe[3].

Additionally, we have determined the boundaries using condition

$$\sum_{i=1}^{n} u_i^{\beta} p_i^{\beta-1} D^{-l_i} \leq \sum_{i=1}^{n} u_i^{\beta} p_i^{\beta}$$

It is evident that equation reduces to the Kraft inequality (1.1) $for \; \beta = 1 and \; u_i = 1 \; for \; every \; i = 1,2, \dots, n.$ The code alphabet's size is D(D ≥ 2).

3. BOUNDS ON THE MEAN CODE LENGTH

Let *L(U)* be the mean code length as defined above. Then under Kraft's inequality, we have

$$H_R^{\beta}(P,U) \leq L(P,U) < D^{\frac{1-R}{R}} H_R^{\beta}(P,U) + \frac{R}{R-1}(1 - D^{\frac{1-R}{R}}).$$

Theorem 3.1 Let $P = \{p_1, p_2, \dots\dots, p_n\}$ be a discrete probability distribution such that $p_i > 0 \; and \; \sum_{i=1}^{n} p_i = 1$, and let $U = \{u_1, u_2, \dots\dots, u_n\}$ be a set of utility values associated with each symbol i .Define the average useful codeword length as:

$$L(P,U) = \sum_{i=1}^{n} u_i p_i l_i.$$

Then the average useful codeword length *L(P,U)* satisfies the inequality

$$H_R^{\beta}(P,U) \leq L(P,U)$$

and

$$H_R^{\beta}(P,U) \leq L(P,U) < D^{\frac{1-R}{R}} H_R^{\beta}(P,U) + \frac{R}{R-1}(1 - D^{\frac{1-R}{R}}).$$

Where $H_R^\beta(P,U)$ is the generalized useful *R-Norm* useful information measure as defined above. Equality in the lower bound holds if and only if $l_i = -\log p_i \; for\ all\ i$, in the limiting case $R \to 1, \beta = 1$

Proof: By Holder's inequality

$$\left(\sum_{i=1}^n x_i^p\right)^{\frac{1}{p}}\left(\sum_{i=1}^n y_i^q\right)^{\frac{1}{q}} \le \sum_{i=1}^n x_i y_i \qquad (3.1.1)$$

$$\forall x_i, y_i > 0, i = 1,2,3,\dots,n \; and \; \frac{1}{p}+\frac{1}{q}=1, p < 1(\neq 0), q < 0 \; or \; q < 1(\neq 0), p < 0.$$

We see that if a positive constant c exists such that

$$x_i^p = c y_i^q \qquad (3.1.2)$$

then the equality holds.

Making the substitution

$$x_i = p_i^{\frac{R\beta}{R-1}}\left(\frac{u_i^\beta}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{R}{R-1}} D^{-l_i}, \quad p = \frac{R-1}{R}$$

$$y_i = p_i^{\frac{R+\beta-1}{1-R}}\left(\frac{u_i^\beta}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{1-R}} \; \& \; q = 1-R$$

Putting these values in (3.1.1), and using (2.3) we get,

$$\left(\frac{u_i^\beta p_i^\beta D^{-l_i\left(\frac{R-1}{R}\right)}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{R}{R-1}}\left(\frac{u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{1-R}} \le \left(\frac{u_i^\beta p_i^{\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right) \le 1$$

$$\Rightarrow \left(\frac{u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{1-R}} \le \left(\frac{u_i^\beta p_i^\beta D^{-l_i\left(\frac{R-1}{R}\right)}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{R}{1-R}}$$

$$\Rightarrow \left[\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{1-R}}\right]^{\frac{1-R}{R}} \le \left[\left(\frac{\sum_{i=1}^n u_i^\beta p_i^\beta D^{-l_i\left(\frac{R-1}{R}\right)}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{R}{1-R}}\right]^{\frac{1-R}{R}}$$

$$\Rightarrow 1-\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{R}} \le 1-\left(\frac{\sum_{i=1}^n u_i^\beta p_i^\beta D^{-l_i\left(\frac{R-1}{R}\right)}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)$$

$$\Rightarrow \frac{1-R}{R}\left[1-\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{R}}\right] \le \frac{1-R}{R}\left[1-\left(\frac{\sum_{i=1}^n u_i^\beta p_i^\beta D^{-l_i\left(\frac{R-1}{R}\right)}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)\right]$$

$$\Rightarrow H_R^\beta(P,U) \le L(P,U)$$

Hence proved

Now,

Let l_i be the positive integer then the inequality

$$-\log_D p_i^R + \log_D \frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta} \le l_i < -\log_D p_i^R + \log_D \frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta} + 1 \qquad (3.1.3)$$

Consider the interval

$$\delta_i = \left[-\log_D p_i^R + \log_D \frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}, -\log_D p_i^R + \log_D \frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta} + 1\right] \qquad (3.1.4)$$

In every δ_i, their exist one positive integer l_i such that

$$0 < -\log_D p_i^R + \log_D \frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta} \le l_i < -\log_D p_i^R + \log_D \frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta} + 1 \qquad (3.1.5)$$

From the left inequality of (3.1.5), we have,

$$-\log_D p_i^R + \log_D \frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta} \le l_i$$

$$\Rightarrow \log\left(\frac{p_i^R}{\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}}\right) \geq l_i$$

$$\Rightarrow \frac{p_i^R}{\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}} \geq D^{-l_i}$$

(3.1.6)

Multiplying both side by $u_i^\beta p_i^{\beta-1}$ and summing over $i = 1,2,\dots,n$ we get (2.3). So there exists a generalized useful code with length $l_i's$.

Let *0<R<1*, then equation (3.1.5) can be written as

$$p_i^{(R-1)}\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1-R}{R}} \leq D^{-l_i\frac{(R-1)}{R}} < p_i^{(R-1)}\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1-R}{R}} D^{\frac{1-R}{R}}$$

(3.1.7)

Multiplying (3.1.7) by $\frac{\sum_{i=1}^n u_i^\beta p_i^\beta}{\sum_{i=1}^n (u_i p_i)^\beta}$ and summing over $i = 1,2,\dots,n$. We obtain the result

$$\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{R}} \leq \left(\frac{\sum_{i=1}^n u_i^\beta p_i^\beta D^{-l_i\frac{(R-1)}{R}}}{\sum_{i=1}^n (u_i p_i)^\beta}\right) < D^{\frac{1-R}{R}}\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{R}}$$

$$\Rightarrow \frac{R}{R-1}\left[1-\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{R}}\right] \leq \frac{R}{R-1}\left[1-\left(\frac{\sum_{i=1}^n u_i^\beta p_i^\beta D^{-l_i\frac{(R-1)}{R}}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)\right]$$

$$< D^{\frac{1-R}{R}}\left\{\frac{R}{R-1}\left[1-\left(\frac{\sum_{i=1}^n u_i^\beta p_i^{R+\beta-1}}{\sum_{i=1}^n (u_i p_i)^\beta}\right)^{\frac{1}{R}}\right]\right\} + \frac{R}{R-1}\left(1-D^{\frac{1-R}{R}}\right)$$

$$\Rightarrow H_R^\beta(P,U) \leq L(P,U) < D^{\frac{1-R}{R}}\, H_R^\beta(P,U) + \frac{R}{R-1}\left(1-D^{\frac{1-R}{R}}\right).$$

Hence proved

APPLICATION

In many practical applications:

1) In medical diagnosis systems, symptoms (events) do not contribute equally to decision making. Assigning higher utility to more critical symptoms leads to a more informative coding strategy.

2) In communication systems with unequal costs (energy or delay) associated with transmitting different messages, utility functions help model this asymmetry.

3) In expert systems where some inferences are more reliable or important than others, the proposed entropy can optimize knowledge encoding.

Using the $H_R^\beta(P,U)$ measure allows us to:

i) Encode symbols in a manner that emphasizes utility-weighted relevance.

ii) Achieve coding bounds that generalize classical Shannon bounds.

iii) Apply fuzziness and parameterized control (via R,β) to adjust coding behavior under different system requirements.

CONCLUSION

In this paper, we introduced a new class of useful information measures called generalized useful R-norm information measures, defined via two parameters R and β , and a utility function U. These measures provide a flexible and powerful framework for quantifying information and uncertain environments.

We derived a noiseless coding theorem for the generalized measure , and obtained bounds on the useful mean codeword length.This result generalizes the classic Shannon noiseless coding theorem, as well as the coding theorems derived for useful entropy and useful R-norm information measures.

The measure provides both theoretical generality and practical utility. Future work may involve extending this framework to noisy channels, source coding with side information, and adaptive coding strategies in machine learning systems.

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